Aggregate Effects of Bureaucratic Startup Costs

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Abstract

Cross-country estimates find increasing government startup fees for creating a business amount to substantial decreases in output. According to World Bank data, over 190 countries performed more than 600 reforms to reduce these costs between 2002 and 2013. Nevertheless, output did not respond as expected in any of these cases. Why? I explore the theoretical channels through which these costs translate into aggregate productivity and output distortions. I then estimate the aggregate impact of bureaucratic startup costs for Spain—a developed economy in which startup fees amount to 17 percent of income per capita and the corresponding procedures take over 80 days to complete. I find the effects of startup costs on output to be sizable, but significantly lower than previously thought to be. In particular, reducing monetary costs to US levels increases output by 1.6 percent, whereas reducing time costs to US levels increases output by 3.7 percent. Additionally I find monetary and time costs have very different effects on entrepreneurial selection, the business productivity distribution, and the business size distribution—an issue previously overlooked in the literature.

1 Introduction

Bureaucratic startup costs are relatively large in many economies compared to US standards, even among OECD countries. Accordingly, a large effort has been

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put forth in the literature to measure them and study their effects. How do legal barriers to business creation affect entrepreneurship and the business size distribution? Through what channels do these distort aggregate productivity and output? Are these effects quantitatively significant? In this paper I try to shed some light on these questions and contrast some of the literature’s previous findings. Controlling for key institutional features—what businesses are affected by these costs and which ones are not—I find the aggregate effects of startup costs are more modest than previously estimated. By modeling different types of legal barriers—upfront fees versus the time spent complying with different procedures—I am also able to reconcile previous contradicting predictions regarding the selection of entrants after a reduction in bureaucratic hurdles. The latter is a result of a novel mechanism in my theory: increasing the time needed to comply with bureaucratic startup procedures poses a fixed cost of entry that increases with an agent’s ability to generate income—the opportunity cost of devoting time to bureaucratic tasks increases. Higher quality entrepreneurial projects are less likely to be pursued, compared to lower quality projects. Instead, imposing a fixed monetary fee on the creation of new firms precludes agents with less profitable projects from entering the market\(^1\). The average quality of projects pursued by entrepreneurs is thus positively affected.

I construct a model of agents choosing between working for a wage or pursuing an entrepreneurial project. Agents face monetary and time costs of entry into entrepreneurship, uninsurable entrepreneurial productivity, and financial constraints. There are two types of technologies in my model economy. A corporate sector, which employs capital and labor using a constant returns to scale technology to produce a consumption and investment good. Productivity in the corporate sector is constant over time. Conversely, agents have access to decreasing returns to scale technologies that use labor and capital. Entrepreneurs operate span of control technologies, and a project’s size is proportional to its productivity. More concretely, agents receive two idiosyncratic shocks at the beginning of each period. First, agents draw a realization of a business idea, which represents the average productivity of an entrepreneurial project. In addition to this, they receive a transitory productivity shock, which makes a project’s productivity each period fluctuate around the business idea. Upon observing both realizations, agents choose their occupation. They can become workers, hired by firms in the economy; they can choose to pursue the business idea realization drawn in that period; or they can continue operating whatever business idea they were operating in the previous period. Entrepreneurial projects face a stochastic life time. Once agents exit entrepreneurship exogenously, they become workers, albeit maintaining their asset holdings.

\(^1\)Hopenhayn (1992) and Clementi and Palazzo (2010) are close examples of industry dynamics models that illustrate this point.
Agents that operate a business idea invest their own assets in the project, plus any capital lent by the market. Lenders, in anticipation of enforcement issues, force entrepreneurs to collateralize debt with their depreciated capital. Financial constraints only depend on an agent’s asset position, not their current business idea nor their transitory productivity. Financial enforcement issues affect entrepreneurs by limiting the potential scale of their projects. An entrepreneur with either a good business idea, hence a large average productivity, or a large transitory productivity shock, maximizes profits at a large scale. A low asset position might nonetheless prevent said agent from obtaining enough funds to operate at the optimal scale. Financial constraints in this economy thus affect agents operating relatively productive projects. The corporate sector is unaffected by financial constraints in my economy. The next two paragraphs describe how financial constraints determine one of the main trade-offs in the model.

Any given business idea can be executed in one of two ways: through self-employment or by setting up a firm. Self-employed entrepreneurs face low bureaucratic startup costs, but are severely constrained in financial markets. Creating a firm, on the other hand, entails higher bureaucratic costs, but grants the project greater access to financial markets. Agents choosing to pursue an entrepreneurial project thus choose between facing higher entry costs or enduring financial constraints. Entrepreneurs face two types of startup costs the period they either take on a new business idea, or modify the type of business they were previously operating. On the one hand, they pay a fixed fee in terms of consumption goods. On the other hand, for a fixed fraction of that same initial period, they are not able to offer their labor in the market, nor produce goods.

Two mechanisms in my model make startup costs together with contract enforcement play a key role in determining aggregate outcomes. First, entrepreneurial selection is decisive in determining the overall productivity of the economy. If complying with startup procedures is costly, very productive individuals might not find it profitable to forgo their previous period’s income in order to upgrade to a better business idea—this effectively lowers business productivity in the economy. Second, bureaucratic startup costs—by distorting entrepreneurs decisions regarding the type of business to operate—affect the financial constraints entrepreneurs face when managing projects. The greater access agents have to funds, the more likely firms will operate at their optimal scales—thus lowering differences in marginal products between businesses.

In order to explore the quantitative implications of my theory, I jointly calibrate the most relevant parameters of my model to related moments I observe in the Spanish economy. I use the World Bank Doing Business Report² and information from the Spanish Ministry of Industry, Energy and Tourism³ to pin

²http://www.doingbusiness.org/
down startup costs. I use micro data on household finances and private firm management from the Spanish Survey of Household Finances (EFF)\textsuperscript{4} to pin down entrepreneurs’ financial constraints in the model, as well as their share of business projects’ income. The EFF also helps me determine the size of publicly traded firms—which are neither subject to the same financial constraints nor startup costs as privately held firms—relative to the set of privately held firms and the productivity distribution of the latter. I use data on the universe of Spanish firms from the official registry\textsuperscript{5} to reflect firm dynamics in my economy.

Three particular modeling choices described previously are meant to capture the crucial trade-offs faced when pursuing an entrepreneurial project for the case of Spain. I preclude my model from overstating the effects of bureaucratic startup costs by i) allowing for the pursuit of an entrepreneurial project through self-employment rather than obliging agents to establish a firm, and ii) modeling privately owned firms separately from the corporate sector. The startup costs measured by the World Bank Doing Business Survey legally do not apply to self-employed businesses in Spain. Furthermore, one can expect larger and publicly held firms to invest in reducing compliance times—whether legally or not. Finally, I model the trade-off between choosing self-employment and establishing a firm as a question of access to financial resources. Lack of access to external finance is consistently quoted as the main concern for entrepreneurs expanding their businesses in Spain\textsuperscript{6}. I also show that, in the EFF data, self-employed entrepreneurs on average have less access to credit and put up greater amounts of collateral than privately held firms.

I conduct the following quantitative experiment: initially, I reduce the monetary cost of establishing a firm to zero—the monetary cost self-employed agents bear when first registering. Next, I reduce only the time cost of establishing a firm to the level of a self-employed entrepreneur’s time cost. Finally, I lower both the monetary and the time cost for firms at the same time. Time costs have a greater impact on aggregates in my economy. Even though lowering both monetary and time costs separately increases output and total factor productivity in steady state, the reduction in time costs increases output by twice as much as reducing monetary costs. There are two channels through which I obtain this result. First, by lowering either cost of establishing a firm for all entrepreneurs, access to credit is effectively cheaper—because established firms face lower fi-

\textsuperscript{4}The Survey of Household Finances (Encuesta Financiera de las Familias) has a similar design to the SCF in the US. In particular, it includes detailed data about households assets, debts, and firms and oversamples wealthier households. http://www.bde.es/bde/en/areas/estadis/Otras_estadistic/Encuesta_Financi/

\textsuperscript{5}http://www.ine.es/jaxi/menu.do?type=pcaxis&path=%2Ft37%2Fp201&file=inebase&L=0

\textsuperscript{6}Two surveys emphasizing this point include Guemes and Coduras (2010) and Iniciador (2010).
nancial constraints than self-employed businesses. As more firms are created, entrepreneurs operate closer to their optimal scales on average. The reason the magnitudes differ is entrepreneurial selection. When monetary costs are lowered, most entering entrepreneurs operate low productivity projects. However, lower time costs incentivize higher productivity entrepreneurs to take on new, better business ideas. Lowering time costs thus has a greater effect on entrepreneurs’ average productivity, leading to the greater output boost.

Bureaucratic startup costs have inspired an array of papers attempting to understand their extent across economies and their effects on them\textsuperscript{7}. De Soto (1989) first attempted to record all procedures needed to follow in order to start a formal business in Peru by actually venturing through the entire process. Following the success of this approach, Djankov, La Porta, Lopez-de Silanes, and Shleifer (2002) introduced a meticulous methodology to measure and compare entry regulations for over eighty countries. The World Bank Doing Business report enriched the approach and adapted it to cover most countries on a yearly basis. Measuring and comparing startup costs across different countries prompted an extensive literature measuring the effects said costs had on outcomes such as productivity, entrepreneurship and growth.

