Start-ups, House Prices, and the Jobless Recovery
- Job Market Paper -

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Start-ups and young firms play a crucial role for US job creation: they grow faster and create more net jobs than older, incumbent firms. Yet since 2007 the number of start-ups in the US has declined by over 20%. Young firm’s below-trend job creation can account for almost all of the persistently high unemployment rate. In this paper I claim that this fact is related to the unprecedented fall in the value of real estate. I construct a model that captures the idea that start-ups require external financing, for which real estate is used as collateral. As the value of this collateral falls, start-up costs increase and the number of new firms declines. I calibrate and compute a quantitative competitive industry model with endogenous entry and exit, firm heterogeneity, labor adjustment costs, and aggregate shocks. It generates a jobless recovery and is able to explain over 80% of the increase and persistence in unemployment since the recession.

JEL: E24; E32; E44; G21; J2; L25; L26

Keywords: Employment; Firm Entry; Start-ups; Labor search; Credit Friction; House Prices; Business Cycles; Jobless Recovery

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1 Introduction

In this paper I argue that the ‘jobless recovery’ can be explained through lower job creation by start-ups (firms of age zero). Figure 1 shows the result of a simple counterfactual exercise. Had employment by start-ups and young firms been equal to its pre-crisis trend, the unemployment rate at the end of 2011 would have been as low as 6.5% instead of 8.5%. The figure also shows that changes in job destruction are not driving the jobless recovery. Even with pre-crisis levels of job destruction the unemployment rate would have been almost as high as we observed. There has been a renewed interest in jobless recoveries due to the slow recovery of the US labor market following the Great Recession: Although GDP growth rates have been positive since the third quarter of 2009, employment has been slow to follow. Only in the first quarter of 2011 did the unemployment rate fall below its end-of-recession level. In the first quarter of 2013, the unemployment rate stood at 7.7%, compared to the 4.8% unemployment rate in the last quarter prior to the recession (Q4 2007). Employment relative to the working age population in mid-2013 was lower than at the height of the financial crisis.

Relatively little is known about who creates - and who destroys - jobs. Every year several hundred thousand new firms are created, providing millions of new jobs. While not all of those firms succeed, those that do remain strong engines of job growth over the coming years. This highlights the importance of studying the labor market’s extraordinary dynamics, resulting from persistent and large heterogeneity across firms: While some firms expand, others contract, firms are born and firms die. At the heart of these dynamics lie start-ups and young firms. Successful start-ups become vibrant young firms which make up the lion’s share of net job creation. A consequence of the prominent role of start-ups is that whenever the inflow of new firms into the economy is interrupted this has adverse effects on job creation. The result can then be a jobless recovery. I will argue that the ‘credit crunch’ and particularly the fall in house prices associated to the recent economic crisis has created such an event. Figure 2 shows the strong correlation between the number of start-ups and the house prices index for all US-recession episodes since 1980. The figure plots the evolution of the HPI and the number of new firms throughout the recession periods. The goal of this paper is to quantitatively assess the importance of the decline in the value of real estate - a major funding vehicle for business formation - as a key reason behind the low number new firms and persistently high unemployment.

To this end I develop a quantitative model of heterogeneous firms that operate in a frictional labor market. Firms must post vacancies that are filled with endogenous probability. Wages are determined through bargaining between workers and firms. Unproductive firms may exit the economy, while new firms can enter. During recessions

\[\text{1Throughout this paper I use the NBER recession dates for my business cycle classifications.}\]
\[\text{2In an important empirical contribution Haltiwanger et al. (2010) show that by controlling for firm age there remains no systematic relationship between firm size and growth.}\]
\[\text{3Over the last 35 years the average number of gross jobs created was around 16 million per year, while 14.4 million jobs per year were destroyed. This respectively corresponds to 17% and 15% of the entire labor force. In other words, over 30% of the labor force is reallocated in a given year.}\]
Figure 1: The actual unemployment rate is plotted in as the blue solid line. The remaining lines show the counterfactual unemployment rates for the following experiments: The green dashed line labeled 'Young Trend' shows unemployment if gross job creation by young firms (age 5 or below) had been equal to its pre-2006 HP-trend. For the red dashdotted line 'Trend JD' I set gross job destruction (JD) after 2009 equal to its pre-2006 HP-trend. Source: Census, BLS, own computations.
firms shed workers and post fewer vacancies, generating a Beveridge-curve relationship between unemployment and vacancies. All agents own a fixed stock of real estate. Entering firms require a one-period loan to finance start-up costs. They obtain this loan from a bank and use their real estate as collateral. Because new entrepreneurs may strategically default, the risk neutral bank efficiently prices interest rates by charging a default premium to compensate for expected losses. In this way changes in the value of collateral feed back to the entry costs of new firms. Adverse conditions on the housing market can constrain the number of start-ups that enter during a recovery. This link between house prices and firm entry can explain why job creation by start-ups decreased before the beginning of the recession in 2007 - a fact that previous models were unable to address. My model generates jobless recoveries if low collateral values prevent some start-ups from entering. Since start-ups have hiring rates over-proportional to their share of output, the link between entry and real estate breaks the strong co-movement in output and unemployment observed in otherwise similar models. Additional propagation comes through labor adjustment costs which are chosen to match key moments of the employment change distribution.

Standard models of the labor market are unable to generate jobless recoveries and sufficient volatility in unemployment and vacancies. The RBC model cannot generate jobless recoveries because shocks are only to aggregate TFP. After a negative shock the reversion to the unconditional mean of TFP increases the marginal benefit of all factor inputs. The Mortensen and Pissarides (1994) search model suffers from the same shortcomings. Furthermore, as pointed out by Shimer (2005) it is unable to generate the volatility in unemployment and vacancies we observe in the data. The competitive industry model (Hoppenhayn (1992) and Hoppenhayn and Rogerson (1993), henceforth (HR)) introduces entry of new firms and therefore appears as a natural starting point for studying start-ups. The HR setup is a frictionless model in which a market-clearing wage is found via the free-entry condition. The general equilibrium effects induced by this condition are quite powerful in this environment, virtually eliminating any selection effects that could result from the composition of entering and exiting firms (see e.g. Lee and Mukoyama (2012)). I therefore depart from the basic HR model in the following respects. First, I add aggregate shocks to the model since I am interested in the business cycle implications of the model. Second, I add a search-and-matching framework where firms fill vacancies with endogenous probability. This allows me to study the implications of the model for unemployment and vacancies and creates a link between new and incumbent firms through labor market tightness. Third, labor adjustment costs are added to the model in order to match the employment change distribution. Finally, I assume that start-ups must borrow the costs of entry. Potential entrepreneurs use real estate to collateralize a fraction of this loan. As the value of housing falls, the interest rate new entrepreneurs pay on the loan increases. This raises their costs of entry and deters some entrepreneurs from entering. Making entry a function of house prices has several advantages. First, there is empirical evidence on the sensitivity of young firms’ hiring behavior with respect to conditions on the credit market. Second, a model with a standard free-entry condition which is not a function of the house price generates entry
Figure 2: Source: BDS and Cash Shiller Home Price Index. HP-filtered. The x-axis shows years/quarters since the respective pre-recession quarter (based on NBER classification).
rates exhibiting excess volatility with respect to the data. The additional dependence on a slow-moving process such as the value of collateral is successful in generating a realistic volatility of entry. Since the focus of this paper lies on entry, achieving realistic entry rates is crucial. An important assumption of my model is that only new firms need to borrow their overhead costs. This is motivated by results of the Survey of Consumer Finances (SCF) which shows that real estate and other personal resources are an important source of business formation, but play a much smaller role for expansions of existing businesses.

My model is then calibrated to match certain cross-sectional data moments, such as the unemployment-vacancy ratio and the firm age- and employment change distributions. I estimate firm-level labor adjustment costs via a simulated method of moments (SMM) approach. The calibrated model can replicate the average life cycle of firms, the positive correlation between productivity and age, and the negative correlation between employment growth and size observed in the data. I find that the model with house prices affecting credit conditions significantly outperforms alternative specifications, particularly because of its ability to generate jobless recoveries. I perform various policy experiments showing that around 80% of the increase and persistence in unemployment since the end of 2006 can be explained by a model with aggregate TFP shocks and changes in the house price index.

