

**AN APPLICATION OF THE TRAMO-SEATS
AUTOMATIC PROCEDURE;
DIRECT VERSUS INDIRECT ADJUSTMENT**

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Tel: +34 91 338 5476. Fax: +34 91 338 5678. The data and software used in this paper can be
freely downloaded from [www.bde.es/Professionals/Econometrics Software](http://www.bde.es/Professionals/Econometrics_Software) .

Abstract

The ARIMA-model-based methodology of programs TRAMO and SEATS is applied for seasonal adjustment and trend-cycle estimation of the exports, imports, and balance of trade Japanese series. The programs are used in an automatic mode, and the results are analyzed. It is shown how the SEATS output can be of help when discriminating among competing models. Finally, the example is used to discuss the important problem of the choice between direct and indirect adjustment of an aggregate. It is concluded that, because aggregation has a strong effect on the spectral shape of the series, and because seasonal adjustment is a non-linear transformation of the original series, direct adjustment is preferable, even at the cost of destroying identities between the original series.

Keywords: Applied Time Series Analysis; Regression - ARIMA models; Seasonal Adjustment; Trend-cycle estimation; Direct and Indirect Adjustment.

1. INTRODUCTION

In the early eighties, the basis of an alternative methodology for seasonal adjustment of time series, the so-called “ARIMA-model-based” (AMB) approach, were set up [see, for example, Burman (1980) or Hillmer and Tiao (1982)]. In essence, the methodology consists of Minimum Mean Square Error (MMSE) estimation (or “signal extraction”) of unobserved components hidden in an observed time series, for which an ARIMA model has been identified (an important precedent is Nerlove, Grether and Carvalho, 1979). Typically, the components (or signals) are the seasonal, trend-cycle, and irregular components, the latter two comprising the seasonally adjusted (SA) series. The three components are assumed mutually orthogonal, and follow linear stochastic processes, with ARIMA-type expressions, usually non-stationary for the case of the trend-cycle and seasonal component. The models for the components aggregate into the ARIMA model identified for the observed series. (A general review is contained in Maravall, 1995.) Estimators of the components are computed via the so-called Wiener-Kolmogorov (WK) filter, as applied to nonstationary series (see Bell, 1984).

It is often the case that, before it can be modeled by an ARIMA process, the series needs prior treatment. Important preadjustments are outlier correction, the removal of calendar, intervention variable, and other possible regression effects, and interpolation of missing values; see, for example, Chang, Tiao, and Chen (1988), Box and Tiao (1975), Chen and Liu (1993), Hillmer, Bell, and Tiao (1983), Gómez and Maravall (1994), and Gómez, Maravall and Peña (1999). Awareness of the need for preadjustment has been growing steadily, and extends beyond model-based signal extraction methods [see, for example, Findley et al (1998)].

The AMB methodology had some appealing features. On the one hand, compliance with the ARIMA model of the observed series would seem to be a good protection against spurious results or model misspecification. On the other hand, the parametric model-based framework could facilitate analysis and inference [see, for example, Pierce (1979, 1980), Bell and Hillmer (1984), Hillmer (1985), Maravall (1987) and Maravall and Planas (1999)]. However, real-world application of the procedure proved elusive, in particular, for large-scale applications; it seemed to require the close attention of time-series analysts and considerable computing resources. This reflected the lack of a reliable and efficient automatic procedure. As a consequence, the AMB methodology remained latent for some years. The appearance of the programs TRAMO and SEATS [Gómez and Maravall (1996)] has changed the situation,

and the AMB methodology is presently being used by many agencies, institutions, and companies throughout the world.

The next section summarizes programs TRAMO and SEATS. Section 3 illustrates their automatic use with Japanese foreign trade series subject to a linear constraint. Section 4 compares direct versus indirect adjustment and two conclusions seem to emerge. First, the superiority of a direct adjustment, even at the cost of breaking balancing constraints. Second, that the better we do the adjustment of disaggregate series, the more likely it is that the discrepancies between direct and indirect adjustment are large.

2. BRIEF DESCRIPTION OF PROGRAMS TRAMO AND SEATS.

TRAMO (“Time series Regression with ARIMA noise, Missing values, and Outliers”) is a program for estimation and forecasting of regression models with errors that follow (in general) nonstationary ARIMA processes, when there may be missing observations in the series, as well as contamination by outliers and other special (deterministic) effects. An important case of the latter is the trading day (TD) effect, caused by the different distribution of week-days in different months. The TD effect can be modeled with one variable that classifies the days as working/non-working, or with 6 variables that capture separate effects for each day of the week.

If B denotes the lag operator, such that $Bx(t) = x(t-1)$, given the observations $y = [y(t_1), y(t_2), \dots, y(t_m)]$ where $0 < t_1 < \dots < t_m$, TRAMO fits the model

$$y(t) = \sum_{i=1}^{n_{out}} \omega_i \lambda_i(B) d_i(t) + \sum_{i=1}^{n_c} \alpha_i cal_i(t) + \sum_{i=1}^{n_{reg}} \beta_i reg_i(t) + x(t) \quad , \quad (2.1)$$

where $d_i(t)$ is a dummy variable that indicates the position of the i -th outlier, $\lambda_i(B)$ is a polynomial in B reflecting its dynamic pattern, cal_i denotes a calendar-type variable, reg_i a regression or intervention variable, and $x(t)$ is the ARIMA error. Regression variables can be entered by the user or built by the program. In particular, a wide class of Intervention Variables (Box and Tiao, 1975), that capture dynamic effects of known special events, is available. The parameter ω_i is the instant i -th outlier effect, α_i and β_i are the coefficients of the calendar and regression-intervention variables, respectively, and n_{out} , n_c and n_{reg} denote the total number of variables entering each summation term in (2.1).

In compact notation, (2.1) can be rewritten as

$$y(t) = z'(t) b + x(t) \quad , \quad (2.2)$$

where b is the vector with the ω, α and β coefficients, and $z'(t)$ denotes a matrix with the columns containing the variables

$$[\text{cal}_1(t), \dots, \text{cal}_{n_c}(t), \lambda_1(B) d_1(t), \dots, \lambda_{n_{\text{out}}}(B) d_{n_{\text{out}}}(t), \text{reg}_1(t), \dots, \text{reg}_{n_{\text{reg}}}(t)].$$

The first term of the r.h.s. of (2.2) represents the effects that should be removed in order to transform the observed series into a series that can be assumed to follow an ARIMA model; thus it contains the preadjustment component.

The ARIMA model for $x(t)$ can be written as

$$\phi(B) \delta(B) x(t) = \theta(B) a(t) \quad , \quad (2.3)$$

where $a(t)$ denotes the white-noise $(0, V_a)$ innovation. (For the rest of the paper, the term “white-noise $(0, V)$ ” will denote a Normally, identically, independently distributed variable, with zero mean and variance V .) The expressions $\phi(B)$, $\delta(B)$, and $\theta(B)$ are finite polynomials in B . The first one contains the stationary autoregressive (AR) roots, $\delta(B)$ contains the nonstationary AR roots, and $\theta(B)$ is an invertible moving average (MA) polynomial. Denoting by s the number of observations per year, in TRAMO-SEATS the polynomials assume the multiplicative form

$$\begin{aligned} \delta(B) &= \nabla^d \nabla_s^{d_s} \quad , \\ \phi(B) &= \left(1 + \phi_1 B + \dots + \phi_p B^p \right) \left(1 + \Phi_1 B^s \right) \quad , \\ \theta(B) &= \left(1 + \theta_1 B + \dots + \theta_q B^q \right) \left(1 + \Theta_1 B^s \right) \quad , \end{aligned}$$

where $\nabla = 1 - B$ and $\nabla_s = 1 - B^s$ are the regular and seasonal difference operators. This ARIMA model will also be referred to as the $(p, d, q) (p_s, d_s, q_s)_s$ model. The model consisting of (2.2) and (2.3) will be called a regression(reg)-ARIMA model.

When used automatically, TRAMO tests for whether the log transformation is needed and for the possible presence of calendar effects. It detects and corrects for three types of outliers namely, additive outliers (AO), transitory changes (TC), and level

shifts (LS)]. An AO outlier consists of an isolated spike, and hence $\lambda(B) = 1$. A LS outlier is a step function, with $\lambda(B) = 1/(1-B)$, and a TC outlier is a spike that gradually disappears, according to $\lambda(B) = 1/(1-0.7B)$. The program also identifies, and estimates by maximum likelihood, the reg-ARIMA model, interpolates missing values, and computes forecasts of the series. It yields estimates and forecasts of the preadjustment component $z'(t)$ and of the series $x(t)$ in (2.2), which is the series that can be assumed the output of a linear stochastic process. This “linearized” or “stochastic” series is equal, therefore, to the interpolated and preadjusted series, and it is the one that will be treated by SEATS.

Program SEATS (“Signal Extraction in ARIMA Time Series”) uses the AMB methodology to estimate unobserved components in series that follow ARIMA models. In SEATS, the unobserved components are the trend-cycle, $p(t)$, seasonal, $s(t)$, transitory, $c(t)$, and irregular, $u(t)$, components as in

$$x(t) = p(t) + c(t) + u(t) + s(t) = n(t) + s(t) \quad , \quad (2.4)$$

where $n(t)$ denotes the SA-series. Broadly speaking, the trend-cycle captures the spectral peak around the zero frequency, the seasonal component captures the spectral peaks around the seasonal frequencies, the irregular component picks up white-noise variation, and the transitory component captures highly transitory variation that differs from white noise. From the ARIMA model for the series, SEATS derives the models for the components, which typically display the following structure. For the trend-cycle and seasonal component,

$$\nabla^D p(t) = w_p(t) \quad , \quad D = d + d_s \quad , \quad (2.5)$$

$$S s(t) = w_s(t) \quad , \quad (2.6)$$

where $S = 1 + B + \dots + B^{s-1}$ denotes the annual aggregation operator, and $w_p(t)$ and $w_s(t)$ are stationary ARMA processes. A model of the type (2.5), with D unit AR roots $B = 1$, will also be denoted an “Integrated of order D ” –or $I(D)$ – model. The transitory component is also a stationary ARMA process, and the irregular component is white noise. The processes $w_p(t)$, $w_s(t)$, $c(t)$, and $u(t)$ are assumed to be mutually uncorrelated. Aggregation of the models for p , s , c , and u yields the ARIMA model (2.3) for the series $x(t)$. The model for the SA-series is obtained from the

aggregation of the models for $p(t)$, $c(t)$, and $u(t)$. Its basic structure is also of the type (2.5), with p replaced by n .