Among the earliest contributions using cross-country data, three are closest to the present paper. Using a general equilibrium model of industry dynamics à la Hopenhayn (1992), Barseghyan and DiCecio (2010) find a 1 percent increase in startup costs—which they construct by lumping time and monetary costs measured by Doing Business—carries along a 0.14 percent decrease in TFP and a 0.21 percent decrease in output per worker. Taking only direct legal costs into account, Barseghyan (2008) finds increasing these by 80 percent of income per capita decreases total factor productivity by 22 and output per worker by 29 percent. Both these papers’ measured aggregate effects surpass the ones I find by an order of magnitude. Finally, Klapper, Laeven, and Rajan (2006) measure the effect of bureaucratic entry costs on overall creation, size, and growth of firms. They use the Djankov et al. (2002) data and firm level data for all industries across various countries. They find that bureaucratic entry costs deter smaller firms from entering an industry. This means entrants are on average bigger, grow more slowly and are less productive in countries with higher entry costs. Crucially, they also lump the monetary and time cost into one—and thus perceive no differential effect of entry costs for different entrepreneurial productivity levels. This finding contrasts with the two papers I describe next.

The relatively low cost entailed by reducing bureaucratic costs and the potentially large gains subsequently prompted an increasing number of institutions and countries to prescribe and pursue these policies. Various of these reforms

\textsuperscript{7}Attempting to write a comprehensive survey is beyond the scope of the paper. Djankov (2009) provides an excellent overview of the literature.
led to specific country studies, mostly measuring their immediate impact on entrepreneurship and productivity. In particular, Seira, Kaplan, and Piedra (2007) and Bruhn (2011) take advantage of the SARE reform in Mexico: the number of procedures, i.e., the time cost of registering a firm, was reduced for a variety of industries on a town level. Seira et al. (2007) find that entry into entrepreneurship increased with the lower costs. Simultaneously, average entrant size increased after the reform. This is in stark contrast to the findings in the cross-country analysis literature, in particular to Klapper et al. (2006). One possible explanation could have been that a greater number of already existing businesses suddenly found formalizing profitable. However Bruhn (2011), exploiting the same policy reform, finds that the entrepreneurs registering mostly were wage earners, not previous informal business owners. This apparent contradiction supports the mechanisms underlying my theory. While a purely monetary fee deters lower productivity individuals from entering entrepreneurship, high time costs have a negative selection effect on entrepreneurs in Mexico. Although the effects these papers find are significant, due to the short time series available, the conclusions do not inform us about the long run. I address that question in this by conducting a steady state analysis within a detailed micro-founded economy.

The structure of my model is closest to the literature on the effects of financial constraints on entrepreneurial selection, wealth accumulation, productivity and output. Some important contributions include among others Quadrini (2000), Cagetti and De Nardi (2006), Buera (2009), and Buera, Kaboski, and Shin (2009). These models provide a natural environment in which to study how the interaction of startup costs with financial constraints affects all outcomes mentioned above. In addition to the model structure itself, the empirical and qualitative analyses complement my own. Agents in my economy face similar frictions and trade-offs to the aforementioned models, with the addition of startup costs and certain other choices due to institutional differences. The literature on financial constraints, wealth accumulation and occupational choice thus provides a benchmark to understand the underlying mechanisms in my theory and how to measure them in the data. Finally, this literature also provides guidance on my quantitative exercise. Many of the calibration strategies employed are valid for my investigation, even though I attempt to improve upon these by taking advantage of micro data and the specific institutional framework in Spain.

The paper is structured as follows. Section 2 presents my data on bureaucratic startup costs, household finances, and privately held firms. I use the data to describe the relevant dimensions of the Spanish economy. In addition, I offer some comparisons to the US, using the Survey of Consumer Finances (SCF). Section 3 describes the model economy. Section 4 describes the calibration process and results. Section 5 concludes.
2 Data

In this section I offer a description of the data I use to document the relation between bureaucratic startup costs, entrepreneurial selection, firm size, and productivity. I also offer a comparison with the US case.

Bureaucratic Startup Costs

The methodology used by the World Bank Doing Business report follows\(^8\) that of Djankov et al. (2002). They study official sources and consult with local legal specialists and government officials to list all procedures (in particular, their number, the time it takes to complete them and the monetary cost associated to them) required when registering a business. For the case of Spain, the costs reported coincide with the costs of establishing a Limited Company (S.L.)\(^9\). Entrepreneurs are assumed to have full information on all procedures and to utilize the services of external agents where necessary to complete them. Importantly, these procedures do not include sector specific requirements nor the necessary steps involved in making the start of operations viable (e.g. contracting utilities, acquiring information on the startup process, etc.). Time is recorded in calendar days and assumes no party involved in the procedures produces unnecessary delays.

One important issue is that of interpreting the data in my economy. I try to introduce the costs above directly into the model, interpreting those numbers as the parameters of the model. I combine the number of procedures and the time each takes to be finalized to compute the total time the entrepreneur spends completing administrative requirements. I introduce this number into the model as the portion of a model's period (assumed to represent one year) the entrepreneur invests in completing legal requirements, \(\xi\). I assume entrepreneurs starting up a business neither work nor produce during the time they are completing administrative tasks. I can introduce the monetary bureaucracy cost directly into the model, given it is measured proportional to income per capita. I introduce it as a one-time fixed cost entrepreneurs pay when starting a new business, \(\Pi_l\). I do not

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\(^8\)Djankov et al. (2002) are very precise about measuring the time it takes to open a business—they account for the order in which the procedures are to be done and have a measure for how long each procedure takes. Up until 2013, the world bank data instead report the number of procedures that have to be completed and the average time each procedure takes to be completed.

\(^9\)In September of 2013 the Spanish government introduced several reforms concerning the registration of firms. Registering certain types of Limited Companies over the internet was made possible under http://portal.circe.es/es-ES/empreendedor/CrearEmpresa/Paginas/CrearEmpresaTiposdeempresa.aspx. I use this resource to check that the costs reported by Doing Business coincide with the costs of registering a Limited Company (S.L.) before the reform.
consider including a paid-in minimum capital requirement, given it would operate
very similarly to the outlays $\Pi_i$.

In contrast to much of the industrial organization or international trade liter-
ature related to entry costs, I am interested in making a relatively clean policy
recommendation. The administrative costs of entry into entrepreneurship (as op-
posed to e.g. market structure- or corruption-related entry costs) are relatively
effort- and costlessly modifiable by authorities. In countries in which these costs
might be very high, the benefit from lowering the costs in terms of entrepreneurial
selection, productivity and GDP might be big. On top of that, I am interested
in exploring how the policy might affect the distribution of consumption in the
economy. These data seem to capture administrative entry costs, directly set by
authorities and isolate them from other entry costs. Therefore, they could serve
to measure the benefits from improving bureaucratic costs, controlling for other
many entry deterrents.

Much of the previous literature either account for the monetary cost alone,
or lump the monetary cost and an imputed time cost (which is equal to the
average wage income forgone in the time it takes to fulfill the legal requirements)
together. My approach is different in that I separate both costs. This is an
important distinction given that in Spain the monetary cost of completing the
paperwork to start a business is comparable to other countries, whereas the time
it takes to fulfill these requirements is disproportionately high. On top of that,
although the monetary costs are equal for all entrepreneurs, the monetary value of
the time costs will vary with how productive agents are. Legal compliance times
will therefore have an effect on entrepreneurial selection that legal fees might not.

Some caveats to using this data stem from the fear that the numbers might
not represent all the relevant time costs I might be interested in capturing. They
exclude the time it takes entrepreneurs to inform themselves of all procedures. If
the laws are very complex, I might be very interested in capturing those effects
and extra time lags. Also, the costs are measured for a very general type of firm. If
costs vary substantially for different firm sizes, industries, or geographic location,
my results might be affected. In addition to this, there might be measurement
error. The authors gather their information by studying laws and consulting with
authorities and private law firms about a hypothetical case. Their estimates do
not reflect having gone through an actual process of setting up a firm, but rather
arbitrary estimates by local experts\textsuperscript{10}.

\textsuperscript{10}Nevertheless, there is plenty of anecdotal evidence that point in the same direction..
<table>
<thead>
<tr>
<th>Procedures</th>
<th>Total days</th>
<th>Monetary Cost (% Income p.c.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>2</td>
<td>2.25</td>
</tr>
<tr>
<td>Canada</td>
<td>2</td>
<td>1.45</td>
</tr>
<tr>
<td>U.S.</td>
<td>4</td>
<td>0.49</td>
</tr>
<tr>
<td>Japan</td>
<td>8</td>
<td>7.5</td>
</tr>
<tr>
<td>France</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>U.K.</td>
<td>6</td>
<td>0.8</td>
</tr>
<tr>
<td>Germany</td>
<td>10</td>
<td>15.69</td>
</tr>
<tr>
<td>Italy</td>
<td>16</td>
<td>20.02</td>
</tr>
<tr>
<td>Spain</td>
<td>11</td>
<td>17.3</td>
</tr>
</tbody>
</table>

Table 1: Time and monetary bureaucratic costs of starting a business—Djankov et al. (2002)

As is clear from table 1, bureaucratic startup costs in Spain are among the highest of all OECD countries. Time costs in Spain grow proportionately more than monetary costs, as compared to some of the economies with lower costs. Interestingly, this is true for other countries with high costs, such as Italy and Germany. Next, I compare entrepreneurial selection and firm characteristics in several European countries, and in particular in Spain, and the US.