This paper contributes to the literature on the role of start-ups over the business cycle, the impact of financial conditions on the real economy, and jobless recoveries. At the basis of the model lies a heterogeneous-firm framework as in HR, to which I propose the extensions discussed above. An important one is the combination of heterogeneous firms with a standard Mortensen and Pissarides (1994) search-and-matching structure. Other papers that have extended the search framework to multi-worker firms include Cooper et al. (2007), Kaas and Kircher (2011), Elsby and Michaels (2013), and Acemoglu and Hawkins (2013). In Cooper et al. (2007) labor adjustment costs are estimated in a heterogeneous firm model with search frictions but their framework does not allow for entry and exit. Kaas and Kircher (2011) augment a simplified HR framework with competitive search. Their model can generate sluggish movements of unemployment following a boom but they rely crucially on a time-varying entry cost and the convexity of the recruiting cost function. Furthermore, firms in Kaas and Kircher (2011) are ex-ante heterogeneous, while in my paper they are ex-ante homogeneous and productivity evolves over time. Elsby and Michaels (2013) introduce the Stole and Zwiebel (1996) bargaining framework to the multi-worker firm environment but do not study entry.

A second important extension to the HR model is the financing friction for new businesses. The paper which is most closely related in this respect is Siemer (2013). Siemer develops a heterogeneous firm model with entry and exit based on Khan and Thomas (2011) in which all firms borrow through optimal lending contracts with financial intermediaries. A financial shock overproportionally increases borrowing costs for small and young firms and reduces entry. The main difference of my model is that it generates jobless recoveries, i.e. underproportional employment growth during recoveries. While in Siemer’s model the financial shock produces a ‘missing generation’ of entrants,
I model a link between the hiring conditions of incumbents and entrants through the endogenous labor market tightness $\theta$. This implies that during a recovery firms benefit from an initially low $\theta$, which increases hiring and lets entry rates return relatively quickly to their pre-recession value. In Siemer’s model the mass of entrants is a 1:1 mapping of the financial shock, implying that entry levels jump back to their unconditional mean once the financial shock has passed. In the data, however, we observe that entry rates continued to be at historically low levels even after financial conditions in the US had returned to “normal”, as measured e.g. by various financial stress indeces. In my setup I model entry costs as a function of the value of house prices (collateral). This helps me to explain why entry rates decreased prior to the recent recession, why they continue to be low relative to their pre-recession trend, and why job creation by incumbent firms recovered before job creation by start-ups.4 Other related work focusing on start-ups includes Coles and Kelishomi (2011), Clementi and Palazzo (2010), and Lee and Mukoyama (2012). Coles and Kelishomi (2011) study single-worker firms with a two-stage entry process. They show that thus replacing the free entry condition in the standard matching framework significantly enhances the aggregate properties of the model. Lee and Mukoyama (2012) study the cyclical properties of entrants vs. exiters but rely on an entry cost parameter which is exogenously pro-cyclical. Clementi and Palazzo (2010) replace the free entry condition of a standard competitive industry model with a fixed mass of potential entrants and show that entry and exit can propagate the effects of aggregate shocks.5 Using a standard free-entry condition Hawkins (2011) finds the opposite result. However, he overstates the cyclicality of entry. To the best of my knowledge the previous literature on heterogeneous firms has not been successful in finding an entry specification that allows for cyclicity in start-up job creation without misspecifying its cyclicality (see e.g. Clementi and Palazzo (2010), Hawkins (2011), Lee and Mukoyama (2012), Berger (2012)). The connection between entry costs and the value of real estate helps to smooth entry rates considerably over the business cycle and generates a realistic degree of fluctuations. Papers which study the link between entrepreneurship and housing collateral empirically are Fort et al. (2013) and Schmalz et al. (2013). Fort et al. (2013) estimate a VAR and conclude that the collapse in housing prices accounts for a significant part of the large decline in job creation by young firms during the recent recession. Liu et al. (2013a) also find a significant effect of house prices on unemployment. Schmalz et al. (2013) empirically link house price shocks to entrepreneurial activity and employment in new firms.

Following the seminal publications by Kiyotaki and Moore (1997) and Bernanke et al. (1999) there now exists a vast theoretical literature on the linkages between the financial sector and the real economy. The impact of credit constraints on macroeconomic outcomes has been studied both in the context of search-and-matching and

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4House Price Indeces such as the All-Transactions House Price Index for the United States by the FHFA clearly shows a decline prior to the end of 2007. The HPI in Q1 2013 stood at 86% of its Q4 2007 value.

5Sedlacek (2011) uses a reduced form specification of the free-entry condition to obtain realistic entry dynamics and reproduce key facts of US firm dynamics.
heterogeneous firm models. A large number of theoretical and empirical papers has found a sizeable effect of credit conditions on the real economy during the recent recession (see e.g. Jermann and Quadrini (2012), Gilchrist and Zakrajšek (2012), and Chodorow-Reich (2013)), but these models do not study entry and exit. In my model start-ups need to borrow in order to pay the entry costs, making firm entry a function of credit conditions. A similar mechanism is modeled in Liu et al. (2013b), where land prices enter a firm’s collateral constraint. As in Chaney et al. (2012) they find that variations in the collateral value have significant effects on investment.

The jobless recovery has been the topic of Galí et al. (2012), Drautzburg (2013), Bachmann (2011), and Berger (2012). In Berger’s model firms lay off unproductive workers during recessions. Differently from my paper, the focus of Berger (2012) is on the intensive margin of job creation. While my mechanism is otherwise complementary to Berger’s, I show that introducing financing costs for entrants can not only generate jobless recoveries, it also significantly contributes to limiting the volatility of the entry rate. Drautzburg (2013) models an occupational choice problem and estimates that approximately one third of the change in start-up job creation following the recent recession can be attributed to higher risk. Bachmann (2011) explains the jobless recovery through a combination of adjustment costs which generate a jobless recovery after a short and shallow recession. For more severe recession episodes such as the 2008/09 case the model cannot reproduce the observed dynamics, however. Galí et al. (2012) argue that the 2008/09 downturn only produced a quantitative change in the relation between GDP and employment. However, by comparing the trajectories of GDP, unemployment, job destruction, the house price index (HPI), and start-up job creation for different recession episodes it becomes clear that series differ substantially compared to the other post-1980 recessions. I show those series in Figures 16 and 17 in Appendix A.1. In particular the link between the HPI and start-up activity (Figure 2) stands out as a particular feature of the 2009/09 recession, as the next section shows.

2 Facts

This section presents some stylized facts about job destruction and job creation, enterprise dynamics, firm survival, and the link between credit conditions and start-ups. Throughout this paper I will refer to firms of age zero as start-ups or entrants, while firms aged one to five years will be referred to as young firms. All firms are employer firms. A start-up is defined as a new firm, not as a new establishment. Unless otherwise noted the data comes from the US Census’ Business Dynamics Statistics (BDS) database. Details regarding all the data used in this paper can be found in the Data Appendix. Robustness checks and additional information about the stylized facts can also be found in the appendix.

Credit constraints in a standard search-and-matching framework were studied by Dromel et al. (2010) and Petrosky-Nadeau (2013), who find that the presence of constraints can impact both the level and the persistence of unemployment. Financial constraints have first been introduced into heterogeneous firm models by Midrigan and Xu (2010), Khan and Thomas (2011) and Siemer (2013).
2.1 Firm Dynamics

The 2008/09 recession episode produced the largest drop in employment since the beginning of the Census’ BDS database in 1977. This was the result of both an increase in gross job destruction and a decrease in gross job creation. I will argue below that persistently low job creation rates are what made the recovery ‘jobless’. In 2008/09 fewer jobs were destroyed than during the 2001 recession. Most of it took place on the intensive margin, that is through downsizings of existing firms. Firm deaths only contributed to around 18% of all gross job destruction since 2008. On the other hand, the years 2008 and 2009 marked the largest decreases in gross job creation in the entire Census data. This is summarized in the first Stylized Fact. It is robust to employing alternative data sources as I show in the appendix.

Stylized Fact 1: The Great Recession was mainly a crisis of low job creation.