It is well-known (see, for example, Maravall, 1995) that, in (2.4), there are a variety of ways in which an additive white noise element can be assigned to the components of the decomposition. This implies a lack of identification amongst the component models. Identification is achieved in SEATS by imposing the “canonical condition” of Box, Hillmer, and Tiao (1978) and Pierce (1978), whereby all additive white noise that can be extracted from the components is assigned to the irregular component. In this way, the variance of the later is maximized, and the rest of the components are as stable as possible, given the stochastic features of the series. (For a discussion of the effect of this assumption, see Pollock, 2003.)

The component estimator and forecast are obtained by means of the WK filter as the MMSE estimators (under the normality assumption, equal to the conditional expectation) of the signal given the observed series. The WK filter is a two-sided, centered, symmetric, and convergent filter. Within the AMB framework, the filter can be given a simple analytical representation. Let the series $x(t)$ follow the ARIMA model

$$\phi(B) x(t) = \theta(B) a(t) , \quad a(t) \sim wn(0, V_a) , \quad (2.7)$$

where $\phi(B)$ also contains now the unit roots. Consider the decomposition of $x(t)$ into “signal plus non-signal” as in $x(t) = s(t) + n(t)$. Let the model for the signal be

$$\phi_s(B) s(t) = \theta_s(B) a_s(t) , \quad a_s(t) \sim wn(0, V_s) ,$$

where $\phi_s(B)$ will contain the roots of $\phi(B)$ associated with the component s . Denote by $\phi_n(B)$ the polynomial in B with the roots of $\phi(B)$ that are not in $\phi_s(B)$ (that is, the AR roots of the non-signal). Then, if $F = B^{-1}$ denotes the forward operator (such that $F x(t) = x(t+1)$), for a doubly infinite series, the WK filter to estimate the signal is given by

$$v_s(B, F) = \frac{V_s}{V_a} \frac{\theta_s(B) \phi_n(B)}{\theta(B)} \frac{\theta_s(F) \phi_n(F)}{\theta(F)} , \quad (2.8)$$

or, equivalently, by the ACF of the stationary ARMA model

$$\theta(B) z(t) = [\theta_s(B) \phi_n(B)] b(t) , \quad b(t) \sim wn(0, V_s / V_a) .$$

(See, for example, Gómez and Maravall, 2001.) The estimator of the signal is obtained through

$$\hat{s}(t) = v_s(B, F) x(t) . \quad (2.9)$$

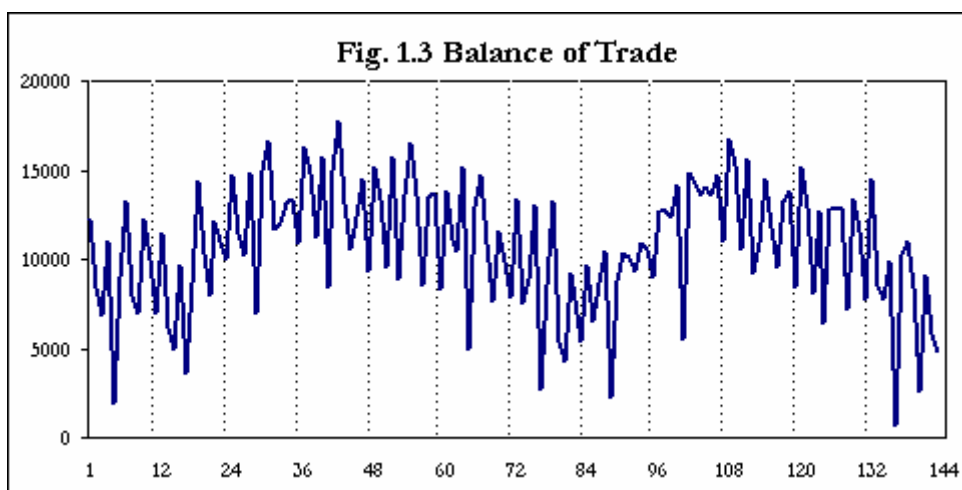
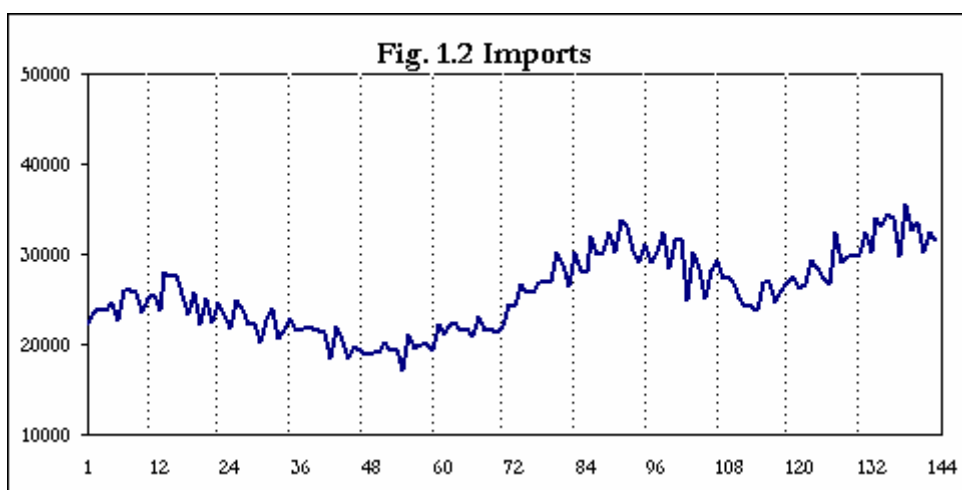
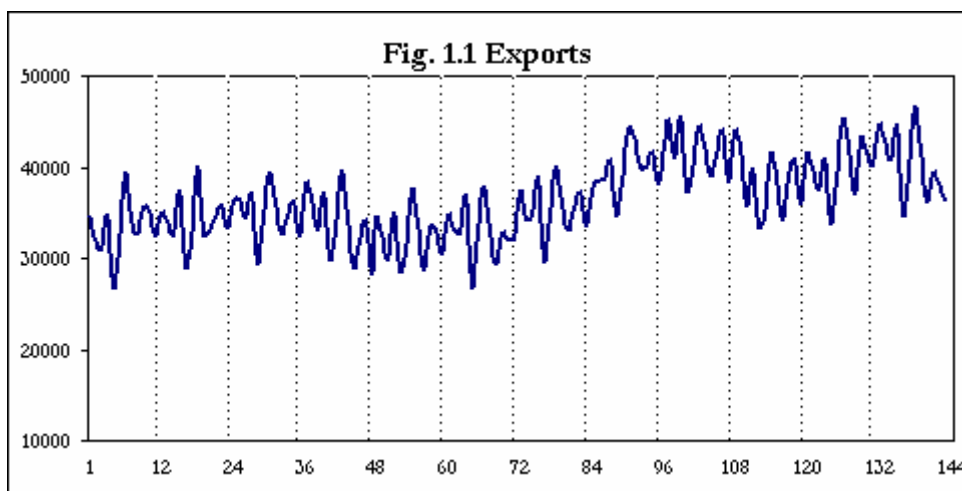
In practice, one deals with a finite series, say, $[x(1), x(2), \dots, x(T)]$. Given that the WK filter converges, for long-enough series, the estimator of the signal for the mid-years of the sample can be considered to be equal to the final estimator (that is, the one that would be obtained with the doubly infinite series). More generally, given the series $[x(1), \dots, x(T)]$, the MMSE estimators and forecasts of the components are obtained applying the two-sided WK filter to the series extended at both ends with forecasts and backcasts (Cleveland and Tiao, 1976). The Burman-Wilson algorithm (Burman, 1980) permits us to obtain the full effect of the doubly infinite filter with just a small number of forecasts and backcasts. The model-based framework is exploited by SEATS to provide Standard Errors (SE) of the estimators and forecasts (the SE are exact if the ARIMA model is correct). Being obtained by using forecasts, the component estimators at the end points of the series will be preliminary, and will suffer revisions as future data becomes available. The model-based framework is also exploited to analyze revisions (size, speed of convergence, etc.) and to provide further elements of interest to short-term monitoring.

When used together, TRAMO preadjusts the series, and SEATS decomposes the linearized series into its stochastic components. The complete final component is equal to the stochastic one estimated by SEATS, plus the deterministic effect associated with that component estimated by TRAMO (for example, an AO outlier will be added to the irregular component, a LS outlier will be added to the trend-cycle, EE will go to the seasonal component, and so on). TRAMO, SEATS, and program TSW, a Windows version that integrates both programs, are freely available at <http://www.bde.es>, together with additional facilities and documentation.

3. AN APPLICATION TO THE JAPANESE FOREIGN TRADE SERIES.

The Japanese exports, imports, and balance of trade monthly series are used to illustrate the automatic functioning of TRAMO-SEATS, as enforced in program TSW

(December 2004 version). The series span the period September 1989 – August 2001 (144 observations) and are displayed in figures 1.1, 1.2 and 1.3.



3.1 Automatic Procedure

The automatic procedure of TRAMO-SEATS requires the prior decision of whether or not to test for the presence of calendar effects, and if so, which TD specification should be used. The different options are controlled by the parameter RSA (see Caporello and Maravall, 2004). The most general case corresponds to the value $RSA = 5$, in which case, a test is performed for the log/level specification and the possible presence of Easter, leap year and trading day effects, using for the latter a 6-variable specification. Then, automatic model identification (AMI) and estimation is performed, jointly with automatic outlier detection and correction.