**Private Firm and Household Finance Micro Data**

Various European countries, such as Spain, Italy, or Germany, share relevant characteristics apart from significant startup costs. Compared to the US, the proportion of self-employed establishments is somewhat greater and the firm size distribution is significantly more skewed to the left. Philippon and Nicolas (2008) show the decline in the proportion of European firms that make it into the FT Global 500 as time goes by—Europe does not seem to be able to produce extraordinarily large, productive firms. Entrepreneurial selection is very different. The proportion of total net worth in the economy held by entrepreneurs is significantly higher in the US, which is indicative of entrepreneurs’ productivity over the longer run. Entrepreneurship rates in contrast—i.e. the proportion of the working age population that owns and operates a business—are at par among European countries and the US. In spite of earlier literature concentrating on bureaucratic startup costs’ effects on development, these facts reveal that Spain, a developed economy, constitutes a good case to study their aggregate effects.

I use the Spanish Survey of Household Finances (EFF\textsuperscript{11} henceforth) to study privately held firms and household finances in Spain. The EFF is conducted by

\textsuperscript{11}Encuesta Financiera de las Familias, http://www.bde.es/bde/en/areas/estadis/Otras_estadistic/Encuesta_Financier/

Bover (2011) details the data collection, design, and contents of the 2008 wave, which I use.
the Bank of Spain (BdE) every three years. It collects information on demographics, income, asset holdings, debt, occupation, and consumption for Spanish households. Its statistical design resembles that of the Survey of Consumer Finances (SCF) conducted in the US. The unit of observation is a household, within a particular calendar year. Crucially, wealthy households are oversampled. This permits the detailed study of household wealth distribution and composition, as well as entrepreneurship. Table 5 shows a clear positive correlation between net worth and entrepreneurship. Furthermore, table 3 states that only a minority of households report owning and managing a firm. The oversampling in the EFF sample thus proves key in identifying the relevant characteristics of entrepreneur households.

Within any given observation in my data, I define a privately held business as a business owned and managed by at least one member of that particular household. Given the unit of observation is a household, I do not observe the complete set of owners of any particular business. To avoid double counting, I weigh each observation of a privately held business by its percentage owned by the corresponding household. I consider only three legal types of businesses: self-employed, limited companies (S.L.), and share companies (S.A.). As figure 6 shows, these three types comprise over 90 percent of Spanish firms. Table 2 compares the number of businesses under each legal category, both in the universe of businesses and in my sample.

<table>
<thead>
<tr>
<th></th>
<th>Self-Employed</th>
<th>S.L.</th>
<th>S.A.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>1,793,897</td>
<td>1,140,820</td>
<td>109,330</td>
</tr>
<tr>
<td>Privately Held</td>
<td>1,665,926</td>
<td>498,301</td>
<td>78,645</td>
</tr>
</tbody>
</table>

Table 2: Main legal forms of privately held firms in Spain. DIRCE and EFF (2008)

The difference between rows in table 2 could be due to two reasons. First, it could represent the number of public businesses. Second, it could partly denote measurement error, as the EFF is designed to represent the wealth distribution of households accurately, not privately owned business holdings. Self-employed businesses, in fact, are by definition privately owned. In order to check the accuracy of the business characteristics distribution in the EFF, I compare the size distribution of self-employed establishments in the EFF to the universe of self-employed establishments from the Spanish firm registry, DIRCE. Figure 2 shows what percentage of all self-employed businesses correspond to each bin of the employee

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12The Spanish nomenclature for the different types of firms is: Autónomos (self-employed), Sociedad Limitada (S.L., for limited companies) and Sociedad Anónima (S.A., for share companies).
13http://www.ine.es/inebmenu/mnu_empresas.htm
size distribution of self-employed businesses, both in the registry of businesses, DIRCE, and the EFF. Even though the total number of self-employed businesses in table 2 doesn’t coincide, the relative size distributions are reasonably similar\textsuperscript{14}.

<table>
<thead>
<tr>
<th></th>
<th>Spain</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Business owners or self-employed</td>
<td>16.7</td>
<td>17.6</td>
</tr>
<tr>
<td>All business owners</td>
<td>15</td>
<td>13.2</td>
</tr>
<tr>
<td>Active business owners</td>
<td>14.2</td>
<td>12.4</td>
</tr>
<tr>
<td>Self-Employed</td>
<td>7.9</td>
<td>7.6</td>
</tr>
<tr>
<td>Self-Employed Business Owners</td>
<td>7.2</td>
<td>5.4</td>
</tr>
</tbody>
</table>

Table 3: Percentage of Entrepreneurs in Population, EFF and SCF respectively

Entrepreneurship levels are similar in Spain and the US. Entrepreneurial selection differs: table 3 shows similar levels of entrepreneurship for different definitions of entrepreneurship\textsuperscript{15} for both economies. Nevertheless, tables 4 and 5 show that entrepreneurs in the US hold a greater portion of the economy’s net worth on average. The greater an agent’s potential productivity, the greater the incentives to accumulate assets to overcome borrowing constraints and operate at an efficient scale. In particular, table 4 shows that entrepreneurs in Spain hold a smaller portion of total net worth in the economy than in the United States.

<table>
<thead>
<tr>
<th></th>
<th>Spain</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Business owners or self-employed</td>
<td>36.4</td>
<td>56.49</td>
</tr>
<tr>
<td>All business owners</td>
<td>35.24</td>
<td>53</td>
</tr>
<tr>
<td>Active business owners</td>
<td>32.2</td>
<td>46.2</td>
</tr>
<tr>
<td>Self-Employed</td>
<td>31.1</td>
<td>43.3</td>
</tr>
<tr>
<td>Self-Employed Business Owners</td>
<td>28.9</td>
<td>39.9</td>
</tr>
</tbody>
</table>

Table 4: Total Net Worth in Economy held by Entrepreneurs, EFF and SCF respectively

Table 5 shows that within the top first and fifth percentile (to a somewhat lesser extent also in the tenth percentile) of the wealth distribution, the proportion of self-employed business owners is significantly lower in Spain than in the US.

\textsuperscript{14}Interestingly, the EFF is relatively skewed to the right. In particular, DIRCE shows no self-employed businesses over the size of 49, whereas some appear in the EFF sample. Although way beyond the scope of this paper, one possible explanation could be households in the EFF reporting informal businesses. The 49 worker threshold is particularly appealing. Similarly to the case of France, studied by Garicano et al. (2013), Spanish labor laws impose a large cost on employers when hiring the fiftieth worker.

\textsuperscript{15}For comparability, I follow the definitions in Cagetti and De Nardi (2006).
Although other measures of entrepreneurship follow a similar pattern, the levels for Spain and the US are not too dissimilar. Figures 8 and 9 show the business size distributions for self-employed and privately held limited companies in Spain and in the US\(^{16}\). Clearly, both business size distributions are skewed to the left in Spain, compared to the US—yet another hint at worse entrepreneurial selection.

<table>
<thead>
<tr>
<th>Wealth Percentile, Top</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
<th>20%</th>
</tr>
</thead>
<tbody>
<tr>
<td>B. owners or self-employed</td>
<td>74.4</td>
<td>80.3</td>
<td>54.9</td>
<td>61.5</td>
</tr>
<tr>
<td>All business owners</td>
<td>74</td>
<td>78.4</td>
<td>54.6</td>
<td>57.9</td>
</tr>
<tr>
<td>Active business owners</td>
<td>67.2</td>
<td>64.5</td>
<td>49.5</td>
<td>50.8</td>
</tr>
<tr>
<td>Self-Employed</td>
<td>28</td>
<td>62</td>
<td>22.3</td>
<td>47</td>
</tr>
<tr>
<td>Self-Employed B. Owners</td>
<td>26.8</td>
<td>54</td>
<td>21.9</td>
<td>39</td>
</tr>
</tbody>
</table>

Table 5: In Percentages

Figure 6 shows a very clear correlation between legal type and business size: smaller businesses are typically self-employed, larger ones limited companies, and the very largest are constituted share companies. Given it is costlier to operate limited or share firm than being self-employed, there have to be incentives to registering as a firm. In table 6 I provide evidence that financial constraints are lower for S.A. than for S.L., which again are less financially constrained than self-employed businesses.

<table>
<thead>
<tr>
<th></th>
<th>Self-Employed</th>
<th>S.L.</th>
<th>S.A.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mortgage</td>
<td>22.85%</td>
<td>18.5%</td>
<td>0.49%</td>
</tr>
<tr>
<td>Other Collateral</td>
<td>7.58%</td>
<td>1.36%</td>
<td>0%</td>
</tr>
<tr>
<td>Collateral Firm Value</td>
<td>18.8</td>
<td>2.31</td>
<td>1.86</td>
</tr>
<tr>
<td>Bank Loans</td>
<td>Personal Only</td>
<td>Personal and Commercial</td>
<td></td>
</tr>
<tr>
<td>Issue Debt</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 6: Types of Loans, Percentages over Entrepreneurs in Debt

Table 7 reflects the bureaucratic hurdles to establishing each legal type of business I consider. Self-employed entrepreneurs can operate at no upfront cost

\(^{16}\)Data for the US is extracted from the 2009 SCF, using the same definitions as for the Spanish data. Even though there are no direct equivalents in terms of different firm type, I directly compare self-employed businesses to sole proprietorships in the US and limited companies (S.L.) to Limited Liability Companies.
within two days of registering with the Spanish tax authorities. Registered firms on the contrary—be it limited or share firms—face much larger overhead costs of formalizing. The more productive a project—and the less financially constrained an entrepreneur—the more likely the founders are to circumvent time costs by externally contracting legal and administrative services. Given the evidence above on the size distribution of different business types and the financial arrangements each business type faces in Spain on average, I will only consider self-employed businesses and limited companies (S.L.) as de facto being subject to the bureaucratic costs measured in the World Bank Doing Business Survey.