Start-ups play a crucial role for the US economy. The main reason for this is their contribution to job creation. While start-ups create around three million new jobs each year the net contribution of incumbent firms is typically negative. The recent recession has left its mark: While net job creation by incumbent firms quickly recovered since the end of the recession, job creation by start-ups in 2011 was at its lowest point since the beginning of the Census BDS series in 1977. At the same time the average size of a start-up has virtually remained unchanged at around 6 employees. This suggests an important extensive margin effect: fewer entrepreneurs start a business. The drop in start-up hiring stands out as a main factor for low job creation since 2008. While gross job creation remained low for all firm ages after the recession trough, the largest decreases in gross job creation occurred among start-ups, followed by the youngest firms. Specifically, in 2011 start-ups created about 700'000 fewer new jobs than in 2007. This is a feature of the ‘Great Recession’ we do not observe to this extent for the other recessions covered by the BDS data. Figure 3 compares changes in absolute gross job creation by firm age relative to the respective pre-recession year across different recession episodes. During the 1980 recessions start-up employment initially increased. During the 2001 recession it decreased but quickly rebounded. The 2008/09 recession was different: Not only was there an unprecedented fall in job creation by start-ups and young firms, this

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7 This holds both in absolute numbers and for the HP-filtered cyclical component. See Figure 14 in Appendix 1 for details.

8 The average since 1977 was 17.66%. A similar point can be made for establishment deaths. The fraction of gross JD from establishment deaths since 2008 was 30.53%, the average since 1977 was 35.38%.

9 This result holds across regions and sectors, as is also discussed in Haltiwanger et al. (2012). They also note, however, that states that were hit hardest by the financial crisis suffered larger decreases in startup employment, a point that I will take up further below.

10 Data for all available age groups is shown. Choosing different base years leaves results virtually unchanged. Furthermore, qualitatively identical results can be obtained by plotting job creation rates or the cohort’s fraction of total job creation (available upon request).
Figure 3: The y-axis shows changes in gross job creation relative to base years 1979, 1989, 1999, and 2007. For age group bins averages are shown. Source: BDS.
decline persisted even after the official end of the recession. Only the recession in the early 1990s bears similarity to the ‘Great Recession’ in the sense that hiring by young firms decreased and remained low until several years after the recession trough. The magnitude of this effect is smaller and the relative effect on start-ups is weaker than in 2008/09, however. Interestingly, apart from the recent downturn the 1990/91 episode was the only recession where house prices were below trend for a prolonged period of time as Figure 17 in Appendix A.1 shows. These stylized facts summarize the above results:

**Stylized Fact 2:** The decrease in job creation was largely due to lower job creation by start-ups and young firms.

Start-ups have employed around 3 million jobs per year since 1977. Coles and Kelishomi (2011) have pointed out, that this number has been relatively inelastic over the cycle.\(^\text{11}\) Using the most recent data the correlation between (the cyclical components of) GDP and job creation by start-ups is 0.356. Job creation by incumbent firms, on the other hand, has a higher procyclicality (0.756).\(^\text{12}\)

**Stylized Fact 3:** Employment in incumbent firms is more strongly procyclical than employment in start-ups.

However, while start-up job creation is less correlated to fluctuations in GDP, it nevertheless shows more volatility over time than gross job creation by established firms. The standard deviation of the cyclical component of job creation over its trend is about 40% larger for entrants than for incumbent firms (0.10 vs. 0.07).

**Stylized Fact 4:** Job creation by start-ups is more volatile than job creation by incumbents.

I divide firms into four size categories, 1-19, 20-99, 100-499, and 500+ employees. The size and age distribution of firms and establishments can be seen in Tables 8 and 9 in the Appendix. It is noteworthy that while over 95% of firms have less than 100 employees, it is large firms that employ almost half of the workforce. The average firm size is 21.43 workers. The distribution of start-ups shows that the vast majority of start-ups (98.1%) are small firms with less than 20 employees.\(^\text{13}\) The age distribution of firms shows that start-ups make up about 11% of all firms. This is an important statistic that my model is going to match. Start-ups and young firms show overproportional employment growth:

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\(^\text{11}\) See Figure 13 in the Appendix for an updated version of a graph used in Coles and Kelishomi (2011).

\(^\text{12}\) These numbers change only marginally by considering alternative subsets of the data. For example the correlation between GDP and job creation by start-ups between 1982-2007 is 0.33 and for gross job creation by incumbents it is 0.72. The correlation between GDP and employment in incumbent firms is 0.5002.

\(^\text{13}\) Very large start-ups are rare and should be treated with caution, as practise shows they are often temporary entities that get folded into other firms later on.
Start-ups employ around 3% of the labor force, yet contribute 18.7% of total job creation. On the other hand, young firms show higher-than-average rates of job destruction. A significant fraction of which is the result of firm exit. These ‘up or out dynamics’ were first described by Haltiwanger et al. (2010). Conditional on survival, young firms grow considerably faster than their mature counterparts. As Figure 4 shows, in the BDS data around 50% of gross job destruction is the result of firm exit during the first three years of a firm’s life. On the other hand, for firms older than 20 years less than 15% of all gross job destruction is the result of firm exit. The total firm exit rate is 8.8% per year.

2.2 Housing and Credit supply

In the wake of the financial crisis there have been numerous initiatives to monitor credit conditions for small businesses. This section will show that after 2007 start-ups have been facing higher costs of obtaining credit. This is important because besides personal wealth, banks are the most important source of funding for start-ups. I present evidence that the sharp drop in the value of real estate, which is a predominant source of collateral for business formation, is connected to the ongoing difficulty for start-ups to obtain financing. State-level regression results indicate that changes in the value of real estate have a negative effect on the number of start-ups, even when controlling for macroeconomic conditions and demand effects.

Start-ups and young firms rely heavily on external liquidity but they face a different initial lending environment and more challenges than mature firms in obtaining credit. Most of this is due to the fact that they do not have an established credit record and typically face restrictions in their access to commercial bonds or other means of financing available to older firms. Why would banks not lend (enough) to start-ups? One reason is the general deterioration in the lending environment of firms.


15 There was an increase in the costs of external finance during the last recession. Both the number and the dollar amount of approved C&I loans fell by around 20% over the course of the crisis. The drop in number and dollar amount of small loans (under $1 Million) was particularly severe. At the end of Q1 2013 the volume of small loans was only 84.84% of its pre-recession value, and their share of all...
however, was the decline of the value of real estate and household net worth, which acts as collateral for initial loans. According to Avery et al. (1998) loans having a personal guarantee account for 55.5% of small business credit dollars. Results from the 2009 and 2010 Surveys of Consumer Finances indicate that personal savings or assets were used as collateral to initiate more than 70% of new businesses, making personal resources the most important funding source of entrepreneurs (Board (2011)). Also Mann (1998), Moon (2009), Dennis Jr. (2010), and Robb and Robinson (2012) highlight the importance of collateralized loans for small business finance. This collateral takes the form of personal assets - mostly real estate. The decrease in the value of real estate has made pledging personal commitments more difficult. Figure 5 shows that net mortgage equity extraction dropped from around 8% of disposable personal income in 2006 to around -6% at the end of 2010, the lowest value on record.\footnote{Thanks to Bill McBride for providing me with his estimates.} Although not all home equity is used for start-up financing, this ‘deleveraging’ by households which accompanied the dramatic decline in household net worth implies that the amount of equity available for start-up equity has been severely curtailed.\footnote{Further evidence comes from FDIC data on used and unused home equity lines which I produce in Figure 21 in Appendix A.1. While unused commitments typically exceed outstanding home equity loans, the 2008/09 recession generated an earlier and steeper decline in unused equity lines. While part of this decline reflects drawdowns of existing lines a large portion represents a reduction of the credit supply by banks, as Bassett et al. (2011) argue in a similar context.}

\begin{figure}
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\includegraphics[width=\textwidth]{figure4.png}
\caption{Gross Job Creation and Job Destruction from Exit and Downsizing by Age. Source: Census 1977-2011. Own computations.}
\end{figure}

C&I loans fell from a pre-crisis average of 32.33\% to 22.39\% in Q1 2013. This decrease in the number of loans was accompanied by an increase in the interest rate spread between smaller and riskier loans and the federal funds rate. Figures 18 and 19 in Appendix A.1 show the spread by loan size and risk.
2.2.1 State-level regressions: The impact of HPI