3.2 Exports Series (E)

The model obtained with $RSA = 5$ contains the three outliers in the first column of Table 1 plus the 6-variable TD specification. The linearized series $x(t)$ in (2.1) follows the ARIMA model

$$(1 + \hat{\phi}_1 B) \nabla \nabla_{12} x(t) = (1 + \hat{\theta}_{12} B^{12}) a(t) \quad , \quad (3.1)$$

with $\hat{\phi}_1 = .432$ ($t = 5.5$) and $\hat{\theta}_{12} = -.795$ ($t = -9.1$), where $a(t)$ is white noise ($0, 1048^2$). (On average, the series is forecast one-month-ahead with a standard error between 2 and 3 % of the mean level of the series for the last three years.) Summary diagnostics are presented in the first row of Table 2. The residuals can be comfortably accepted as zero-mean, uncorrelated, and Normally distributed; they do not contain residual seasonality, nor heteroscedasticity in variance, and their signs are randomly distributed. Figure 2.1 displays the residuals; Figure 2.2, the residual Autocorrelation Function (ACF), and Figure 2.3, the linearized series and the preadjustment component.

Table 1. Outliers (In parenthesis: t-values.)

Date	Exports	Imports	Balance of Trade
1/91	-----	LS (-3.7)	-----
4/91	-----	AO (-5.2)	-----
1/94	AO (3.5)	-----	-----
4/97	-----	-----	LS (3.5)
2/98	-----	LS (-4.0)	-----
11/98	LS (-3.9)	-----	-----
2/99	-----	-----	AO (-4.3)
12/00	-----	AO (3.2)	-----
5/01	LS (-3.4)	-----	-----

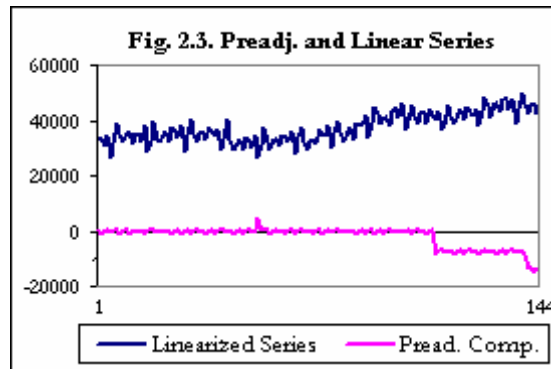
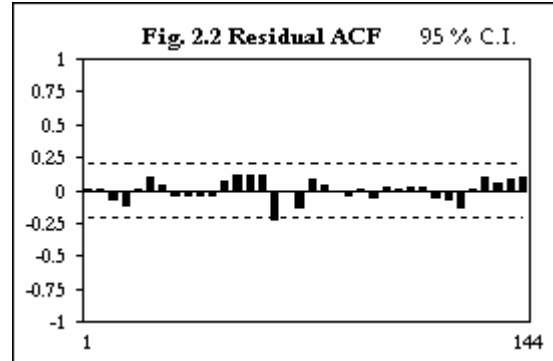
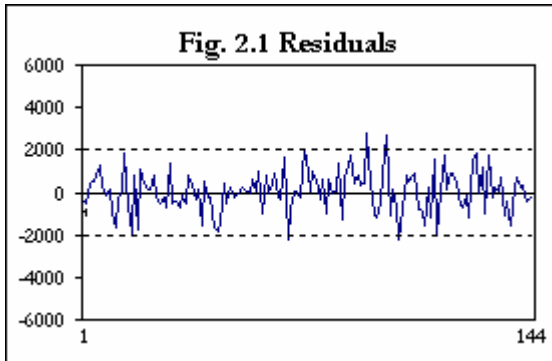
Table 2. ARIMA fit: Summary diagnostics

	$t(\mu_a)$	$Q_a(24)$	N_a	$t_a(\text{skew})$	$t_a(\text{kur})$	$Q_{as}(2)$	$Q_{a2}(24)$	$t_a(\text{runs})$
Exports	.5	22.8	.1	.1	-.3	1.0	24.3	-.6
Imports	-.0	29.3	.4	-.6	.0	4.7	19.2	-.2
Balance	-.8	21.9	.1	.0	.3	2.0	20.0	.4
Exports (alternative)	-.1	11.9	.9	-.3	-.9	1.9	20.2	.9
CV (95%)	$ t < 2$	$Q < 34$	$N < 6$	$ t < 2$	$ t < 2$	$Q < 6$	$Q < 37$	$ t < 2$

Notes to Table 2:

- 1) $t(\mu_a)$ is the t-value associated with H_0 : the mean of residuals = zero.
- 2) $Q_a(24)$ is the "portmanteau" Ljung-Box test for residual autocorrelation, computed with 24 autocorrelations, asymptotically distributed (a.d.) as $\chi^2(22 \text{ d.f.})$.
- 3) N_a is the Behra-Jarque test for Normality of the residuals, a.d. as $\chi^2(2 \text{ d.f.})$.
- 4) $t_a(\text{skew})$ is the t-value associated with H_0 : skewness (residuals) = 0.
- 5) $t_a(\text{kur})$ is the t-value associated with H_0 : kurtosis (residuals) = 3.
- 6) $Q_{as}(2)$ is the Pierce test for the presence of seasonality in the residual autocorrelation, (approximately) a.d. as $\chi^2(2 \text{ d.f.})$.
- 7) $Q_{a2}(24)$ is the McLeod and Li test on linearity of the process versus bilinear or ARCH-type structures, a.d. as $\chi^2(24 \text{ d.f.})$.

- 8) t_a (runs) is the t-value associated with H_0 : signs of the residuals are random.
 9) The approximate 95% critical value for each test is given in the last row.



SEATS decomposes the linearized exports series that follows model (3.1) into components, which also follow ARIMA-type models, namely,

$$\nabla^2 p(t) = (1 + .019B - .981B^2) a_p(t) = (1 - .981B)(1 + B) a_p(t) ;$$

$$S s(t) = (1 + 1.171B + 1.615B^2 + 1.542B^3 + 1.413B^4 + 1.202B^5 + .931B^6 + .629B^7 + .427B^8 + .239B^9 - .094B^{10} - .268B^{11}) a_s(t) ;$$

$$(1 + .432B) c(t) = (1 - B) a_c(t) ;$$

$$(1 + .432B) \nabla^2 n(t) = (1 - .982B - .001B^2 + .002B^3) a_n(t) ;$$

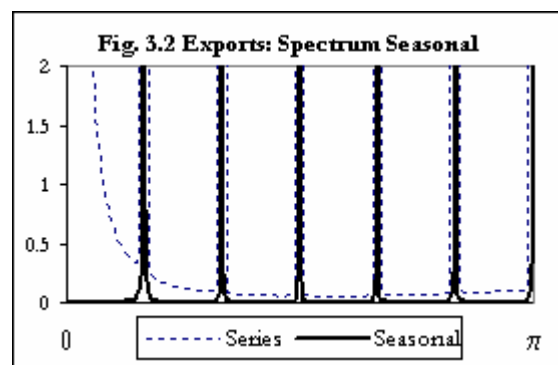
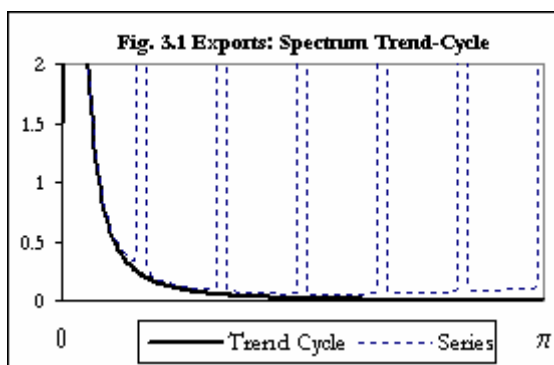
$$u(t) = w.n. ;$$

where $a_p(t)$, $a_s(t)$, $a_c(t)$, and $u(t)$ are mutually uncorrelated white noises with the variances given in Table 3.

Table 3. Standard Deviation of Component Innovation

	Trend-cycle	Seasonal Component
Exports	322	119
Imports	391	61
Balance	254	291
Exports (alternative)	336	146