<table>
<thead>
<tr>
<th>Autónomos (Self-Emp)</th>
<th>S.L. (LLC)</th>
<th>S.A. (Corp.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monetary Costs</td>
<td>None</td>
<td>See</td>
</tr>
<tr>
<td>Time Costs</td>
<td>2 days</td>
<td>Table 1</td>
</tr>
</tbody>
</table>

Table 7: Main legal forms of privately held firms in Spain

3 Model Economy

The model economy is populated by a measure one continuum of ex-ante identical households. There is a single, homogeneous good which is consumed and used as capital to produce. There is no aggregate uncertainty in the economy.

Preferences

Agents maximize their expected, discounted lifetime utility of consumption. The instantaneous utility function of agents is of the CRRA type,

\[ u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma} \]

where \( \sigma \) is the coefficient of relative risk aversion and each future period is discounted at a rate \( \beta \).

Shocks

Agents in this economy face two types of uninsurable, idiosyncratic shocks. \( \theta \) represents business ideas that occur to individuals at the beginning of every period. We call entrepreneurs’ project choice for the current period \( z' \). Entrepreneurs can choose to implement new business ideas received each period, \( z' = \theta \), or continue the project they ran in the preceding period, \( z' = z \). \( \theta \) is correlated with the previous period’s project choice \( z \) and is i.i.d. across entrepreneurs. In addition
to this, entrepreneurs receive a productivity shock $\varepsilon$ every period. $\varepsilon$ has mean one and is i.i.d. across periods and agents in the economy.

We can think of $z'$ as representing a firm’s average productivity, while $\varepsilon$ represents transitory fluctuations around $z'$.

**Legal Type**

There are two legal forms non-corporate firms can adopt in the economy$^{17}$: entrepreneurs can choose to operate as self-employed, or they can register their business as a limited company ($l = 1$).

**Startup Costs**

Agents who decide to start a firm in the non-corporate sector have to face two types of startup costs: first, a cost denominated in consumption goods, $\varsigma + \Pi_l$, where $\Pi_l$ is a bureaucratic fee the government collects from each new entrepreneur to spend on government consumption and $\varsigma$ represents all market-related entry barriers. New entrepreneurs also have to face the fact that establishing their business takes up a fraction $\xi_l$ of a period. During this time, they neither work nor produce. Note the subscript on $\Pi_l$ and $\xi_l$, denoting startup costs differ depending on the legal type the entrepreneur chooses to take on.

**Exogenous Exit**

Entrepreneurial projects have a probability $\pi$ of surviving. Entrepreneurs exiting exogenously keep their asset holdings, but start the next period as workers: $(a, z', l, \theta, \varepsilon) = (a', 0, 0, \theta_w, 0)$, where $\theta_w$ represents not receiving a productive business idea.

**Financial Arrangements**

In order to produce, entrepreneurs need to rent capital. Machines can be bought with one’s own assets, or by taking out loans. Lenders in this economy only observe borrowers’ asset holdings, not the quality of their projects in terms of agents’ business ideas $z'$ and transitory productivity $\varepsilon$.

$^{17}$As discussed in section [insert reference], figure [insert reference] shows the main types of privately held firms prevailing in Spain. Self-employed entrepreneurs (autónomos) and Limited Companies (S.L.) make up [insert figure]% of all privately held firms and [insert figure]% of firms with at most 50 employees.
Lenders know ex ante that due to enforcement issues, if an entrepreneur defaults on their loan, only a fraction $\phi_l$ of the depreciated capital of the firm can be claimed by lenders: 

$$(1 + r)(k - a) \leq \phi_l(1 - \delta)k$$

Note that the collateral constraint depends on the legal status of a firm, $l$. In order to collateralize loans, entrepreneurs are not lent more than a fraction of the firm’s capital. Firms’ size is thus limited by the asset holdings of their managing proprietors:

$$k \leq \frac{1 + r}{1 + r - \phi_l(1 - \delta)}a$$

Note that financial constraints are a function of the entrepreneur’s wealth only, independently of their business idea $z'$. This means that there might be individuals with great business perspectives who won’t be able to produce at an optimal scale, due to lack of enough wealth to collateralize their needed debt levels. Agents are not lent resources in order to smooth consumption.

If there were no borrowing constraints in the economy, startup costs would not matter, as agents might simply borrow against future income to start a business. Ergo, entrepreneurial selection would not be affected by startup costs.

**Technology**

**Corporate sector**

The corporate sector is composed of a representative price taking firm that uses capital and labor to produce the same homogeneous good as the entrepreneurial sector.

$$F(K, N) = AK^\alpha N^{1-\alpha}$$

The corporate sector does not face financial constraints when choosing their optimal levels of factor demands. The corporate firm’s first order conditions are:

$$r + \delta = \alpha A \left(\frac{K}{N}\right)^{\alpha-1} \quad (1)$$
$$w = (1 - \alpha) A \left(\frac{K}{N}\right)^\alpha \quad (2)$$

Note therefore how the corporate sector’s capital-labor ratio will therefore optimally be:

$$\frac{K}{N} = \frac{\alpha w}{1 - \alpha r + \delta}$$
I include the corporate sector to make sure I capture the appropriate magnitude of the effects of startup costs in the economy: say the entrepreneurial sector were very small in terms of total output and factor employment. Then, not assuming the corporate sector would overstate my results.

Non-corporate sector

Entrepreneurs can implement business ideas \( z' \) in this economy by using capital and labor to produce a consumption and investment good, using the following technology:

\[
f(z'\varepsilon, k, n) = z'\varepsilon(k^n n^{1-\alpha})^{1-\nu}
\]

where \( z' \in \{ z, \theta \} \) is the chosen project, \( \varepsilon \) is the transitory productivity shock, \( k \) and \( n \) the amounts of capital and labor used in production, while \( \nu \) and \( \alpha \) are the returns to scale of the different outputs. Note that production in concave in all of the inputs, in particular in managerial input \( z'\varepsilon \). This is the key to having a distribution of firm sizes in the economy. Entrepreneurs solve the following static problem to maximize profits each period:

\[
\max_{n,k} z'\varepsilon (k^n n^{1-\alpha})^{1-\nu} - (1 + r)(k - a) - wn + (1 - \delta)k
\]

s.t. \( \phi_l (1 - \delta)k \geq (k - a)(1 + r) \)

The optimal choices of labor and capital are:

\[
k(a, z'\varepsilon, l') = \begin{cases} 
\left[ \frac{(1-\nu)z'\varepsilon\alpha}{r+\delta} \right]^\frac{1}{\nu} \frac{(r+\delta)(1-\alpha)}{\alpha^\nu l} \frac{1}{1+r-\phi_l(1-\delta)a} & \text{if } k \leq \frac{1+r}{1+r-\phi_l(1-\delta)a} a \\
\frac{1}{1+r-\phi_l(1-\delta)}a & \text{if else} 
\end{cases} \tag{3}
\]

\[
n(a, z'\varepsilon, l') = \left[ \frac{(1-\alpha)(1-\nu)z'\varepsilon}{w} k^{\alpha(1-\nu)} \right]^\frac{1}{\nu(1-\nu)+\nu} \tag{4}
\]

The first order conditions also reveal the following relation for unconstrained firms:

\[
\frac{k}{n} = \frac{\alpha}{1 - \alpha} \frac{w}{r + \delta}
\]

Thus, no matter what firm a manager owns, the capital-labor ratio will be equal in every financially unconstrained establishment.
Timing of the Model

Figure 1 illustrates a model period, which represents a natural year in my calibration. A model period starts. Agents observe their asset positions $a$, their previous period’s business $z$, its legal type $l$, and receive an idea shock $\theta$ and a transitory productivity shock $\varepsilon$. At that point, they choose occupation: worker ($z' = 0$) or entrepreneur. If they choose to be an entrepreneur, they choose what business idea to pursue in the current period $z' \in \{z, \theta\}$ and the legal form of their business ($l' \in \{Aut, SL\}$). Finally, they pick the corresponding capital $k(a, z', l')$ and labor $n(a, z', l')$ amounts to hire. Once production has occurred in the economy, all agents receive their corresponding incomes, to be described below. Finally, agents choose how much to consume $c$ and what asset level $a'$ to carry onto the next period. As the model period ends, agents die with probability $(1 - \pi)$, giving birth to offspring that conserve their parents’ asset positions, but start with no previous business experience ($z = 0$) and no business idea ($\theta = 0$).