I test the hypothesis of a positive link between the value of real estate and the labor market by combining state-level data on house prices and establishment birth. Table 1 shows various state-level regressions. I use establishment births from the BLS Business Employment Dynamics (BDM) as the dependent variable (BIRTH). Although this is establishment-level and not firm-level data I use this dataset because of its quarterly frequency. The data is available from Q2 1993-Q2 2013. The main explanatory variable is the state-level HPI, which comes from the Federal Housing Finance Agency (FHFA). As additional controls I use two alternative cyclical indicators: the state-level unemployment rate (UE) and state-level personal income (PI). I use personal income as a cyclical indicator because state-level GDP is only available on an annual basis. All variables have been HP-filtered. I am controlling for year- and state-effects and use cluster-robust standard errors in all regressions. Summary statistics for all variables can be found in Appendix A.0. The first column in Table 1 shows a simple regression of BIRTH on HPI. The HPI is positively correlated with the number of new establishments at the state-level. This relationship is robust to controlling for cyclical indicators: Columns (2), (3), and (4) control for personal income and unemployment, which are both significant at the

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I removed the states AK, DC, DE, HI, ND, SD, VT, WV, and WY from the analysis because of an FHFA warning. The HPI from those states have been derived from fewer than 15,000 transactions over the last ten years. Using a fixed-effect estimator leaves the results virtually unchanged. The same is true for using the variables in levels or logs instead of the cyclical component of the HP-filtered data. Results are available upon request.
5%-level and have the expected sign. Column (4) controls for both UE and PI jointly. Here, the state-level unemployment rate is no longer significant at the 10%-level. The last stylized fact summarizes the above results:

<table>
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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
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<tbody>
<tr>
<td>HPI</td>
<td>11.9366*</td>
<td>9.4346*</td>
<td>10.2039*</td>
<td>8.7394*</td>
</tr>
<tr>
<td></td>
<td>(2.32)</td>
<td>(2.36)</td>
<td>(2.04)</td>
<td>(2.14)</td>
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<tr>
<td>PI</td>
<td>0.0153***</td>
<td>0.0149***</td>
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<tr>
<td></td>
<td>(13.98)</td>
<td></td>
<td></td>
<td>(14.67)</td>
</tr>
<tr>
<td>UE</td>
<td>-87.2835*</td>
<td>-38.4972</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(-2.58)</td>
<td></td>
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<td>(-1.13)</td>
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<tr>
<td>cons</td>
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<td>96.9491***</td>
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<td>(-1.87)</td>
<td>(5.27)</td>
<td>(-0.62)</td>
<td>(-0.69)</td>
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<td>N</td>
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<tr>
<td>r²</td>
<td>0.0567</td>
<td>0.0775</td>
<td>0.0590</td>
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Dependent variable: Establishment Birth. * t statistics in parentheses.
All regression include year- and state dummies.
Source: BLS, FHFA, BEA.
All series are quarterly and have been HP-filtered with \( \lambda = 1600 \).

Stylized Fact 5: During the 2008/09 recession the financing environment for start-ups deteriorated. Credit supply by commercial banks decreased, and the value of real estate - widely used as collateral - fell.

This section has produced five stylized facts about job creation and destruction. One, high unemployment is mainly driven by low job creation figures. Two, a large part of the decrease in job creation was due to the behavior of the youngest firms. Start-ups constitute the single largest contributor to net job creation. It is more volatile but less cyclical than job creation by incumbents. Job creation by start-ups has taken a prolonged dive since the onset of the recent crisis. Five, there was a decrease in the availability of external finance for start-ups. Credit supply by commercial banks dropped during the 2008/09 recession, partly because declining property values diminished the value of collateral. I now present my model which figures a collateral channel and uses exogenous variation in the value of collateral to replicate the Stylized Facts presented above.
3 The Model

The economy consists of a fixed mass of workers and entrepreneurs. There is a competitive bank which provides start-up financing and is jointly owned by all agents. Each worker and each entrepreneur owns one unit of housing $h$, the price of which is $q^h$. Housing provides utility to all agents and can serve as collateral when entrepreneurs finance start-up loans. Workers can supply labor and consume their income, either from wages or home production. Entrepreneurs own the production process which utilizes labor to generate a single consumption good. Output is a function of labor and two types of profitability: idiosyncratic and aggregate. Shocks to profitability can be interpreted as changes in productivity or demand. Both types of profitability evolve persistently over time. Time is discrete and a period refers to one quarter.

The labor market is frictional. To hire unemployed workers firms must post vacancies $v$ which are filled with endogenous probability. Labor is perfectly divisible. Following the standard search and matching literature a matching function captures those frictions. It is denoted as $m(U, V) = \mu U^\gamma V^{1-\gamma}$. Its inputs are the unemployment rate $U$ and the vacancy rate $V$. Vacancies posted by firms are filled with probability $H(\theta) = m/V$. An unemployed worker finds a job with probability $\phi(U, V) = m/U$. The ratio $\theta \equiv V/U$ is a sufficient labor market statistic to compute the vacancy-filling and job-finding rates in this economy. Employed workers may lose their job if the entrepreneur they are matched with exits or decides to reduce employment in his production site. The worker takes both the job-finding rate and the job-destruction rate as exogenous. The workers’ compensation for their labor input is specified through a bargaining process between the entrepreneur and the worker, where the entrepreneur has all the bargaining power.$^{20}$

A fixed cost to production guarantees that firms exit when they receive a sufficiently low profitability draw. New firms that enter the economy must pay start-up cost $c_e$. To finance $c_e$, new firms obtain an intra-period loan from the bank. A fraction of the loan can be secured by collateral, for which agents use their real estate $h$. Changes in the value of collateral $q^h$ lead to variations in the effective cost of entry $\tilde{c}_e$ and hence in the number of firms that enter the economy. Shocks to $q^h$ are exogenous. I estimate a process of $q^h$ and its cross-correlation coefficient with aggregate TFP from the US data.

The timing of events in my model is based on the setup in Hopenhayn and Rogerson (1993): Prior to the realizations of new aggregate and idiosyncratic shocks, firms decide whether to continue operating or exit. For new entrants exiting implies that intra-period loans are defaulted on. Then the aggregate state realizes and new firms enter the economy without knowing their idiosyncratic productivity draw. The idiosyncratic shocks realize and all firms decide on their desired employment level. Bargaining takes place between workers and entrepreneurs, after which production occurs, and compensations are paid. The model is now explained in more detail.

$^{20}$This is following Cooper et al. (2007).
3.1 Workers

Workers can either be employed or unemployed. They derive utility $\varphi(h)$ from housing independently of their employment status. When they are unemployed they receive an outside option $b(a)$, which can vary with the aggregate state $a$. This outside option reflects the returns to home production. With probability $\phi(U, V)$ an unemployed worker is able to find a job, thus becoming employed next period. We can write the value of being unemployed as

$$W^u(a, h) = Z(b(a) + \pi^b) + \varphi(h) + \beta E_{a'|a}[\phi(U, V)W^e(a', h) + (1 - \phi(U, V))W^u(a', h)], \quad (1)$$

where $Z(\cdot)$ describes the worker’s utility from consumption and redistributed profits made by the bank $\pi^b$. The term $\varphi(h)$ describes utility from housing. The discount factor is $\beta$, and $\phi(\cdot)$ is the job finding rate which depends on the current unemployment rate $U$ as well as the number of vacancies $V$. The utility function $Z(\cdot)$ is assumed to be strictly increasing and concave. For simplicity I assume that there is no disutility from labor. The expectations operator in (1) is taken over the future values of unemployment and unemployment.

By contrast, when a worker is currently employed he receives a compensation $\omega$ as defined by the state-contingent contract. With (endogenous) probability $\delta$ the worker loses his job and receives the value of unemployment $W^u(a', h)$ next period. With the remaining probability he continues to be employed.

$$W^e(a, h) = Z(\omega(a) + \pi^b) + \varphi(h) + \beta E_{a'|a}[(1 - \delta)W^e(a', h) + \delta W^u(a', h)] \quad (2)$$

3.2 Entrepreneurs

Entrepreneurs own the production process. Income from firms constitutes the entrepreneurs’ only source of income and they consume all profits within the period.\(^{21}\)

Entrepreneurs are assumed to be risk-neutral. They produce using a production technology $F(e)$, where $e$ represents the number of workers. The production function has the properties $F_\epsilon(e) > 0$ and $F_{ee}(e) < 0$, meaning it exhibits decreasing returns to labor, which might arise from fixed factors such as capital or materials, from imperfect substitutability for consumers of the goods produced by different firms or from managerial span-of-control as in Lucas (1978). At the end of a period entrepreneurs decide whether to continue operation or exit, based on their expectation of future shocks.\(^{22}\) At the same time new entrepreneurs enter the economy. After the realization of uncertainty, entrepreneurs make hiring and firing decisions. A fraction $\chi$ of the workforce is separated exogenously (quits) each period. Given the state vector the entrepreneurs and

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\(^{21}\)See e.g. evidence in Moskowitz and Vissing-Jorgensen (2002) who show that entrepreneurial risk is not diversified and that dividends from the firm are the only source of income for owners.