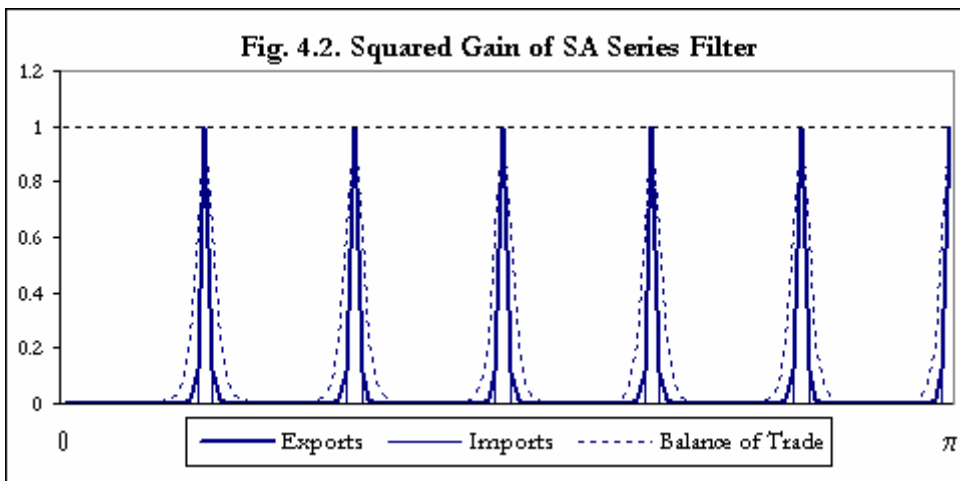
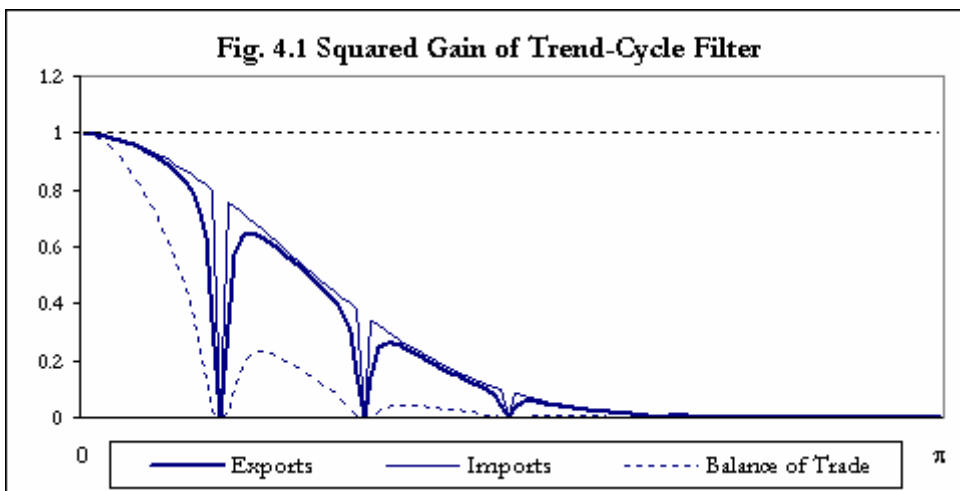
The trend-cycle follows thus an IMA (2,2) process, and factorization of the MA polynomial reveals the factor $(1+B)$, associated with a spectral zero at the π -radians frequency. This zero is implied by the already mentioned “canonical property”, used for identification. Figure 3.1 shows the monotonically decreasing trend-cycle spectrum. The seasonal component is a nonstationary ARMA (11,11) process, with the AR polynomial given by the annual aggregation operator ($S = 1+B+\dots+B^{11}$); its spectrum is given in Figure 3.2, and the spectral zero is located between the last two harmonics. The transitory component picks up the AR factor $(1+.323 B)$, which would otherwise contaminate the seasonal component with an undesirable short-term highly erratic variation, and follows a stationary ARMA (1,1) model, with the spectral zero for the zero-frequency. The irregular component is simply white noise. The distinction between a transitory and an irregular component is due to the fact that extracting a white-noise irregular element facilitates testing (see Maravall, 1987). Their behavior, however, is similar and, from a practical point of view, both components could be added. The resulting component also follows an ARMA (1,1) model.



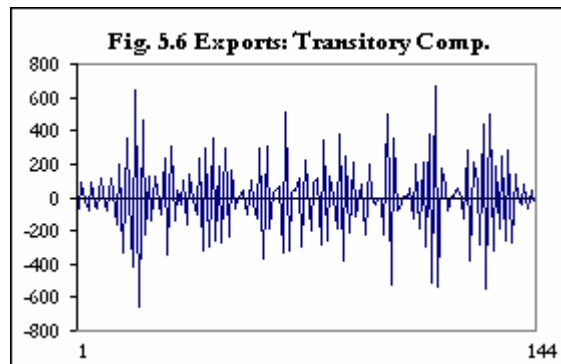
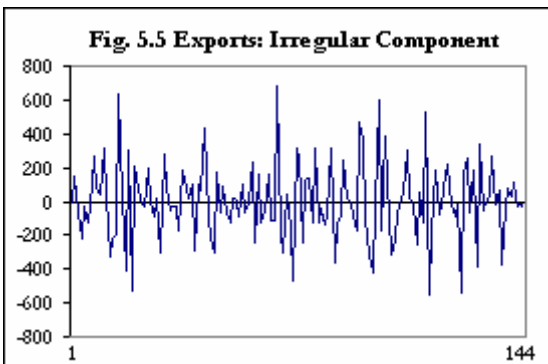
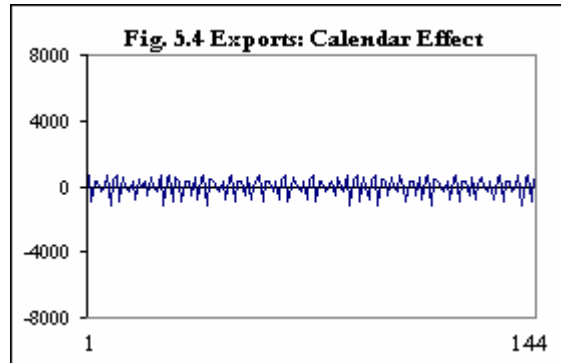
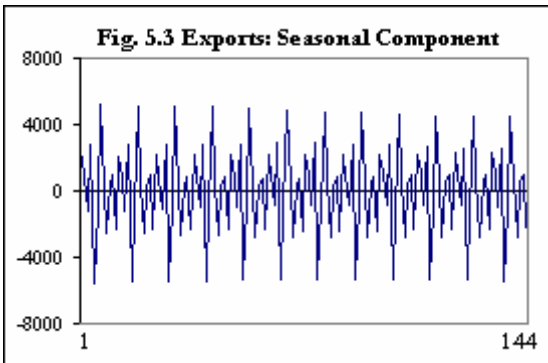
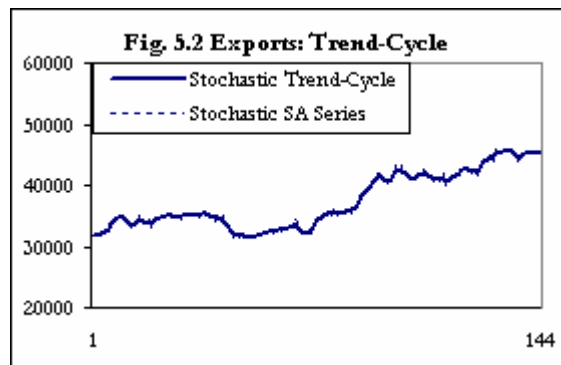
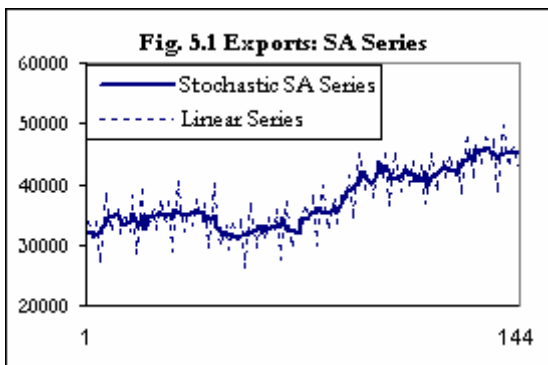
An important feature of a filter is its Squared Gain (SG). Denote by $g_z(\omega)$ the (pseudo) spectrum of the variable $z(t)$, with the frequency ω expressed in radians. From (2.9) it is obtained that

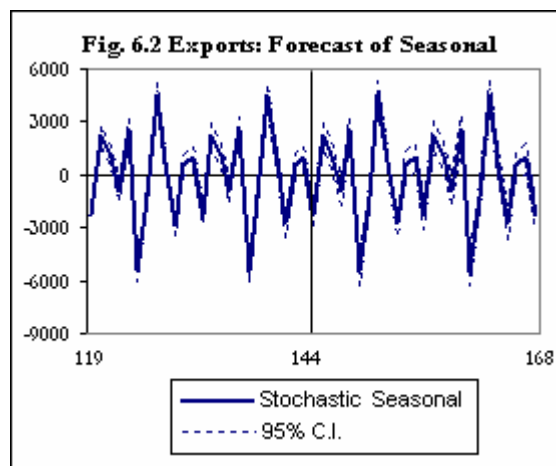
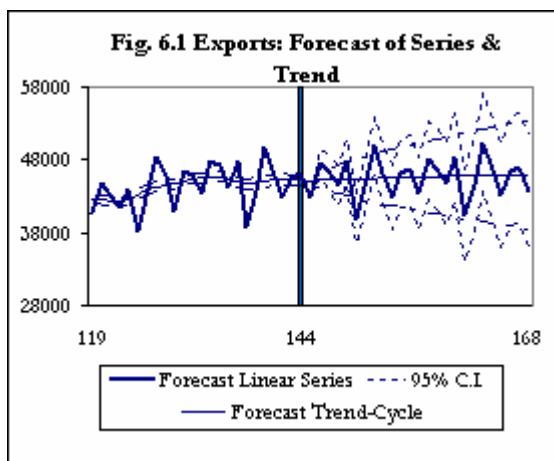
$$g_{\hat{s}}(\omega) = [\tilde{v}(\omega)]^2 g_x(\omega) = SG(\omega) g_x(\omega).$$

where $\tilde{v}(\omega)$ is the Fourier Transform of (2.8). Therefore, the squared gain determines, from the series spectrum, which frequencies will be used to estimate the signal. Figures 4.1 and 4.2 show the squared gains of the filters that estimate the trend-cycle and seasonal components. The latter only removes variation in a small neighborhood of the seasonal frequencies; the former also removes high frequency variation. The differences between the filters for the same component are due to the fact that the AMB approach tailors the filter to the ARIMA model for the observed series.



The estimators of the different components are given in figures 5.1 – 5.6. Figures 5.1 and 5.2 reveal the relative importance of the seasonal variation and the smaller contribution of the transitory/irregular component. (The trend-cycle still exhibits some short-term variation.) The seasonal component (Figure 5.3) is rather stable, and the irregular and transitory components (Figures 5.5 and 5.6) contain highly erratic and transitory noise. Figures 6.1 and 6.2 present the last two years of the original series, trend-cycle, and seasonal component, and the two-year-ahead forecast functions with the associated 95% probability intervals.





Some properties of the decomposition achieved are presented in the first row of tables 3 to 6. Table 3 shows that the seasonal component is relatively stable (with a small innovation variance), while the trend-cycle is subject to larger stochastic shocks. As mentioned before, the 2-sided filters imply revisions in the estimators, which are an important part of the estimation error in the preliminary estimator. Of particular interest is the error in the concurrent estimator (that is, the estimator of the signal for the most recent period). As seen in Table 4, the estimation error of the concurrent SA-series estimator is smaller than that of the trend-cycle, and the revision that the estimator will suffer is also smaller. On the other hand, Table 5 shows that the trend-cycle estimator will converge faster to the final one. Table 6 indicates that the series contains highly significant seasonality, which shows up not only for historical estimation, but also in preliminary estimation and forecasting.