Recursive Formulation of Agents’ Problems

Agents observe their asset positions $a$, their previous business idea $z$, legal type of their previous business $l$, an idea shock $\theta$, and a transitory productivity shock $\varepsilon$. Next, they decide whether to open a new businesses (which might imply implementing a new business idea $\theta$, switching the legal type of their business $l$, or both), maintain the business they managed the previous period $z$, or work for some firm for a wage $w$.

The value to an agent that chooses not to continue with the previous period’s project is $V_n(a, z, l, \varepsilon, \theta)$, which depends on the value of starting a new business $V_{nb}(a, z, l, \varepsilon, \theta)$ and the value of being a worker $V_w(a)$:

$$V_n(a, z, l, \varepsilon, \theta) = \max\{V_{nb}(a, z, l, \varepsilon, \theta), V_w(a)\}$$

As described above, agents start a new business whenever they choose to either implement a new business idea (which yields $V_z(a, l, \varepsilon, \theta)$), to switch their business’ legal type (leading to $V_l(a, z, l, \varepsilon)$) or do both simultaneously (which leads to $V_{lz}(a, l, \varepsilon, \theta)$). The value of starting a new business $V_{nb}(a, z, l, \varepsilon, \theta)$ is therefore the most profitable out of these three options:

$$V_{nb}(a, z, l, \varepsilon, \theta) = \max\{V_z(a, l, \varepsilon, \theta), V_l(a, z, l, \varepsilon), V_{lz}(a, l, \varepsilon, \theta)\}$$

Finally, an agent that was an entrepreneur in the previous period can choose to continue running the same business, which yields value $V_{cb}(a, z, l, \varepsilon)$ or not to continue with the previous period’s activity and obtain $V_n(a, z, l, \varepsilon, \theta)$:

$$V(a, z, l, \varepsilon, \theta) = \max\{V_{cb}(a, z, l, \varepsilon), V_n(a, z, l, \varepsilon, \theta)\}$$
This structure permits me to keep track of who sets up a new firm, in which case they would have to pay the startup costs. Next, I describe each of the problems faced by agents according to their occupational choices.

Workers

Workers maximize current utility plus the expected value of not continuing with the current project in the future. In the case of workers this is trivial, since they are not managing any project by definition. That is why $z' = 0$.

$$V_w(a) = \max_{c, a'} \{ u(c) + \beta [\pi E_{\theta', \varepsilon'} V_n(a', 0, 0, \varepsilon', \theta') + (1 - \pi) V_w(a')] \}$$

s.t. $c + a' \leq (1 + r) a + w$

$a' \geq 0$

$log \theta' = \rho_\theta \log z' + \zeta', \zeta' \sim i.i.d. N \left( \mu_\theta, \sigma_\theta^2 \right)$

$log \varepsilon' \sim i.i.d. N \left( 0, \sigma_\varepsilon^2 \right)$

$s = (a, z, l, \varepsilon, \theta)$

Their budget constraint states that they consume and save while inelastically supplying a unit of labor in the market, receiving asset income. Workers are constrained from borrowing.

Continuing Businesses

Continuing business owners receive an idea shock $\theta$, but choose to continue with their previous project $z$. Thus, $z' = z$ and $l = l'$.

$$V_{cb}(a, z, l, \varepsilon) = \max_{c, a', k, n} \{ u(c) + \beta [\pi E_{\theta', \varepsilon'} V_k(a', z, l, \varepsilon', \theta') + (1 - \pi) V_w(a')] \}$$

s.t. $c + a' \leq \varepsilon' z (k^n (1 - \alpha))^{1 - \nu} - (1 + r) (k - a) - wn + (1 - \delta) k$

$\phi_l (1 - \delta) k \geq (k - a)(1 + r)$

$a' \geq 0$

$log \varepsilon' \sim i.i.d. N \left( 0, \sigma_\varepsilon^2 \right)$

$log \theta' = \rho_\theta \log z' + \zeta', \zeta' \sim i.i.d. N \left( \mu_\theta, \sigma_\theta^2 \right)$

Entrepreneurs consume and save for the next period out of the profits made and depreciated capital. Their firms' sizes are financially constrained as described above. They are not allowed to borrow for consumption smoothing purposes.
New Businesses

Entrepreneurs who manage new businesses time face a similar problem to entrepreneurs with continuing businesses, with the addition of start up costs and the fact that the business might be new due to three different reasons:

First, an entrepreneur might choose to use a newly received business idea, $z' = \theta$ and $l' = l$:

$$V_z(a, l, \varepsilon, \theta) = \max_{c, a', k, n} \{ u(c) + \beta \left[ \pi E_{\theta', \varepsilon'} V(a', \theta, l, \varepsilon', \theta') + (1 - \pi) V_w(a') \right] \}
\text{s.t.}
\begin{align*}
c + a' + \Pi l + \varsigma &\leq (1 - \delta)k + (1 - \xi l)\varepsilon\theta(k^\alpha n^{1-\alpha})^{1-\nu} \\
-(1 + r)(k - a) - wn &\quad \phi l(1 - \delta)k \geq (k - a)(1 + r) \\
a' &\geq 0 \\
\log \varepsilon' &\sim \text{i.i.d. } N(0, \sigma_{\varepsilon}^2) \\
\log \theta' &\sim \text{i.i.d. } N(\mu_\theta, \sigma_\theta^2)
\end{align*}$$

Alternatively, an entrepreneur might choose to continue using their previous idea, $z' = z$, but change the business' legally type, $l' \neq l$:

$$V_l(a, z, l, \varepsilon) = \max_{c, a', k, n} \{ u(c) + \beta \left[ \pi E_{\theta', \varepsilon'} V(a', z, l', \varepsilon', \theta') + (1 - \pi) V_w(a') \right] \}
\text{s.t.}
\begin{align*}
c + a' + \Pi l + \varsigma &\leq (1 - \delta)k + (1 - \xi l)\varepsilon z(k^\alpha n^{1-\alpha})^{1-\nu} \\
-(1 + r)(k - a) - wn &\quad \phi l(1 - \delta)k \geq (k - a)(1 + r) \\
a' &\geq 0 \\
\log \varepsilon' &\sim \text{i.i.d. } N(0, \sigma_{\varepsilon}^2) \\
\log \theta' &\sim \text{i.i.d. } N(\mu_\theta, \sigma_\theta^2)
\end{align*}$$

Finally, an agent might choose to operate a new idea $z' = \theta$ under a different legal setup $l' \neq l$:

$$V_{lz}(a, l, \varepsilon, \theta) = \max_{c, a', k, n} \{ u(c) + \beta \left[ \pi E_{\theta', \varepsilon'} V(a', l, \varepsilon', \theta') + (1 - \pi) V_w(a') \right] \}
\text{s.t.}
\begin{align*}
c + a' + \Pi l + \varsigma &\leq (1 - \delta)k + (1 - \xi l)\varepsilon \theta(k^\alpha n^{1-\alpha})^{1-\nu} \\
-(1 + r)(k - a) - wn &\quad \phi l(1 - \delta)k \geq (k - a)(1 + r) \\
a' &\geq 0 \\
\log \varepsilon' &\sim \text{i.i.d. } N(0, \sigma_{\varepsilon}^2) \\
\log \theta' &\sim \text{i.i.d. } N(\mu_\theta, \sigma_\theta^2)
\end{align*}$$
Note how bureaucratic costs enter the problem. On the one hand, \( \Pi_l \) is a cost denominated in units of consumption good that enters all entrepreneurs’ problems equally, regardless of their productivity. On the other hand, \( \xi_l \) represents the fraction of a period it takes an entrepreneur to deal with bureaucratic requirements. It will entail a greater opportunity cost of opening a business for more productive entrepreneurs. It will therefore have a negative effect on entrepreneurial selection \( \Pi_l \) will not. New entrepreneurs face a second type of entry cost in units of consumption good, \( \varsigma \). This summarizes all other types of entry barriers due to market structure issues.

**Income as a function of assets**

Profits for entrepreneurs are increasing in asset holdings for low values of asset holdings, due to borrowing constraints that impede entrepreneurs from producing at their optimal scale. Once entrepreneurs overcome borrowing constraints, profits become independent of asset holdings, conditional on entrepreneurial ability. \( \Pi \) and \( \varsigma \) lower income for entrepreneurs for any assets holdings level by the sum of both costs. This affects all entrepreneurs in the same way. \( \xi \) on the other hand flattens entrepreneurial profits. Furthermore, it proportionally affects profits of more productive entrepreneurs more than profits of less productive entrepreneurs. This creates a greater opportunity cost of taking on new, more productive projects for more talented individuals.