\(^{22}\)As in Hopenhayn and Rogerson (1993), since there is no additional information gained between periods, the exit decision is taken at the end of a period. This is mainly a computational convenience. Since I have one-period loans in my model the end-of-period exit decision is necessary to obtain default in the same period the loan was issued.
the workers bargain over a compensation \( \omega(a, \varepsilon, e) \). The firm’s state vector at time \( t \) is \( (a, \varepsilon, e, \theta) \), where \( \theta \) reflects labor market tightness, as explained in more detail below. The profit function is given by

\[
\pi(a, \varepsilon, e) = a\varepsilon F(e) - e\omega(a, \varepsilon, e) - \Gamma - \mathcal{C}.
\]  

Output is affected by two multiplicative profitability shocks \( a, \varepsilon \). While the former is an aggregate shock, the latter affects only idiosyncratic profitability. The term \( \Gamma \) is a fixed cost of operation to induce exit in low profitability states.\(^{23}\) \( \mathcal{C} \) defines a cost function given by

\[
\mathcal{C} = -F_v v^1 + c_v v^2 - F_f f^1 + c_f f^2.
\]

The indicator function \( 1^+ \) is equal to one if the firm is hiring and equal to \( 1^- \) if the firm is firing. The number of vacancies posted is \( v \) and the amount of fired workers is \( f \). There are two types of costs connected to hiring. One is a fixed cost \( F_v \). The other is a quadratic cost \( c_v \). The respective cost associated to firing are given by \( F_f \) and \( c_f \).

The Employment decision A firm that is operation when idiosyncratic profitabilities are realized is called an incumbent, or ‘continuing’ firm. This firm employed \( e_{-1} \) workers last period and faces a shock \( x \), where \( x = (a, \varepsilon) \) consists of the aggregate and idiosyncratic productivity realization. Also part of the firm’s state vector is the aggregate labor market tightness \( \theta \). This determines how effective the firm can hire new workers. In order to compute the expected value of \( \theta' \) firms require knowledge about \( \Gamma \), the joint distribution over \( (n, \varepsilon) \) and its law of motion. This is described in detail below. The state vector is summarized by \( s = (x, e_{-1}; \theta) \). The value function for a continuing firm is denoted \( Q^c(s) \). Because there are fixed costs to variations in employment, the entrepreneur faces a discrete choice problem within the period. He can decide between posting vacancies, remaining inactive, and firing workers. Vacancies must be reposted each period. The value \( Q^c(s) \) is thus given by the maximum of the values of posting vacancies, firing, and inaction.

\[
Q^c(s) = \max\{Q^v(s), Q^n(s), Q^f(s)\}
\]

The three Bellman equations will now be described in turn. Because the entrepreneur can choose to exit at the end of the period, the continuation value in each case is given by the maximum of the expected value of continuing and exiting. The value of exit is \( Q^x(e) \) and will be described below. The value of posting vacancies \( Q^v \) is given by

\[
Q^v(s) = \max_v \pi(a, \varepsilon, e) + \beta E_x \max\{Q^v(x', e'; \theta'), Q^x(e)\},
\]

and the evolution of employment is given by the number of quits and the vacancy filling rate \( H(\theta) \)

\[
e = e_{-1}(1 - \chi) + H(\theta)v,
\]

\(^{23}\)The entrepreneur’s problem is stated net of housing utility and net of redistributed bank profits because in the baseline model these values do not affect incumbent entrepreneurs’ optimal decision. In Appendix A.4 I outline a model where the price of collateral also enters the incumbent entrepreneur’s problem.
The value of firing workers is

\[ Q^f(s) = \max_f \pi(a, \varepsilon, e_{-1}(1 - \chi) - f) + \beta E_x \max\{Q^c(x', e'; \theta'), Q^x(e)\}. \] (6)

Lastly, the value of inaction is given by

\[ Q^h(s) = \pi(a, \varepsilon, e_{-1}(1 - \chi)) + \beta E_x \max\{Q^c(x', e'; \theta'), Q^x(e)\}. \] (7)

Here \( E_x \) denotes the expectation conditional on the current value of \( x \). The maximum operator nested on the right-hand side of all three Bellman equations reflects the fact that a firm can make a decision about exiting before the next period. Since this is decided before the realization of new information the choice can be made in the current period. Conditional on this period’s employment choice the entrepreneur must evaluate the expected value of being active next period, given by \( E_x [Q^c(x', e'; \theta')] \), and compare this to the present discounted value of exiting, given by \( Q^x(e) \). This value is defined below. The policy function for employment will be denoted \( \phi^e(s) \). The employment policy function will be characterized by different cutoff values in the \((x, e_{-1})\) space. For a given \( e_{-1} \) there exists a region of inaction over the values of the idiosyncratic shock due to the presence of fixed costs. An example is given in Figure 23 in Appendix A.2. For values higher than a cutoff profitability, the firm hires new workers, while for values below a lower cutoff profitability workers are shed. Note that changes in employment do not take ‘time-to-build’ because I want to rule this out as a driver of jobless recoveries.

**The Wage Contract** We can now define the optimal wage contract between workers and entrepreneurs. The contract specifies \( w(S) \), the compensation for a worker in a firm with state \( S \), where \( S = (a, \varepsilon, e, \theta) \) is the firm’s state vector. A simplifying assumption is that entrepreneurs are able to make take-it-or-leave-it offers, i.e. the workers have zero bargaining power.\(^{24}\) The firm thus chooses the wage subject to the worker’s participation constraint. This is given by \( W^f(a) \geq W^u(a) \). It says that the employed workers’ outside option must be at least as large as the remuneration offered by the contract. In equilibrium the participation constraint will hold with equality, implying \( Z(w(S)) + \varphi(h) = Z(b(a)) + \varphi(h) \), or \( w(a) = b(a) \).\(^{25}\) This is a simple way in which the model generates movements in the wage without the complexity of adding aggregate labor demand as an additional state variable. I assume the following functional form for the outside option \( b(a) = b_0a^{b_1} \). The parameter \( b_0 \) is part of the model calibration, while \( b_1 \) is estimated from the data. Importantly, \( b_1 < 1 \).

\(^{24}\)As in Cooper et al. (2007) and many other papers this assumption is employed to facilitate the computation of the optimal contract. See Elsby and Michaels (2013) and Acemoglu and Hawkins (2013) for a different approach based on Stole and Zwiebel (1996). Kaas and Kircher (2011) introduce a competitive search procedure. This simplification does not change my results qualitatively as long as the elasticity of the bargained wage with respect to aggregate profitability is not larger than 1, for which to the best of my knowledge no evidence exists. In Appendix A.4 I show some intuition for a model with an alternative bargaining rule based on Stole and Zwiebel (1996).

\(^{25}\)Formally, the profit maximizing contract results from the following optimization problem: \( \hat{\pi}(a, \varepsilon, e) = \max_{w(S)} axF(e) - ew(S) \) subject to \( W^u(a) \geq W^u(a) \).
The Exit Decision  At the end of a period, before any new information about the exogenous shocks arrives, an incumbent entrepreneur has to decide whether he wants to continue operating or exit next period. The exit decision is thus based on the expected future value of the firm, which ensures that a firm will never post vacancies and exit in the same period. If the entrepreneur decides to exit, he will reduce the amount of workers to zero (paying the firing costs for the remaining workers) and generate zero revenue. However, he avoids paying the fixed cost of operation. Any outstanding debt obligations are defaulted on. The value of exiting is given by

$$Q^x(e) = 0 - F_f - C_f e \leq 0.$$  

(8)

This formulation implies that once a firm has decided to exit, it can not re-enter the market. All future profits are zero. The firm decides to exit whenever the expected value of continuing its operation is below the expected value of exiting with the current stock of employment carried over from the last period, $e$.

$$E_{a', \epsilon' | a, \epsilon} [Q^c(a', \epsilon', e, \theta')] - Q^x(e) < 0.$$  

(9)

Here $F$, the fixed cost of operation, induces exit for low realizations of $\epsilon$ since $Q^x(e)$ is always non-positive. The associated exit policy function will be denoted $\phi^x(s)$ and takes a value of one if the firm exits, and zero otherwise. Because $Q^c(x)$ is increasing in $\epsilon$, for a given $e$, $a$, and $\theta$ the exit policy function is characterized by a threshold productivity level $\bar{\epsilon}^x$ below which a firm exits. This threshold is defined as the lowest realization of $\epsilon$ such that the expected value of continuing exceed the value of exiting.