Table 4. Estimation Standard Errors: Concurrent Estimator

	Total Estimation Error		Revision Error	
	Trend-cycle	SA series	Trend-cycle	SA series
Exports	486	367	352	260
Imports	470	83	312	51
Balance	609	561	464	399
Exports (alternative)	516	410	374	292

Table 5. Convergence of Estimators: Percentage Reduction in Revision Error Variance

	After 1 year of additional data		After 5 years of additional data	
	Trend-cycle	SA series	Trend-cycle	SA series
Exports	62	20	85	68
Imports	90	17	92	83
Balance	79	45	98	96
Exports (alternative)	60	25	88	77

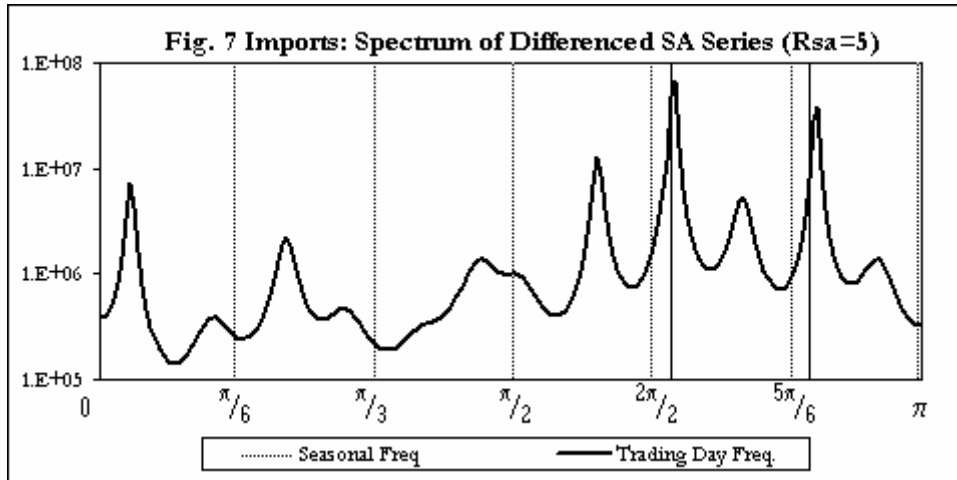
Table 6. Significance of Seasonality: Number of Months per Year with Significant Seasonality (95% level)

	Historical Estimator	Preliminary Estimator (last year)	Forecasts (next year)
Exports	12	11	10
Imports	10	9	9
Balance	9	9	8
Exports (alternative)	11	11	10

3.3 Import Series (I)

Starting with $RSA = 5$, the 6-variable TD specification is not significant. However, Figure 7 displays the spectrum of the differenced SA series without TD effect removed, computed as in Soukup and Findley (1999). The spectrum evidences an important TD effect in the SA series. Using the one-variable specification ($RSA=4$), a significant TD effect is detected. The model obtained, however, contains 6 outliers, a bit too many for 144 periods. Raising the threshold for outlier detection to $VA = 3.5$, the results are most satisfactory. Logs are selected and a model of the type (3.1), with $\hat{\phi}_1 = .380$ ($t=4.5$) and $\hat{\theta}_{12} = -.983$ ($t=-62$), is obtained. As seen in Table 2, the tests are comfortably passed. The preadjustment component consist of the 4 outliers

of Table 1 and the parsimonious TD effect (with $t = 11.1$). The SE of the 1-period-ahead forecast is in the order of 3.7% of the level of the series.



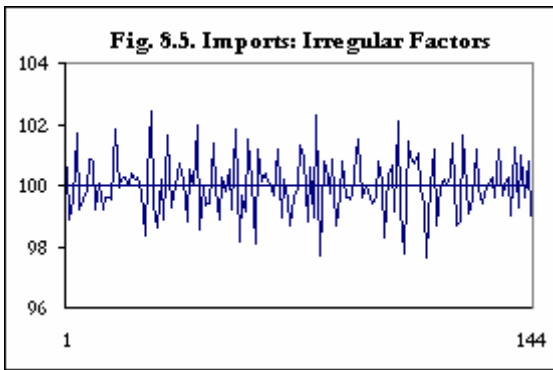
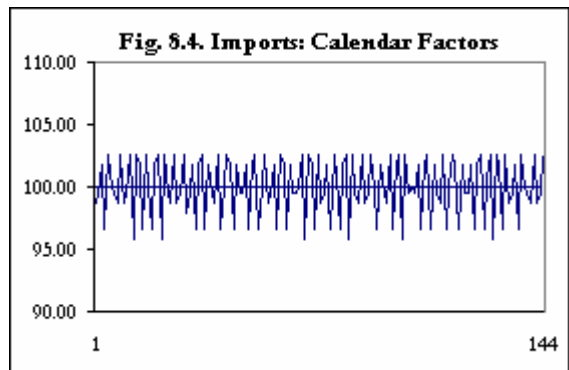
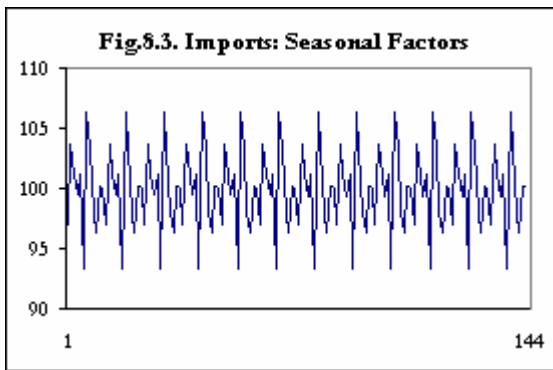
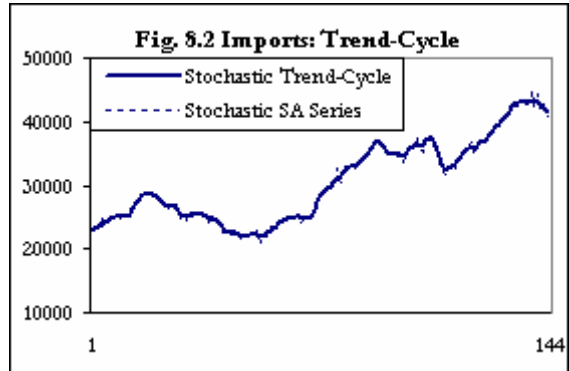
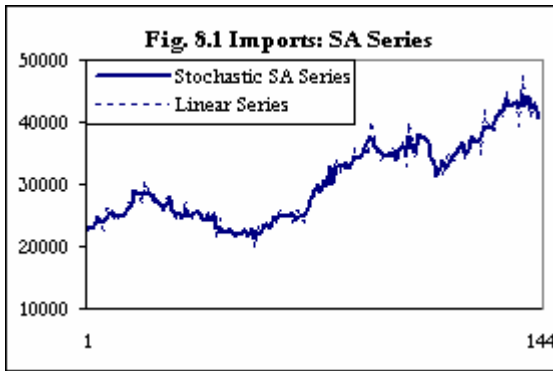
As with the exports series, SEATS decomposes the model into trend-cycle, seasonal, transitory, and irregular components. The model for the trend-cycle component is the IMA(2,2) model

$$\nabla^2 p(t) = (1 + .001B - .999B^2) a_p(t) = (1 - .999B)(1 + B) a_p(t) \quad ,$$

and $Ss(t)$ follows an MA(11) process; the transitory component is an ARMA(1,1) process, the irregular component is white noise, and the model for the SA series is given by

$$(1 + .380B) \nabla^2 n(t) = (1 - .991B) a_n(t) \quad ,$$

with the innovation variances of Table 3. The squared gains of the WK filters are shown in figures 4.1 and 4.2. Some characteristics of the decomposition (the SD of the components innovation, the SE of the estimation error, the size and convergence of revisions in the concurrent estimator, and the significance of seasonality) are given in the second row of Tables 3 to 6. Figures 8.1 to 8.6 present the stochastic decomposition of the linearized series. Seasonality is weaker for imports than for exports, although it is more stable.



3.4 Balance of Trade Series (BT)

Trying $RSA = 5$, directly yields results that are satisfactory. The model obtained contains the 2 outliers of Table 1, plus the 6-variable TD effect, as the preadjustment component. The ARIMA model is given by

$$\nabla \nabla_{12} x(t) = (1 + \theta_1 B) (1 + \theta_{12} B^{12}) a(t) , \quad (3.2)$$