**Equilibrium Definition**

A steady state recursive competitive equilibrium for this economy consists of value functions \( V(a, z, l, \varepsilon, \theta) \), \( V_n(a, z, l, \varepsilon, \theta) \), \( V_{ab}(a, z, l, \varepsilon, \theta) \), \( V_{cb}(a, z, l, \varepsilon, \theta) \), \( V_z(a, l, \varepsilon, \theta) \), \( V_l(a, l, \varepsilon, \theta) \), \( V_{lz}(a, \varepsilon, \theta) \), and \( V_w(a) \); policy functions \( z'(a, z, l, \varepsilon, \theta) \), \( l'(a, z, l, \varepsilon, \theta) \), \( k_i(a, z', l', \varepsilon) \), \( n_i(a, z', l', \varepsilon) \), \( c_i(a, z', l', \varepsilon) \), and \( a_i'(a, z', l', \varepsilon) \), for \( i \in \{cb, z, l, lz, w\} \); the wage rate \( w \) and interest rate \( r \); capital and labor demands \( K \) and \( N \) from the corporate sector; a function \( \Psi(\mu) \) mapping the space of households’ distribution \( \mu \) into the next period distribution, and an invariant distribution \( \mu^* \) such that:

- Given \( \{w, r\} \), the policy rules \( c_i(a, z', l', \varepsilon) \) and \( a_i'(a, z', l', \varepsilon) \), for \( i \in \{cb, z, l, lz, w\} \), and \( z'(a, z, l, \varepsilon, \theta) \) and \( l'(a, z, l, \varepsilon, \theta) \) solve the agents’ problems associated to \( V_{cb}(a, z, l, \varepsilon, \theta) \), \( V_z(a, l, \varepsilon, \theta) \), \( V_l(a, z, l, \varepsilon, \theta) \), and \( V_w(a) \) respectively, while \( k_i(a, z', l', \varepsilon) \) and \( n_i(a, z', l', \varepsilon) \) for \( i \in \{cb, z, l, lz, w\} \) solve each type of entrepreneurs’ static profit maximization problems.

- Given \( \{w, r\} \), the corporate sector chooses \( \{K, N\} \) to maximize profits.
• Capital and labor markets clear:

\[
\sum_{\theta} \sum_{\varepsilon} \sum_{l} \sum_{z} \int_a k_i(a, z', l', \varepsilon) \mu(a, z, l, \varepsilon, \theta) da + K = \sum_{\theta} \sum_{\varepsilon} \sum_{l} \sum_{z} \int_a a \mu(a, z, l, \varepsilon, \theta) da
\]

\[
\sum_{\theta} \sum_{\varepsilon} \sum_{l} \sum_{z} \int_a n_i(a, z', l', \varepsilon) \mu(a, z, l, \varepsilon, \theta) da + L = \sum_{\theta} \sum_{\varepsilon} \sum_{l} \sum_{z} \int_a \{z'(a, z, l, \varepsilon, \theta) = 0\} \mu(a, z, l, \varepsilon, \theta) da
\]

• All bureaucratic fees paid by entering entrepreneurs are spent by the government, while all other monetary entry costs $\varsigma$ are lost:

\[
\sum_{\theta} \sum_{\varepsilon} \sum_{l} \sum_{z} \int_a \{0 < z'(. \land (z'(. \neq z \lor l'(.) \neq l))\} \int_a (\Pi_{\theta'} + \varsigma) \mu(a, z, l, \varepsilon, \theta) da = G + \text{Cost}
\]

• The distribution $\mu^*(a, z, l, \varepsilon, \theta)$ is a fixed point of the mapping $\Psi(\mu(a, z, l, \varepsilon, \theta))$:

\[
\mu'(a', z', l', \varepsilon', \theta') = \Psi(\mu(a, z, l, \varepsilon, \theta)) = \sum_{\theta} \sum_{\varepsilon} \sum_{l} \sum_{z} \int_a \mu(a, z, l, \varepsilon, \theta)[\pi I (a', z', l'|a, z, l, \varepsilon, \theta) P(\theta'|z') P(\varepsilon')
\]

\[
+ (1 - \pi) I_0(a', 0, 0|a, z, l, \varepsilon, \theta)] da
\]

Where $I (a', z', l'|a, z, l, \varepsilon, \theta)$ determines the deterministic policy choices made by agents once they observe their state variables and $P(\theta'|z')$ is the conditional probability of receiving a certain realization of an idea shock in the next period, given the current idea choice. $\pi$ is the survival probability of agents. $I_0(a', 0, 0|a, z, l, \varepsilon, \theta)$ represents offspring inheriting parents' asset positions, but not their businesses.

4 Quantitative Exercise

In this section I discuss my calibration strategy for the model and conduct a comparative statics policy exercise on the startup costs faced by agents wanting to establish their businesses as limited companies (S.L.).

Targeted Moments

All the data I use for Spain correspond to the year 2009\(^{18}\), which corresponds to the period of time the 2008 EFF wave was collected\(^{19}\).

\(^{18}\)As does the data I use above for the U.S. economy.

\(^{19}\)Again, see Bover (2011) for details.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Capital Income Share</td>
<td>0.27</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation</td>
<td>0.06</td>
</tr>
<tr>
<td>$\xi_{Aut}, \xi_{SL}$</td>
<td>Time cost</td>
<td>0.08 / 0.33</td>
</tr>
<tr>
<td>$\Pi_{Aut}, \Pi_{SL}$</td>
<td>Monetary Cost</td>
<td>0 / 0.65</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Risk Aversion</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 8: Exogenous parameters

All parameters in table 8 are set invariable to every solution iteration of the model, with the exception of $\Pi_{SL}$: as the World Bank Doing Business Survey reports monetary bureaucratic costs as percentage of income per capita, I guess an initial value for $\Pi_{SL}$; solve the model, and check whether I hit the appropriate ratio. Otherwise, I iterate on $\Pi_{SL}$ accordingly. The capital income share and depreciation rate I compute from the Spanish national accounts using standard methods in the quantitative literature. The coefficient of relative risk aversion for households I set to a value of 2, following Hall (2009). Finally, I assume a period in my model corresponds to a natural year. $\xi_{Aut}$ and $\xi_{SL}$ are set correspondingly to the fraction of a year that startup procedures are measured to take in Spain.

Interest Rate

The interest rate is taken from the Bank of Spain’s Interest Rate Statistics.\(^{20}\) In particular, I use table 19 3.12, which reports the average interest rate on loans up to one year to the general public for other uses. I compute the average for 2011.

Firm Size, Entry and Exit

Data for the firm size distribution by employment and firm entry and exit by size come from the Spanish Centralized Firm Directory (DIRCE\(^{21}\)) maintained by INE. They compile their data from various sources, including tax and social security data.

Baseline Calibration

I will start by numerically exploring the solution to the model. In order to do so, I set the parameters in the model to those outlined in Tables 8 and 8. In addition to these values, I assume nine possible values for the business idea shock

\(^{21}\)http://www.ine.es/inebmenu/mnu_empresas.htm
distribution and the transition matrix \( \Gamma \) outlined below.

\[
\theta \in \{0, 2.32, 2.44, 2.71, 2.86, 3.01, 3.18, 3.35\}
\]

\[
\Gamma_{\theta} = \begin{pmatrix}
0.23 & 0.39 & 0.29 & 0.08 & 0.01 & 0.00 & 0.00 & 0.00 \\
0.07 & 0.27 & 0.39 & 0.22 & 0.04 & 0.00 & 0.00 & 0.00 \\
0.01 & 0.11 & 0.33 & 0.37 & 0.15 & 0.02 & 0.00 & 0.00 \\
0.00 & 0.028 & 0.17 & 0.38 & 0.31 & 0.09 & 0.01 & 0.00 \\
0.00 & 0.00 & 0.05 & 0.24 & 0.40 & 0.24 & 0.05 & 0.00 \\
0.00 & 0.00 & 0.01 & 0.09 & 0.31 & 0.38 & 0.17 & 0.03 \\
0.00 & 0.00 & 0.00 & 0.02 & 0.15 & 0.37 & 0.33 & 0.11 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.04 & 0.22 & 0.39 & 0.27 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.01 & 0.08 & 0.29 & 0.39 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
\end{pmatrix}
\]

Realizations of the transitory productivity shock can take on three possible values, as outlined below:

\[
\varepsilon \in \{0.74, 1, 1.44\}
\]

\[
\Gamma_{\varepsilon} = \begin{pmatrix}
0.23 & 0.39 & 0.29 & 0.08 \\
0.07 & 0.27 & 0.39 & 0.22 \\
0.00 & 0.00 & 0.00 & 0.00 \\
\end{pmatrix}
\]

Both shock vectors and transition matrices are the result of discretizing an AR(1) process for log shocks using the procedure developed in Tauchen (1986), assuming the following processes:

\[
\ln\theta' = \rho_\theta \ln z + \eta', \eta' \sim N(\mu_\theta, \sigma_\theta^2)
\]

\[
\ln\varepsilon' \sim i.i.d. N(0, \sigma_\varepsilon^2)
\]