Definition. The threshold productivity level $\bar{\epsilon}^x$ below which a firm exits is defined as

$$\left\{ \begin{array}{ll} \bar{\epsilon}^x_t = \inf \{ \epsilon \in S : E_{a', \epsilon' | a, \epsilon} [Q^c(a', \epsilon', e, \theta')] \geq Q^x(a', e_{-1}) \} & \text{or} \\ \bar{\epsilon}^x_t = 0 & \text{if this set is empty} \end{array} \right..$$  

(10)

Each period a fraction $F(\bar{\epsilon}^x)$ of new entrants exits, while the remaining fraction continues operating. The cutoff $\bar{\epsilon}^x$ is (weakly) decreasing in $a$, and (weakly) increasing in $\theta$ and $e_{-1}$. The intuition for this is straightforward: Because $a$ is persistent, an increase in $a$ raises the expected value of the continuing firm. At the same time the increase in $a$ has no effect on the value of exit. Increases in $\theta$ decrease the firm’s value and hence increase $\bar{\epsilon}^x$. An increase in $\theta$ lowers the number of workers a firm that posts vacancies is able to hire, but has no effect on $Q^x(\cdot)$. The cutoff $\bar{\epsilon}^x$ can never decrease in $\theta$ because the effect of $\theta$ on the firm’s value function $Q^c(s)$ is less or equal to zero. Because the adjustment costs are increasing in $e_{-1}$, everything else equal a higher employment stock has a positive effect on $\bar{\epsilon}^x$.

Note that no additional information is revealed between the end of the current period and the time of the exit decision. Therefore the firm can determine in period $t$ whether it will choose to exit in period $t + 1$. This insight makes the computation of the problem easier and brings the timing of the exit decision in line with the default decision by entrants.
**The Entry Process** At the beginning of each period there is a continuum of ex-ante identical potential entrants. The entry decision is made before the idiosyncratic profitability is known. Entrants do not pay a fixed cost of operation $\Gamma$ in the first period. Instead, to enter, potential entrants must pay a start-up cost $\tilde{c}_e$, which they compare to the expected value of entry $Q_e$. The cost $\tilde{c}_e \equiv c_e \cdot \tilde{R}$ consists of a positive physical entry cost $c_e$ times the interest rate charged by the bank, $\tilde{R}$ (defined below). If the value function $Q^e$ is known, the value of entry gross of entry costs is given by the value of an incumbent firm evaluated at zero employment and the expected initial productivity draw

$$Q^e(a, \theta) \equiv \int_{\epsilon} Q^e(a, \epsilon_0, 0, \theta) d\nu.$$ 

Once an entrepreneur has decided to enter he receives an initial profitability draw $\epsilon_0$ from a distribution $\nu$, which may differ from the distribution of incumbents productivity draws. After the initial period, profitability evolves identically to that of all other incumbent firms. Employment in start-ups is given by the amount of successful hires, $e = H(\theta)\nu$. The value of entry is increasing in $a$ and decreasing in $\theta$. Total start-up job creation is $\int_{\epsilon} \phi^e(a, \epsilon_0, 0, \theta) d\nu$. Firms entering in period $t$ have mass $M_t$, which is pinned down via a free-entry condition. Free entry requires that the cost of entry be equal to the value of entry.

$$\tilde{c}_e = Q^e(a, \theta). \quad (11)$$

**Proposition 1.** There is a unique $M_t$ which solves (11).

The proof can be found in Appendix A.2. The logic is that as $M_t$ increases, labor market tightness $\theta$ goes up since more firms are hiring. This negatively affects $Q^e$ since a firm needs to post more (costly) vacancies to fill the same number of jobs. On the other hand, $\tilde{c}_e$ is increasing in $\theta$ as the next section will show. This is a result of the exit threshold $\bar{\epsilon}_x$ which is increasing in the labor market tightness. With $Q^e$ monotonically decreasing and $\tilde{c}_e$ monotonically increasing in $\theta$ the intersection where (11) holds is unique.

### 3.3 The Bank

The bank is owned by all agents in the economy and behaves competitively, i.e. makes zero profits. To pay the entry cost $c_e$ new firms must obtain a loan from the bank. Firms that are still in operation at the end of the period pay back the loan plus any interest payments that accrued. Entrepreneurs can use their real estate as collateral to secure part of the loan. This can be thought of as a shortcut for the idea that in reality some loans are completely secured by real estate while others are not. Putting down

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27I restrict attention to the case where $c_e < c^*$, where $c^* > 0$ is a number such that if $c_e \geq c^*$ no positive entry rates exist and the equilibrium is one of no firms. In the numerical solution of the model it will furthermore be the case that $c_e \geq \Gamma$, meaning that entrants have to pay a cost higher than the fixed cost of operation.
collateral for a loan is desirable because uncollateralized loans are risky for the bank, while collateralized loans are not. A start-up entrepreneur may strategically choose to exit and hence walk away from his obligations before the loan has to be repaid. Therefore, the bank efficiently prices interest rates by charging a default premium on the uncollateralized fraction of the loan in order to compensate itself for expected losses.\footnote{This is similar to the mechanism in Townsend (1979) and Bernanke \textit{et al.} (1999) where the bank faces a costly state-verification problem. In my model state-verification is costless but in case of default the bank is unable to recuperate any fraction of the initial loan because wages are paid before the intra-period loan is reimbursed. I choose this timing of events in order to eliminate the default dimension from the worker-firm bargaining problem.} The collateralized fraction of the loan is riskless for the bank, hence the intra-period interest rate for it is 1. The fraction of the loan that can be collateralized depends on the value of real estate, $q^h$. The diagram in Figure 6 illustrates the structure of the loan.

**Figure 6: The Intra-period Loan.** For the collateralized fraction of the loan the intra-period interest rate is 1. The uncollateralized part includes a positive default risk for which the bank charges a no-default interest rate larger than unity.

**Interest Rates and the Value of Collateral** Default occurs with positive probability, i.e. whenever a borrowing firm chooses to exit. In that case the bank claims the collateral which was used to secure the loan. No repayment is received for the uncollateralized fraction of the loan because - as can be seen from (8) - profits are non-positive if a firm exits. Payment of collateralized loans can always be enforced by the bank in case of default, hence the intra-period interest rate for this part of a loan is equal to the risk-free rate 1. This corresponds to the bottom area in Figure 6. The remaining fraction of the loan is \textit{not} secured by collateral and the bank charges a loan rate $\hat{R} \geq 1$. Since the bank is perfectly competitive the loan rate is determined by a zero-profit condition $\hat{R}(1 - F(\bar{\epsilon}^x)) = 1$. This implies that the risk-neutral bank receives the same expected return as the risk-free rate, which is 1. The total loan rate paid on $c_o$ is denoted $\tilde{R}$ and
is given by a combination of the risk-free rate and $\hat{R}$. Proposition 2 defines it.

**Proposition 2.** The loan rate $\tilde{R}$ is given by

\[
\begin{align*}
\tilde{R} &= \frac{q}{\varepsilon} + \left(\frac{c - q}{c}\right) \cdot \hat{R} & \text{if } q^h < c_e \\
\tilde{R} &= 1 & \text{if } q^h \geq c_e,
\end{align*}
\]

where

\[\hat{R} = \left(\int_{\tilde{e}}^{\infty} d\nu\right)^{-1}.
\]

The proof can be found in the appendix. The intuition for the result is that if $q^h \geq c_e$ the new entrepreneur can fully collateralize his loan, which implies that he pays the risk-free rate on the intra-period loan. If $q^h < c_e$ he receives the risk-free rate only on a fraction $\frac{q^h}{c_e} < 1$ of the loan. The difference $c_e - q^h$ of the loan $c_e$ has to be borrowed at the risky interest rate $\hat{R}$. This rate is increasing in the probability of receiving an initial profitability draw below $\tilde{e}$. If an entrant would never choose to exit, then the integral $\int_{\tilde{e}}^{\infty} d\nu = 1$ and $\hat{R} = \tilde{R} = 1$. Changes in $\hat{R}$ are a key driver for the dynamics of the model because changes in the cost of entry have important effects on the number of entrants and hence on job creation and unemployment. Since $\tilde{e}$ is (weakly) decreasing in $a$ and (weakly) increasing in $\theta$ it follows that both $\hat{R}$ and $\tilde{R}$ are (weakly) decreasing in $a$ and (weakly) increasing in $\theta$. Furthermore, the effective loan rate $\tilde{R}$ is (weakly) decreasing in $q^h$.