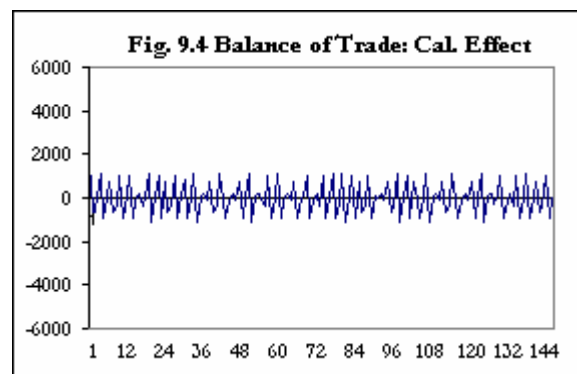
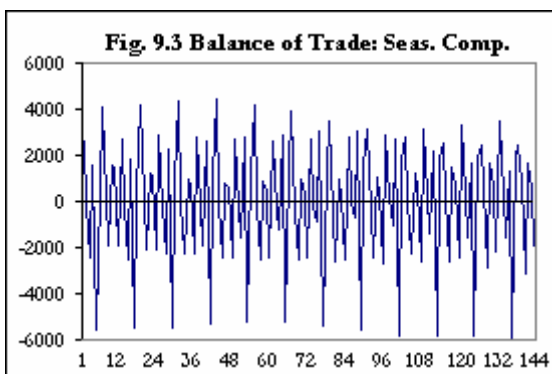
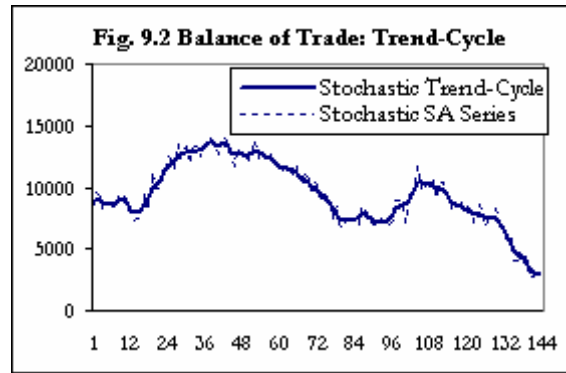
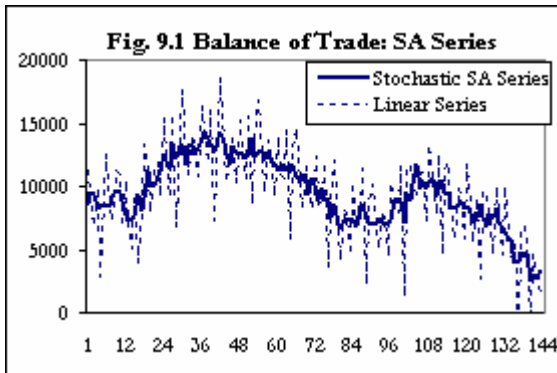
with $\theta_1 = -.452$ ($t = -5.4$), $\theta_{12} = -.533$ ($t = -5.5$), and residual variance equal to 1250^2 (which represents a SE of the 1-period-ahead forecast of about 12% of the mean level of the series for the last three years). The third row of Table1 contains the summary diagnostics of the fitting. The ARIMA model decomposition yields

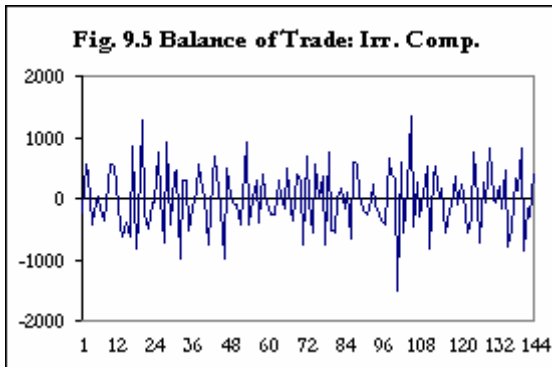
$$\nabla^2 p(t) = (1 + .051B - .949B^2) a_p(t) = (1 - .949B)(1 + B) a_p(t) ,$$

for the trend-cycle, an MA(11) process for $S_s(t)$, $u(t)$ white noise, and a SA-series that follows the IMA(2,2) model

$$\nabla^2 n(t) = (1 - 1.413B + .441B^2) a_n(t) = (1 - .464B)(1 - .949B) a_n(t) .$$

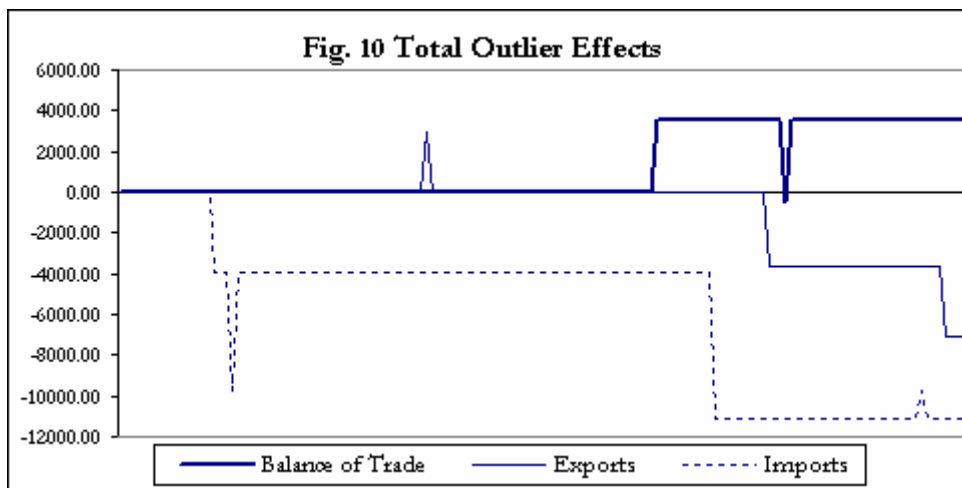
The squared gains of the WK filters are also given in figures 4.1 and 4.2, and some features of the decomposition are given in Tables 3 to 6 (third column). Figures 9.1 to 9.5 present the estimators of the stochastic components, and seasonality is seen to play a dominant role in the short-term movements of the series.





3.5 Some Remarks on the Models

Concerning preadjustment, the models for exports, imports and the balance of trade series contain 3, 4, and 2 outliers, that is, on average, one outlier every 4 years. The number of outliers is not excessive, and none of them is exceedingly large. The 9 outliers are displayed in Figure 10; it is noteworthy that none of the outliers is shared by two of the series. The calendar effect is highly significant in all three cases, although imports require a parsimonious specification. While the export and the balance of trade series are better modeled with an additive decomposition, for imports a multiplicative one is preferable.



As for the stochastic series, exports and imports both follow a model of the type (3.1), with similar coefficients, and they display stable seasonal components. Noticing that, for moderate values of ϕ , $(1 + \phi B)^{-1}$ is not far from $(1 - \phi B)$, model

(3.2) is not far from model (3.1). But it implies a seasonal component that is noticeably more unstable. For the three series, the trend-cycle follows an IMA(2,2) model, with the root $(1+B)$ present in the MA polynomial (implying a spectral zero for the highest frequency $\omega = \pi$), and the second root very close to $(1-B)$. Near cancellation of this last root with one of the differences shows that the model is close to a “canonical IMA(1,1) plus drift” model.

$$\nabla p(t) = (1+B) a_p(t) + \mu \quad .$$

The model for the SA exports and imports series is, very approximately, an ARIMA (1,2,1) model practically indistinguishable from the model

$$(1+\phi B) \nabla n(t) = a_n(t) + \mu \quad , \quad (3.3)$$

while for the balance of trade series, it is practically indistinguishable from

$$\nabla n(t) = (1+\theta B) a_n(t) + \mu \quad , \quad (3.4)$$

with ϕ and θ of opposite signs and moderate value.

For the number of observations considered and the short-term nature of the analysis, the I(2) models

$$(1+\phi B) \nabla^2 n(t) = (1-.99) a_{nt} \quad ,$$

$$\nabla^2 n(t) = (1+\theta B) (1-.99) a_{nt} \quad ,$$

will yield results close to the I(1) models (3.3) and (3.4), respectively. However, it does not follow that the I(1) formulations are preferable. Both contain the same number of parameters; the I(2) models lose the first observation, but, implicitly, they allow for a slowly adaptive mean in (3.3) and (3.4) (or, in other words, for a slowly adaptive slope in the trend of the series). Because of this added flexibility, the I(2) models may be preferable (the close to non-invertible MA root causes no numerical problem).

Comparing the standard deviation of the component innovations, Table 3 shows that the imports series contains the most stable seasonal component, while the balance of trade series presents the most unstable one. Table 4 shows that the

estimation and revision errors for the SA series and for the trend-cycle are largest for the balance of trade series although, as shown in Table 5, convergence of the preliminary estimator for the exports and imports series is relatively slow. Table 6 indicates that seasonality is highly significant for the three series.

It is often the case that identification of the ARIMA model does not yield a clear-cut unique solution. When the decomposition of the series is a relevant concern, comparison of the SEATS results may help in the selection. As an example, if one imposes leap year to the TD effect on the exports series (using $RSA = 3$, $ITRAD = 7$ instead of $RSA = 5$ as input) the results are slightly different, mainly as a consequence of the correlation between the calendar-based variables. Model (3.1) is obtained, with parameter estimates $\hat{\phi}_1 = .323$ and $\hat{\theta}_{12} = -.742$. Three outliers are also detected, although one of them is different. The TRAMO results pass all tests comfortably, and the Bayesian Information Criterion and residual SE for the first and second models are 14.23 and 14.24, and 1045 and 1040, respectively. From a fitting point of view, both models could be accepted. However, if we compare the results of SEATS, Tables 3, 4, and 5 show that, although the differences are small, the second model yields less stable trend-cycle and seasonal components, larger estimation errors, and larger revisions. On the other hand, it presents a slight increase in the speed of convergence of the revision in the concurrent estimator. Altogether, the SEATS results point towards the choice of the first model.

4. DIRECT VERSUS INDIRECT ADJUSTMENT.

Direct adjustment of the three series with TRAMO-SEATS run in a basically automatic mode yields sensible decompositions in the three cases. These are univariate decompositions and they ignore possible relationships amongst the original series. Yet, in this case, an important relationship has been ignored: by construction, the balance of trade series is the difference between exports and imports

$$BT(t) = E(t) - I(t) . \tag{4.1}$$

Thus another obvious way to obtain the SA series for BT is an indirect adjustment, whereby the SA imports series are subtracted from the SA exports series.

4.1 The Choice of a Procedure: General Remarks

Because of its relevance to many applications, the question of whether adjustment should be direct or indirect is an old one, and yet the issue has not been resolved. Geweke (1978) showed that, under a MMSE criterion, within the class of linear methods, and assuming the joint distribution of the components is known, indirect adjustment is preferable in theory. From an applied point of view, his result was of limited interest. First, the levels of aggregation can be very many, and it is typically the case that the first level of aggregation is made up of highly irregular and non-linear series. In fact, it has often been pointed out that more aggregated series tend to be more linear (an expected effect of the Central Limit Theorem). Moreover, Ghysels (1997) and Ghysels and Osborn (2001) relax the assumption of complete knowledge of the full distribution and show that Geweke's result may well not hold. Therefore aggregation should often precede adjustment.