All parameters in table 9 are calibrated jointly, meaning there is no one to one mapping between any given parameter and a certain targeted moment. Nevertheless, certain moments are affected more predominantly by some parameters, which prompts me to design the calibration strategy I outline next. I start with the parameters and moments with the clearest relationships. I target the risk free interest rate described in the previous section in order to find a suitable value for agents’ discount factor in my model. I use the exit rate of non-corporate firms to extract information on the exogenous exit rate for business in my model. The model assumes workers not employed by privately held firms are absorbed by the corporate sector, hence the employment share of privately held firms contains information on corporate total factor productivity. The level of non-bureaucratic entry costs is set to approximate the average size of entrants, relative to the average size of survivors. The idea behind this is that higher monetary entry costs deter lower-productivity firms—hence smaller, due to the span of control
assumption—from entering the market. I use the ratio of private firm owners to workers in the private sector to extract information on the span of control parameter in my economy. As entrepreneurs’ production functions become less concave, profits increase and more agents will become entrepreneurs as opposed to workers.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Object</th>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount Factor</td>
<td>Risk free interest rate</td>
<td>0.031</td>
<td>0.025</td>
</tr>
</tbody>
</table>
| $\phi_{Aut}, \phi_{SL}$ | Financial Constraints | Collateral
                  |                                    | 18.8 / 2.31 | 0.003/0.001 |
| $\nu$            | Span of Control         | Private Firm Owners
                  |                                    | 0.13    | 0.00     |
| $A$              | Relative Size of Corp.  | Employment share from EFF             | 35.74%  | 6%       |
| $\mu_\theta$     | Idea Shock Mean         | Average Firm
                  |                                    | 2.87    | 8.245    |
| $\sigma_\theta$  | Idea Shock Stdev.       | Biggest Firm
                  |                                    | 171     | 189.735  |
| $\rho_\theta$    | Idea Shock Persistence  | Income Gini Coeff.                    | 0.303   | 0.045    |
| $\sigma_\varepsilon$ | TFP Shock Stdev.       | #Autonomos
                  |                                    | 4.56    | 9938489  |
| $\varsigma$      | Other Entry Costs       | #S.L. / $<50$
                  |                                    | 0.192   | 0.023    |
| $\pi$            | Exit Rates              | Aggregate Exit Rate                   | 0.118   | 0.928    |

Table 9: Jointly calibrated parameters and corresponding targets

As is standard in the literature since Lucas Jr (1978), I use the information contained in the firm size distribution to pin down entrepreneurs’ productivity distribution in my model: I use the idea shock mean and standard deviation to match the ratios of the average to smallest firm and largest to smallest firm in my data, respectively. In order to pin down the persistence parameter for the business idea shock, I’d ideally use panel data to observe individuals’ occupations and income streams. Such data is difficult to obtain for the case of Spain. Given my cross-sectional data, the strategy I adopt is to target the income Gini coefficient in my economy: the lower the persistence in the business idea data generating process, the higher the dispersion in the cross-sectional ergodic distribution of business idea shocks in my economy. This leads to a wider firm size distribution and thus, again due to the span of control assumption, to higher income disparities. Finally, I use the fact that I observe limited liability companies with less than fifty employees to pin down the dispersion of the transitory productivity shock in my model. A small limited liability firm in my model is likely to be operating a relatively productive business idea at a low transitory productivity shock draw. The lower the dispersion of the transitory productivity shocks, the lower the probability an entrepreneur with a productive business idea will produce at a small scale in a given period.

The only parameters left to calibrate are the financial enforcement parameters or collateral constraints for self-employed and limited liability firms. Although

---

22Habla del componente de panel de la EFF, de la muestra continua de vidas laborales.
the literature has approached this parameter in several ways, most authors have used aggregate data to shed light on it. I instead use data on privately held firms’ financial obligations. I set each legal type’s collateral constraint parameter to replicate the average collateral over firm value ratio I observe in the EFF: as collateral constraints become tighter for firms, they need to put up a larger share of their loans as collateral. In my model, as entrepreneurs become more constrained for any given level of asset holdings, they will manage smaller businesses, thus lowering the collateral to firm value ratio.

Policy Experiment

In this section I conduct three policy experiments. First, I lower bureaucratic fees for limited companies $\Pi_{SL}$ to US levels, then I lower the waiting time to open a limited company $\xi_{SL}$ again to US levels, and finally I will compute the outcome of an economy with both startup costs at US levels.

I measure the effects of each policy on various outcomes: total output, which is the sum of output produced by the corporate sector and all the privately held businesses; total factor productivity which I measure as in a growth accounting exercise\(^{23}\); unconditional average productivity of entrepreneurs\(^{24}\), which captures entrepreneurial selection through privately held firms’ productivity; size-weighted average entrepreneur productivity\(^{25}\), which in comparison to unconditional average productivity, captures the extent to which financial constraints distort productive resource allocations; the ergodic distribution of operated projects in the economy; and finally the percentage of agents that receive a more productive business idea shock than their current project and decide to implement it.

<table>
<thead>
<tr>
<th>$\xi_{SL} = 0.008, \Pi_{SL} = 0.27$</th>
<th>$\xi_{SL} = 0.33, \Pi_{SL} = 0$</th>
<th>$\xi_{SL} = 0.008, \Pi_{SL} = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP p.c.</td>
<td>+3.70%</td>
<td>+1.63%</td>
</tr>
<tr>
<td>TFP</td>
<td>+2.47%</td>
<td>+0.66%</td>
</tr>
<tr>
<td>Uncond. Avg. Productivity</td>
<td>+1.98%</td>
<td>-0.34%</td>
</tr>
<tr>
<td>Size-weighted Avg. Prod.</td>
<td>+2.43%</td>
<td>+1.17%</td>
</tr>
</tbody>
</table>

Table 10: Percentage deviation from benchmark ($\xi_{SL} = 0.33, \Pi_{SL} = 0.27$)

\(^{23}\)Effectively, I add all output in the economy, and all productive factors, assume a constant returns to scale production function for the entire economy, and back out total factor productivity as if all was produced by a single representative firm. Mathematically, $\text{TFP} = \frac{Y_C + Y_e}{(K_C + K_e)(N_C + N_e)}$, where $C$ subscripts denote the corporate sector and $e$ subscripts denote entrepreneurs.

\(^{24}\)Unconditional average productivity is measured as $\sum_a \sum_{z'} \sum_l \sum_{\varepsilon} \sum_a \int_a z'(a,z,l,\varepsilon,\theta) \mu(a,z,l,\varepsilon,\theta)da$ $\sum_a \sum_{z'} \sum_l \sum_{\varepsilon} \sum_a \int_a z'(a,z,l,\varepsilon,\theta) \mu(a,z,l,\varepsilon,\theta)da$ $\sum_a \sum_{z'} \sum_l \sum_{\varepsilon} \sum_a \int_a z'(a,z,l,\varepsilon,\theta) \mu(a,z,l,\varepsilon,\theta)da$

\(^{25}\)Size-weighted average productivity is measured as $\sum_a \sum_{z'} \sum_l \sum_{\varepsilon} \sum_a \int_a z'(a,z,l,\varepsilon,\theta) \mu(a,z,l,\varepsilon,\theta)da$. 

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Several interesting results become clear from table 10. Lowering both time costs $\xi_{SL}$ and monetary costs $\Pi_{SL}$ for limited companies, has significant effects on output, overall productivity, and entrepreneurial selection. Decreasing the time it takes to register limited companies though, has greater impact on both output and overall productivity in the economy. Furthermore, the unconditional average productivity of operating entrepreneurs rises significantly. New businesses are on average more productive than the previously existing ones. In contrast, lowering monetary costs slightly lowers unconditional average productivity—entering entrepreneurs operate on average low productivity projects. In spite of the worsened selection, the economy obtains productivity and output gains as lowering limited companies’ registration costs permits more entrepreneurs to access greater outside funding. The size-weighted average productivity of entrepreneurs increases even when the unconditional average doesn’t, as entrepreneurs operate closer to their optimal scales. Figures 10 and 11 illustrate this point: lower monetary costs alone has little impact on the ergodic distribution of operated projects in the economy, so most productivity gains must come from a more efficient distribution of investment resources.

Combining a decrease in time and monetary costs interestingly does not have additive results. Although output, overall productivity


5 Conclusion

Bureaucratic impediments to firm creation are surprisingly high even among various OECD countries, which I document using the World Bank Doing Business report data. The low cost of lowering these have made them a policy priority in recent years. I study how the two most common forms of business entry regulations distinctively affect entrepreneurial selection. In particular, I distinguish between the effects of upfront monetary fees, and time-consuming procedures. Upfront fees deter lower-productivity projects from being carried out, while time-consuming bureaucratic hurdles have a negative selection effect on entrepreneurial entry. This entrepreneurial selection channel translates into economy-wide business productivity and output distortions.

Incurring a fixed monetary cost is only profitable for projects with a high enough expected future profit stream. Time-consuming procedures on the other hand, oblige the entrepreneur to forgo producing income while carrying these out—the opportunity cost of starting a business is thus increasing with an individual’s profit generating capacity. Monetary startup costs ameliorate entrepreneurial selection—but lower overall aggregate productivity and output moderately through general equilibrium effects. Time costs on the other hand deter high productivity businesses from operating, which translates into significantly lower aggregate pro-
ductivity and output. The message of the paper is clear: preemptive bureaucratic regulations—if absolutely necessary—should minimize the time needed to abide by them, as their adverse aggregate effects are sizable.

This paper belongs to an exciting line of ongoing research. Further work remains to be developed in at least two broad areas. On the one hand, further exploring the effects bureaucratic hurdles might have on entire range of aggregate outcomes. For example in labor markets—specifically on job creation and unemployment—or the informal economy and its fiscal consequences. On the other hand, bureaucratic startup costs interact in rich ways with other relevant, separately studied policy instruments—such as labor and corporate taxation, other labor market regulations or size-dependent policies. Understanding the complementarity between these is crucial in achieving the corresponding policy goals in the most efficient way.
Appendix

A Figures

Timing of the model

Figure 1: Timing of the model
Figure 2: Self-Employed Percentage of Businesses in each bin

Figure 3: Percentage of all firms in bin over total number of firms. DIRCE and US Census
Figure 4: Productivity by firm size, relative to average productivity. OECD

Figure 5: FT Global 500 Companies by Region of origin, percentages.
Figure 6: Percentage of self-employed, SL, and SA per size bin. DIRCE

Figure 7: Over sum of Aut. (Self-Employed) and SL (LLCs) per bin, EFF
Figure 8: Percentage of Sole Proprietorships in bin over total, EFF and SCF respectively

Figure 9: Percentage of LLCs in bin over total, EFF and SCF respectively
Policy Experiment

Figure 10: Ergodic Occupation Distribution

Figure 11: Decision to Switch to New Idea
B  Procedures Solving the Economy

Algorithm to Solve the Economy Numerically

Given the definition of equilibrium and the properties of the first order conditions found above, I design the following algorithm to solve the economy numerically:

1. Guess an initial capital-labor ratio in the economy \( \left( \frac{K}{N} \right) \). Given the first order conditions of the firm, Equations (1) and (2), this pins down the wage and interest rate \{ \omega, r \}.