### 3.4 Equilibrium

**The distribution over incumbent firms** In the absence of aggregate shocks (as in Hopenhayn and Rogerson (1993)) it is possible to solve for a stationary distribution of incumbent firms $\lambda^*$. Although my model incorporates aggregate shocks it is useful to spell out the transition of the firm distribution here, since the non-stochastic simulation method is based on it. The distribution over incumbent firms in period $t$ is given by $\lambda_t$. The mass of entering firms shall be denoted $M_t$. I will drop the time subscripts for notational convenience. The transition from any $\lambda$ to $\lambda'$ will be written as $\lambda' = T(\lambda, M)$. The operator $T$ is linearly homogeneous in $\lambda$ and $M$ jointly. This implies that if we doubled the amount of firms in this economy and doubled the amount of entrants the resulting distribution would be unchanged.

Assuming that some initial distribution $\lambda_0$ exists and given the policy functions for employment and exit we can now write the law of motion of the distribution over incumbent firms. For any set $(e, x) \in E \times X$, where $E$ and $X$ respectively denote the employment and exogenous shock space the law of motion for $\lambda$ can be written as
\[
\lambda'((e \ x)' \in E \times X) = \int_{x \in x'} \int_{E \times X} (1 - \phi_x(x, e; \theta)) \times 1_{\{\phi_e(x, e; \theta) \in e'\}} \times F(dx' | x) \lambda(dx) \\
+ M \int_{x \in x'} \int_{0 \times X} \times 1_{\{\phi_e(x, 0; \theta) \in e'\}} \times F(dx' | x) \nu(dx). \quad (12)
\]

This defines the operator \(T\). For the case without aggregate shocks \(x = \varepsilon\) and a stationary distribution \(\lambda^*\) exists.\(^{29}\)

**Endogenous and Exogenous processes** The law of motion for the labor market tightness \(\theta\) follows the law of motion

\[
\theta' = \Pi(a, a', \lambda).
\]

The knowledge of \(\Pi\) requires the joint distribution over employment and idiosyncratic profitability, which is (theoretically) infinitely-dimensional. I follow the approach developed by Krusell and Smith (1998) described in the following section. Briefly, the approach consists of postulating a functional form for \(\Pi\) which entrepreneurs use to make their optimal decisions. From a subsequent simulation of the model one can check the consistency between the actual law of motion of \(\theta\) and the one predicted by the guess of \(\Pi\). The resulting equilibrium must be such that \(\Pi\) must track the evolution of \(\theta\) very accurately. This is explained in more detail below.

Unemployment in the model follows

\[
U' = (1 - U)\delta(U, V) + (1 - \phi(U, V))U,
\]

where \(\delta(U, V)\) is the separation rate and \(\phi(U, V)\) describes the job-finding rate. I assume that the logarithms of both \(a, \varepsilon,\) and \(q^h\) follow autoregressive processes.

\[
\ln a_t = \rho_a \ln a_{t-1} + v_{a,t} , \quad v_a \sim N(0, \sigma_a)
\]

\[
\ln \varepsilon_t = \rho_\varepsilon \ln \varepsilon_{t-1} + v_{\varepsilon,t} , \quad v_\varepsilon \sim N(0, \sigma_\varepsilon)
\]

\[
q^h_t = \rho_q q^h_{t-1} + v_{q,t} , \quad v_q \sim N(0, \sigma_q)
\]

The initial productivity of entrants is determined by a draw from \(v_q \sim N(0, \sigma_q)\) and then evolves according to (14). In the simulation I enforce a correlation coefficient between \(q^h\) and \(a\) obtained from the data.

\(^{29}\)Equation (12) can be most easily read by fixing an exogenous state \(x'\), then integrating over the space of incumbents \((E \times X)\) and selecting those for whom the policy function \(\phi_e(\cdot)\) prescribes \(e'\). The term \(F(dx'|x)\) defines the probability that a firm with current productivity \(x\) has productivity \(x'\) next period. This is multiplied with \(\lambda\) to obtain the mass of these firms. The second term refers to entrants, who have mass \(M\). Their initial employment is equal to zero and they cannot exit in the same period as they enter, otherwise the structure is identical. The stationary equilibrium with entry and exit is given by \(\lambda^* = (I - \pi')^{-1}(\pi' \ast E)\), where \(\lambda\) is the distribution over incumbents, \(\pi\) is the transition matrix and \(E\) is the distribution over entrants.
Equilibrium  For a given $\lambda_0$ a recursive competitive equilibrium consists of (i) value functions $Q^v(a,\epsilon,e_{-1};\theta)$ and $Q^c(a,\epsilon_{-1};\theta)$, (ii) policy functions $\phi^e(a,\epsilon,e_{-1};\theta)$ and $\phi^c(a,\epsilon,e_{-1};\theta)$, (iii) bounded sequences of non-negative negotiated wages $\{w_t\}_{t=0}^\infty$, and interest rates $\{R_t\}_{t=0}^\infty$, unemployment $\{U_t\}_{t=0}^\infty$, vacancies $\{V_t\}_{t=0}^\infty$, incumbent measures $\{\lambda_t\}_{t=0}^\infty$ and entrant measures $\{M_t\}_{t=0}^\infty$ such that (1) $Q^v(a,\epsilon,e_{-1};\theta)$, $\phi^e(a,\epsilon,e_{-1};\theta)$, and $\phi^c(a,\epsilon,e_{-1};\theta)$ solve the incumbent’s problem, (2) $\{w_t\}_{t=0}^\infty$ satisfies the worker’s participation constraint, and $\{R_t\}_{t=0}^\infty$ is given by the bank’s zero-profit condition, (3) labor market tightness $\{\theta_t\}_{t=0}^\infty$ is determined by the ratio of vacancies $\{V_t\}_{t=0}^\infty$ over unemployment $\{U_t\}_{t=0}^\infty$, (4) the measure of entrants is given by the free-entry condition (11), (5) $\lambda_t$ evolves according to (12).

3.5 Calibration

I calibrate the model at quarterly frequency. The steady state equilibrium without aggregate shocks matches US non-farm establishment level data. All parameter values together with their calibration targets are listed in Table 2. The parameters can be divided into two groups. The first group consists of parameters that are either taken from the existing literature or backed out given static calibration targets. The second group of parameters is estimated with a simulated method of moments (SMM) procedure. The first group includes the discount factor $\beta$, the curvature of the profit function $\alpha$, the value of leisure parameters, the parameters governing the evolution of the aggregate states, as well as the parameters of the matching function. $\beta$ and $\alpha$ are taken from the literature.