Perhaps the most serious limitation of Geweke's analysis is the fact that seasonal adjustment, as enforced in the vast majority of cases, implies a non-linear procedure. Non-linearity shows up because of a variety of reasons. First, if the series (x_1, \dots, x_T) is adjusted at time T , with a standard two-sided filter, both ends of the adjusted series will contain preliminary estimators that are generated by different stochastic processes (Bell and Martin, 2004; Gómez and Maravall, 2001). Thus the adjusted series can be seen as generated by a time-varying-parameter ARIMA-type process. (This non-linear effect was detected by Granger, Ghysels and Siklos, 1996). More relevantly, most methods used at present (such as TRAMO-SEATS or X12ARIMA) imply many choices, informed by testing procedures of the 0-1 type (examples are the choice of a multiplicative or additive adjustment; the detection and correction of outliers; the possible presence of calendar effects). Thus, for test statistics that are close to the critical value, the addition of a new observation may imply a drastic model change. For example, trading-day variables may disappear from the model; and this may affect outliers, and so on. Moreover, an outlier that is clearly significant in one series may become not-significant when buried in the wider aggregate. Alternatively, observations for the same period that approach –but do not reach– the critical value for outlier detection in the disaggregate series may provide a significant outlier in the aggregate series.

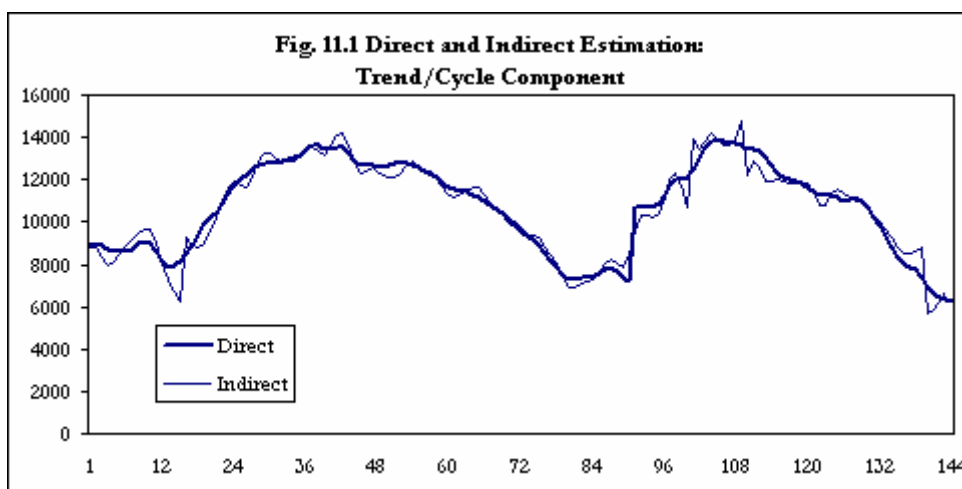
As a consequence, the dilemma of direct versus indirect adjustment has not been resolved, despite the fact that the two adjustments may differ substantially. The absence of a definitive solution has fostered a pragmatic approach among users: choose the solution that yields the SA series with the more desirable properties.

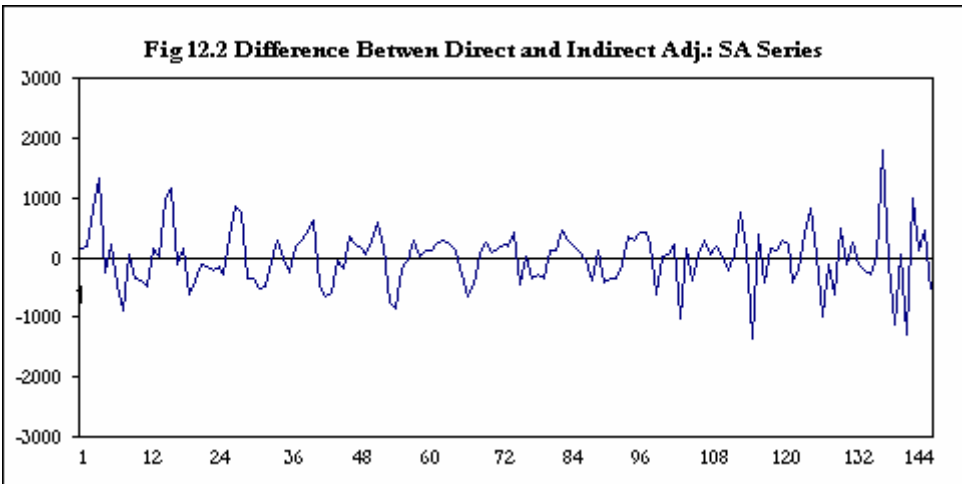
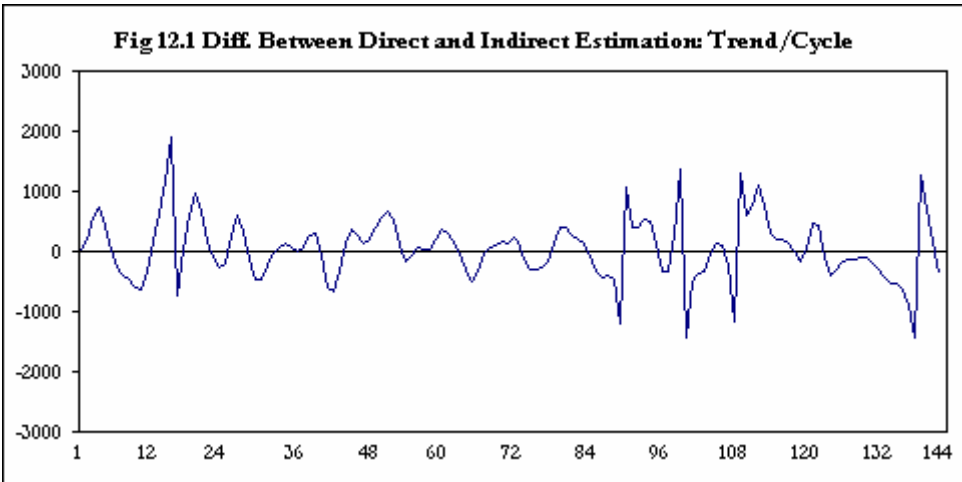
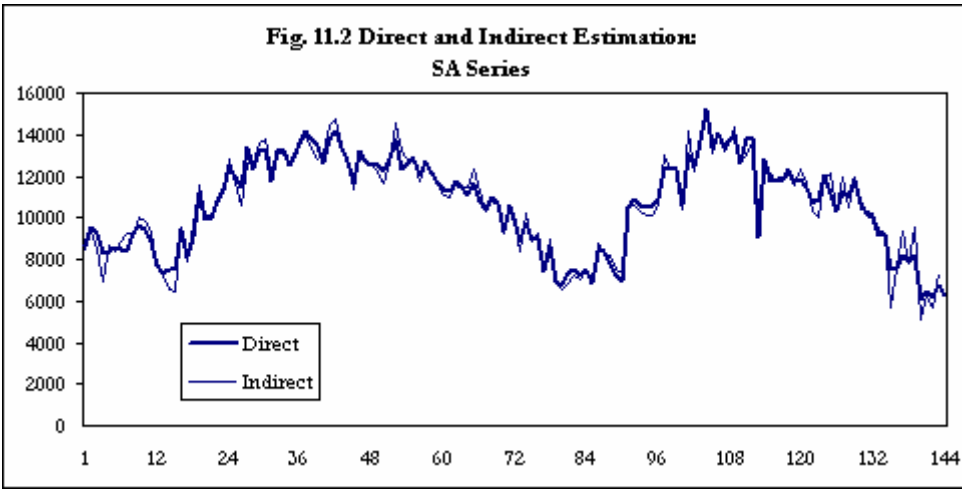
However, considerable ambiguity remains concerning what are the desirable properties of an adjustment. Hood and Findley (2001) suggest some important criteria, such as a check on residual seasonality and/or calendar effect, and –when none is found- a comparison of the stability of the seasonal component and of the size of the revisions in the adjusted series. Astolfi, Ladiray and Mazzi (2001) propose some additional criteria, such as smoothness of the trend, a comparison of the cyclical peaks and troughs, and a test for idempotency, namely, is the SA series unaffected by being seasonally adjusted?

The pragmatic approach has some drawbacks. Besides the lack of a general agreement on the desirable properties of an adjusted series, the direct and indirect approaches may both pass the most obvious checks (for example, lack of residual seasonality) and they may provide contradictory results in the others (for example, smaller revisions may characterize a procedure that fails the idempotency check). Moreover, the addition of a few observations may change the ranking and considerable instability may follow.

4.2 The Empirical Application

The SA series and the trend-cycle obtained with direct and indirect adjustment of the balance of trade series are displayed in figures 11.1 and 11.2. For both components, the difference between the two adjustments are given in Figure 12.1 and 12.2. The mean of the differences can be assumed zero and the two standard deviations are close, although the differences in the SA series are a bit smaller. The direct estimators are considerably smoother, most notably for the trend-cycle case.