2. Given prices \{ \omega, r \}, solve the households’ problems, which yield the corresponding policy functions \( z'(a, z, l, \varepsilon, \theta), l'(a, z, l, \varepsilon, \theta), c_i(a, z', l', \varepsilon) \) and \( a'_i(a, z', l', \varepsilon) \), for \( i \in \{ cb, z, l, lz, w \} \). The solution to the entrepreneurs’ policy functions \( k_i(a, z', l', \varepsilon), n_i(a, z', l', \varepsilon) \) is given by the analytical expressions in Equations (3) and (4) respectively.

3. Find the ergodic distribution \( \mu^*(a, z, l, \varepsilon, \theta) \) of assets, prior projects, prior legal types, transitory productivity shocks and idea shocks by finding the matrix \( \Psi \) that represents the mapping \( \mu'(a', z', l', \varepsilon', \theta') = \Psi(\mu(a, z, l, \varepsilon, \theta)) \).

4. Given \( \mu^*(a, z, l, \varepsilon, \theta) \), compute aggregate asset supply \( \sum_{\theta} \sum_{\varepsilon} \sum_{l} \sum_{z} \int_a a \mu(a, z, l, \varepsilon, \theta) da \) and labor supply \( \sum_{\theta} \sum_{\varepsilon} \sum_{l} \sum_{z} \int_a 1 \{ z'(a, z, l, \varepsilon, \theta) = 0 \} \mu(a, z, l, \varepsilon, \theta) da \) in the economy.

5. Compute the corporate capital and labor demands implied by market clearing:

\[
\hat{K} = \sum_{\theta} \sum_{\varepsilon} \sum_{l} \sum_{z} \int_a a \mu(a, z, l, \varepsilon, \theta) da - \sum_{\theta} \sum_{\varepsilon} \sum_{l} \sum_{z} \int_a k_i(a, z', l', \varepsilon) \mu(a, z, l, \varepsilon, \theta) da
\]

and

\[
\hat{N} = \sum_{\theta} \sum_{\varepsilon} \sum_{l} \sum_{z} \int_a 1 \{ z'(a, z, l, \varepsilon, \theta) = 0 \} \mu(a, z, l, \varepsilon, \theta) da \\
- \sum_{\theta} \sum_{\varepsilon} \sum_{l} \sum_{z} \int_a n_i(a, z', l', \varepsilon) \mu(a, z, l, \varepsilon, \theta) da
\]
6. Compare the implied capital labor ratio \( \left( \frac{K}{N} \right) \) to the initial guess \( \left( \frac{K}{N} \right) \).

Given \( \left( \frac{K}{N} \right) \) is a decreasing function of \( \left( \frac{K}{N} \right) \), use a one-dimensional bisection algorithm to find a new guess \( \left( \frac{K}{N} \right)^{j+1} \) until a fixed point is found\(^{26}\).

### Finding the Ergodic Distribution of State Variables

In order to find the ergodic distribution \( \mu^*(a, z, l, \varepsilon, \theta) \) of assets, prior projects, prior legal status, transitory productivity shocks and business idea shocks in the economy, I take advantage of the first order markov chain structure of the shocks by writing the state space as a row vector \( S \) with dimension equal to the product of the number of possible legal statuses \( N_l \), the number of possible personality projects \( N_\theta \), the size of the asset grid \( N_a \), the number of possible transitory productivity shock realizations \( N_\varepsilon \) and the number of possible business idea shocks \( N_\theta \). Finding the ergodic distribution of the state space then boils down to finding the one period transition matrix for the economy \( \Psi \), and computing the limiting state space \( S_\infty \) as as fixed point of the equation \( S' = \Psi S \). Finding the transition matrix \( \Psi \) for the state space of the economy involves the following steps:

1. Find the deterministic policy function \( I(a', z', l'| a, z, l, \varepsilon, \theta) \) that determines the positions on next period’s state space \( S' (a', z', l', \varepsilon, \theta) \) given each position in the current state space \( S(a, z, l, \varepsilon, \theta) \). For every possible position on the state space, the deterministic policy function is represented by a vector with the same dimensions as the state space of the economy. It will contain zero entries for all positions except for \( N_\varepsilon \cdot N_\theta \) positions, representing the value of \( I(a', z', l'| a, z, l, \varepsilon, \theta) \) for every possible combination of value of \( (\varepsilon', \theta') \).

2. Using the transition matrix for shocks \( \Gamma_{\varepsilon, \theta} = P(\varepsilon', \theta'| \theta) \), weight the deterministic policy functions in step 1, so that each entry represents the probability of transitioning into a certain state space position \( S (a', z', l', \varepsilon', \theta') \), given an initial state space position \( S (a, z, l, \varepsilon, \theta) \), i.e. \( P(a', z', l'| a, z, l, \varepsilon, \theta) = I(a', z', l'| a, z, l, \varepsilon, \theta) \cdot P(\varepsilon', \theta'| \theta) \). Call this matrix \( \Psi_I \). It is the state space transition agents face in case they survive a period, which occurs with probability \( \pi \).

3. With probability \((1 - \pi)\) agents die, in which case their offspring inherit an asset position \( a \) determined a period before by \( I(a', z', l'| a, z, l, \varepsilon, \theta) \), but receive \((\varepsilon, \theta) = (0, 0)\). This transition is represented by a matrix \( \Psi_0 \) with the same dimension as \( \Psi_I \), for which each row is filled with zeros except for

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\(^{26}\) An equilibrium in the economy is not guaranteed for any parameter configuration.
the position $I_0(a', z', l'|a, 0, 0, 0, 0)$ representing $S(I(a'|a, z, l, \varepsilon, \theta), 0, 0, 0, 0)$. Note that given $z' \in \{0, z, \theta\}$, offspring are necessarily born with $z' = 0$.

4. Finally, the state space transition matrix agents face is $\Psi = \pi \Psi_I + (1 - \pi) \Psi_0$.

**Value Function Iteration Algorithm**

In order to solve for households’ value functions $V_{cb}(a, z, l, \varepsilon)$, $V_z(a, l, \varepsilon, \theta)$, $V_l(a, z, \varepsilon)$, and $V_{lz}(a, \varepsilon, \theta)$, and their corresponding policy functions $z'(a, z, l, \varepsilon, \theta)$, $l'(a, z, l, \varepsilon, \theta)$, $k_i(a, z', l', \varepsilon)$, $n_i(a, z', l', \varepsilon)$, $c_i(a, z', l', \varepsilon)$, and $a_i'(a, z', l', \varepsilon)$, for $i \in \{cb, z, l, lz, w\}$, I design the following algorithm: $V(a, z, l, \varepsilon, \theta)$, $V_n(a, z, l, \varepsilon, \theta)$, $V_{nb}(a, z, l, \varepsilon, \theta)$.

1. Start with a guess for $V_w(a)^j$ and $V(a, z, l, \varepsilon, \theta)^j$, find $E_{\theta} V(a, z, l, \varepsilon', \theta')^j$.

2. Solve for $V_{cb}(a, z, l, \varepsilon)^{j+1}$, $V_z(a, l, \varepsilon, \theta)^{j+1}$, $V_l(a, z, \varepsilon)^{j+1}$, and $V_{lz}(a, \varepsilon, \theta)^{j+1}$, given $V_w(a)^j$ and $E_{\theta} V(a, z, l, \varepsilon', \theta')^j$.

3. Solve for $V_{nb}(a, z, l, \varepsilon, \theta)^{j+1}$ given $V_z(a, l, \varepsilon, \theta)^{j+1}$, $V_l(a, z, \varepsilon)^{j+1}$, and $V_{lz}(a, \varepsilon, \theta)^{j+1}$.

4. Solve for $V_n(a, z, l, \varepsilon, \theta)^{j+1}$ given $V_w(a)^j$ and $V_{nb}(a, z, l, \varepsilon, \theta)^{j+1}$, find $E_{\theta} V_n(a, z, l, \varepsilon', \theta')^{j+1}$.

5. Solve for $V(a, z, l, \varepsilon, \theta)^{j+1}$ given $V_n(a, z, l, \varepsilon, \theta)^{j+1}$ and $V_{cb}(a, z, l, \varepsilon)^{j+1}$.

6. Solve for $V_w(a)^{j+1}$ given $E_{\theta} V_n(a, z, l, \varepsilon', \theta')^{j+1}$ and $V_w(a)^j$.

7. Check for the difference between iterations $j$ and $j + 1$.

8. If small enough, stop.

9. Otherwise, start at 1. using iteration $j + 1$. 
References