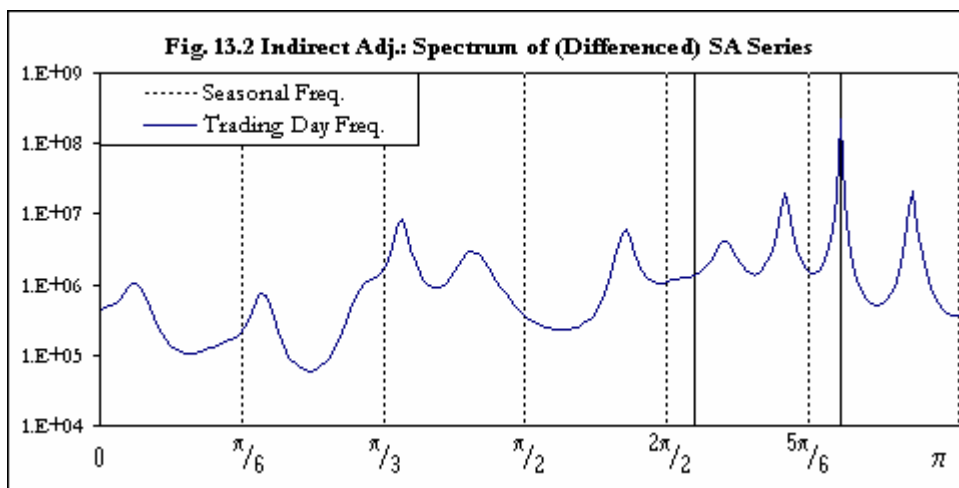
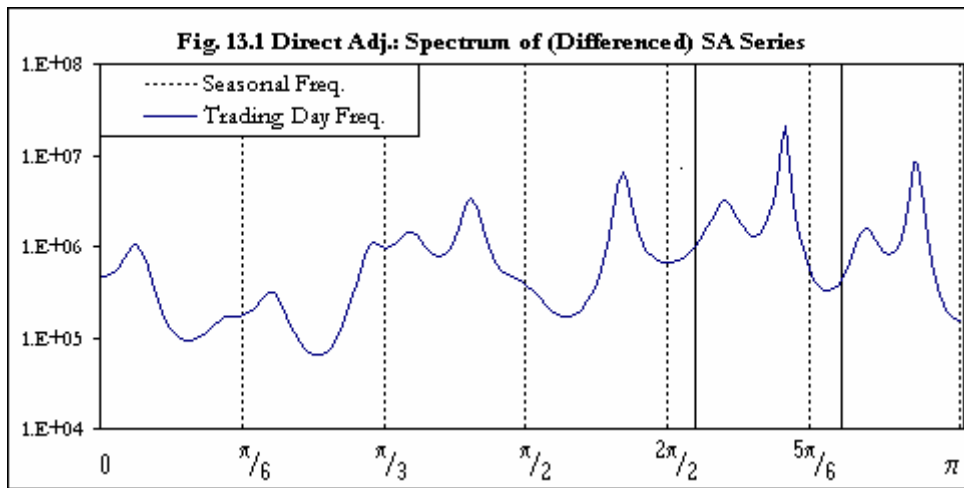


It is well-known that if exactly the same linear filter is applied to the disaggregate series, direct and indirect adjustment will yield the same result. Using, for example, as the fixed filter the one implied by the ARIMA model for the balance of trade series the relation (4.1) also holds for the adjusted series. (This is practically the same model as the one popularized by Box and Jenkins (1970) (the so-called Airline Model) and, as shown by Cleveland and Tiao (1976), a model for which X11 would also provide reasonable results). However, it is easy to see that residual seasonality and trading-day effects remain in the adjusted disaggregate and aggregate series. Furthermore, the residuals obtained by applying the model to the three series are highly correlated, revisions in the adjusted series increase and the idempotency property does not hold (i.e., adjustment of the SA series yields a new series).

The discrepancy between direct and indirect adjustment can be due to differences in the log/level transformation applied to the original series, in the preadjustment component that transforms the original series into the stochastic one, and in the filters applied to the stochastic series. In the example we consider, first, while the export and balance of trade series are modeled in levels, imports are modeled in logs. Second, preadjustment consists of two types of corrections: one, for trading day effect; the other, for outliers. Somewhat remarkably, although no outlier is shared by (at least) two of the series, as Figure 12.2 shows, these poor aggregation properties of the outliers have a moderate impact on the difference between the direct and indirect method. Third, the filters for the stochastic export and import series are rather similar and the main difference with respect to the filter applied to the balance of trade series is that the latter is aimed at capturing a more moving seasonality.

Comparing the results of the direct and indirect adjustment for the example considered, no residual seasonality is found in the SA series obtained with either of the two procedures: the autocorrelations for lags 12 and 24 of the stationary transformation of the SA series are, in both cases, small and negative, the spectrum shows no peaks around the seasonal frequencies (Figures 13.1 and 13.2), and the non-parametric Kendall and Ord (1990) test for the presence of seasonality has a p-value above .5 in the two cases. Moreover, running TRAMO-SEATS (with RSA=5) on the two SA series, seasonal adjustment leaves the two series unchanged, and hence the idempotency checks are passed. As for the cyclical peaks and troughs, Figure 11.1 shows directly that, for periods longer than 2 years, the cyclical signals are in phase and in agreement. On the other hand, Figure 13.2 evidences a peak for the higher trading-day frequency in the indirect adjustment case. Furthermore, I have

computed the revisions in the SA series for the central periods $t = 49, \dots, 84$ by proceeding in a “routine” manner. Thus, every January, the complete model was re-identified and, for the rest of the year, kept fixed (including prior outliers,) allowing for detection and correction for outliers in the incoming data. The means of the two series of 36 revisions can be assumed zero; the RMSE is 402.0 for the case of direct adjustment (extremely close to the theoretical value of 399.4 provided in the SEATS output) and 500.4 for indirect adjustment. Considering that direct adjustment also provides smoother SA and trend-cycle series, the pragmatic approach would point towards the choice of direct adjustment.



4.3 A Final Remark: The Concept of Seasonality

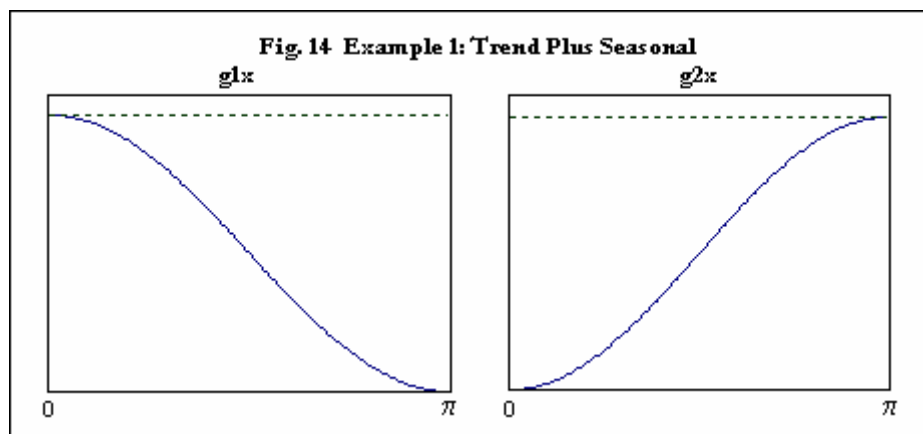
Improvements in seasonal adjustment methods have introduced strongly nonlinear elements to the procedure. Given that nonlinear transformations cannot be expected to respect linear constraints in the original data, direct and indirect adjustment may yield considerably different results. This cannot be interpreted as a shortcoming of the adjustment method, and it may even happen that the better the individual series are adjusted, the larger the difference becomes. Although a universally accepted definition of seasonality is still missing, ultimately it is a univariate concept: the only way we can judge at present adjustment of a set of series is by looking at them individually, to see if seasonality has been properly removed from each. Furthermore, because, for most macroeconomic series, full disaggregation is impossible, indirect adjustment will always start at some relatively high level of aggregation. Thus, preservation of constraints would seem a hopeless task, and direct adjustment would appear to be the appropriate procedure. This conclusion is reinforced with two examples that illustrate the conceptual problems that may appear if the SA series are computed with an indirect adjustment procedure. The examples are extremely simple, but they illustrate how interactions between disaggregate series can invalidate even the meaning of an indirect seasonal adjustment.

Example 1: Assume that we define seasonality simply as a peak around a seasonal frequency (and, accordingly, a trend as a peak around the zero frequency). Consider seasonal adjustment of the series $X(t)$, equal to the sum of the two series $x_1(t)$ and $x_2(t)$ observed every semester, that follow the models

$$\begin{aligned} x_1(t) &= a(t) + a(t-1) \quad , \\ x_2(t) &= b(t) - b(t-1) \quad , \end{aligned}$$

with $a(t)$ and $b(t)$ denoting two uncorrelated series of white-noise innovations, with variances $V_a = V_b = 1$. The spectra of the two series are equal to $g_1(\omega) = (1 + \cos \omega)/\pi$, and $g_2(\omega) = (1 - \cos \omega)/\pi$, for $-\pi \leq \omega \leq \pi$. As shown in Figure 14, the first spectrum presents a peak for $\omega = 0$, and decreases monotonically until it becomes zero for $\omega = \pi$. The second spectrum presents a peak for $\omega = \pi$ (the seasonal frequency) and, moving to the left, decreases monotonically until it becomes zero for $\omega = 0$. Thus, in this very simple model, x_1 and x_2 can be seen as a trend and a seasonal component, respectively. As a consequence, the SA x_1 series is $x_1(t)$ itself (there is no seasonality to remove), and the SA x_2 series is always zero,

given that $x_2(t)$ is a seasonal component. Therefore, the aggregate SA series obtained with indirect adjustment is equal to $x_1(t)$. However, given that $g_1(\omega) + g_2(\omega) = \text{constant}$, the aggregate series $X(t)$ is a white-noise series. As such, it contains no seasonality, and the SA $X(t)$ series should simply be the series itself ($= x_1(t) + x_2(t)$). As a consequence, indirect adjustment would not make sense and direct adjustment would be the proper answer.



Example 2: Assume that a country, divided in 50 regions, holds an important book fair every April, and that this fair has an effect on the total book sales for the country (although concentrated in the region where it is held). Furthermore, assume that the fair is hosted by a succession of regions (chosen in an alphabetical order, a random order, or in whatever manner). The SA regional series of book sales would not have the peaks due to the fair removed, because they are not seasonal (they would occur on average every 50 years). Therefore, the SA aggregate series that indirect adjustment yields would contain a relevant peak every April. Again, indirect adjustment produces nonsense and direct adjustment would yield the correct answer. This book fair example, which I first presented at a European Central Bank seminar, was further discussed by Compton (2002) and was one of the reasons that convinced the Bank of England (2003) to relax the imposition on the SA series of the additivity constraints among the original ones.

To conclude, the dilemma “direct versus indirect adjustment” seems to have a clear answer. At any level of aggregation, direct adjustment should be

used. If need be, a note could explain that, because aggregation modifies the dynamic structure of the series, and because seasonal adjustment is a non-linear transformation of the original series, aggregation constraints between the series cannot be expected to be preserved.

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