I fit AR(1)-processes to the data to back out the persistence and innovation parameters for $a$ and $q^b$. For $a$ I use US output from 1977-2011, while for $q^b$ I use the purchase-only HPI from 1977-2011. Both series are HP-filtered. The correlation between output and HPI is 0.628. I enforce this correlation coefficient onto the simulated processes. Recall that workers’ value of leisure is $b(a) = b_0a^{b_1}$. To estimate $b_1$ I use (HP-filtered, seasonally adjusted) average weekly wages from the Quarterly Census of Employment and Wages (QCEW) between 2001 and 2011. The correlation between the cyclical component of this series and GDP is 0.49, which is almost identical to the value used in Cooper et al. (2007). I calibrate $b_0$ to match an average firm size of 21.43 from the BDS data. Regarding the parameters of the matching function, I assume a constant returns to scale function which takes the form

$$m = \mu U^\gamma V^{1-\gamma} = \mu V \theta^{-\gamma},$$

where $\theta = \frac{1}{U}$ measures labor market tightness. The job-finding rate of a worker is defined as $\phi = m/U$, which given the functional form for the matching function takes the form $\phi = \mu \theta^{1-\gamma}$. Similarly the vacancy-filling rate for firms, $H = m/V$ takes the form $H = \mu \theta^{-\gamma}$. Based on BLS data the average unemployment rate over the time of my sample (1977-2010) was 6.3%, which serves as my target for the steady state. I target a vacancy-filling probability of 0.71, in line with empirical evidence in Den Haan et al. (2000), Pissarides (2009), Shimer (2012), and Elsby and Michaels (2013). The same studies suggest a steady-state value of $\theta = 0.70$. The matching elasticity $\gamma$ is set to

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30In the economy with aggregate shocks this equilibrium is boundedly rational because the law of motion for $\theta$ is approximated.
<table>
<thead>
<tr>
<th>Calibrated Parameters</th>
<th>Symbol</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Factor</td>
<td>$\beta$</td>
<td>0.99</td>
<td>implies $r^{ann} = 4%$</td>
</tr>
<tr>
<td>Curvature of profit function</td>
<td>$\alpha$</td>
<td>0.65</td>
<td>Cooper et al. (2007)</td>
</tr>
<tr>
<td>Autocorrelation of $a$</td>
<td>$\rho_a$</td>
<td>0.95</td>
<td>Output 1977-2011</td>
</tr>
<tr>
<td>Standard deviation of $\nu_a$</td>
<td>$\sigma_{\nu_a}$</td>
<td>0.05</td>
<td>Output 1977-2011</td>
</tr>
<tr>
<td>Autocorrelation of $q^H$</td>
<td>$\rho_q$</td>
<td>0.9565</td>
<td>HPI 1977-2011</td>
</tr>
<tr>
<td>Standard deviation of $\nu_q$</td>
<td>$\sigma_{\nu_q}$</td>
<td>0.08</td>
<td>HPI 1977-2011</td>
</tr>
<tr>
<td>Correlation $q^H$ and $a$</td>
<td>$\rho_{q,a}$</td>
<td>0.628</td>
<td>same as above</td>
</tr>
<tr>
<td>Base wage</td>
<td>$b_0$</td>
<td>0.9</td>
<td>Average firms size 21.43</td>
</tr>
<tr>
<td>Sensitivity of outside option to $a$</td>
<td>$b_1$</td>
<td>0.49</td>
<td>BLS QCEW</td>
</tr>
<tr>
<td>Matching elasticity</td>
<td>$\gamma$</td>
<td>0.6</td>
<td>Pissarides and Petrongolo (2001)</td>
</tr>
<tr>
<td>Match efficiency</td>
<td>$\mu$</td>
<td>0.5732</td>
<td>$H = 0.71, \theta = 0.7$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Estimated Parameters</th>
<th>Symbol</th>
<th>Value</th>
<th>Calibration Target / Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed costs vacancies</td>
<td>$F_v$</td>
<td>0.01</td>
<td>Inaction in $\Delta e$</td>
</tr>
<tr>
<td>Quadratic costs vacancies</td>
<td>$c_v$</td>
<td>0.005</td>
<td>Small changes in $\Delta e$</td>
</tr>
<tr>
<td>Fixed costs firing</td>
<td>$F_f$</td>
<td>0.01</td>
<td>Inaction in $\Delta e$</td>
</tr>
<tr>
<td>Quadratic costs firing</td>
<td>$c_f$</td>
<td>0.005</td>
<td>Small changes in $\Delta e$</td>
</tr>
<tr>
<td>Fixed costs of operation</td>
<td>$\Gamma$</td>
<td>3.3</td>
<td>Firm Exit Rate 8.8% (BDS)</td>
</tr>
<tr>
<td>Autocorrelation of $\varepsilon$</td>
<td>$\rho_{\varepsilon}$</td>
<td>0.97</td>
<td>Firm size distribution</td>
</tr>
<tr>
<td>Standard deviation of $\varepsilon$</td>
<td>$\sigma_{\varepsilon}$</td>
<td>0.02</td>
<td>Distribution of $\Delta e$, JC</td>
</tr>
<tr>
<td>Std dev of initial productivity</td>
<td>$\sigma_{\nu}$</td>
<td>0.02</td>
<td>Start-up Fraction of JC = 18.7%</td>
</tr>
</tbody>
</table>

Table 2: Parameter Values. The first block consists of calibrated parameters, the parameters in the second block consists were estimated via SMM.
My target for the vacancy-filling rate together with a choice of $\gamma$ implies a matching efficiency parameter of $\mu = 0.5732$.

The cost parameters in $C$ and the parameters governing the idiosyncratic profitability process are consistently estimated via SMM. This entails finding the vector of structural parameters $\Theta$ which minimizes the (weighted) distance $L(\Theta)$ between data moments and moments of the model. The distance is defined as

$$L(\Theta) = \left( \Gamma^D - \Gamma^M(\Theta) \right) \Xi \left( \Gamma^D - \Gamma^M(\Theta) \right)' ,$$

where $\Gamma^D$ are data moments and $\Gamma^M(\Theta)$ are moments from a simulation of the model, given parameters $\Theta$. The weighting matrix is $\Xi$. I solve the dynamic programming problem and generate policy functions given a parameter vector $\Theta$. From the simulation of the model I then obtain $\Gamma^M(\Theta)$. The algorithm finds the parameter vector $\Theta$ which minimizes $L(\Theta)$. The parameter vector is $\Theta = (F_f, c_f, F_v, c_v, \Gamma, \rho_\varepsilon, \sigma_\varepsilon, \sigma_\nu)$. I restrict the model such that $F_f = F_v$ and $c_f = c_v$, i.e. hiring and firing costs are symmetric. The moments $\Gamma^D$ chosen to estimate $\Theta$ are motivated by Cooper et al. (2012) and Berger (2012) and are reported in the column 'Data' in Table 3. The first four moments are derived from the distribution of employment changes for continuing establishments using Census BDS data between 1985-1999. The first row reports the inaction rate, i.e. the fraction of establishments that did not undergo any employment change over the course of one year. The high value suggests that fixed costs of labor adjustment are important. The second column $|\Delta e| \leq .1$ reports the fraction of ‘small’ employment changes of under 10% in absolute value. Rows 3 and 4 report large positive and negative employment changes of over 30%. These large changes are very prevalent in the data, indicating large changes in firm-level productivity over time. Row 5 is the firm exit rate from the BDS data between 1977 and 2011. From the same data comes the fraction of gross job creation through firm birth, which is around 19%. Both $\Theta$ and $\Gamma^D$ consist of six (unique) elements, but there exists no direct mapping between them. The following can be said about the identification, however: The fixed costs $F_f$ and $F_v$ play a crucial role for generating inaction, while the quadratic costs are identified through small employment changes, $|\Delta e| \leq .1$. The quadratic costs also play an important role for generating exit among large plants. The operational overhead cost $\Gamma$ is used to pin down the exit rate. Start-up job creation largely depends on the initial productivity draw, whose variance is governed by $\sigma_\nu$. The persistence of the idiosyncratic shock $\rho_\varepsilon$ is crucial for determining the shape of the size and age distributions and affects the frequency of employment adjustments. The variance of $\varepsilon$ is important for large adjustments and the size distribution of firms. It indirectly affects all moments in $\Gamma^M(\Theta)$. The stationary model is further discussed below.

---

31 This is based on a survey by Pissarides and Petrongolo (2001). Cooper et al. (2007) estimate this parameter to be .36, Hall (2005b) finds 0.72.

32 Equilibrium is enforced during all of these estimations, meaning that the entrepreneur’s beliefs about $\theta$ are consistent.
<table>
<thead>
<tr>
<th></th>
<th>Data Moments</th>
<th>Model Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta e = 0$</td>
<td>0.38</td>
<td>0.36</td>
</tr>
<tr>
<td>$</td>
<td>\Delta e</td>
<td>\leq .1$</td>
</tr>
<tr>
<td>$\Delta e &gt; .3$</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>$\Delta e &lt; -.3$</td>
<td>0.14</td>
<td>0.16</td>
</tr>
<tr>
<td>Exit Rate</td>
<td>8.8%</td>
<td>6.3%</td>
</tr>
<tr>
<td>Start-up JC</td>
<td>18.7%</td>
<td>17.0%</td>
</tr>
</tbody>
</table>

Table 3: Data Moments and SMM estimates. Column 3 estimates the benchmark model using symmetric adjustment costs (AC) for hiring and firing. The employment change numbers are taken from Berger (2012) who uses LBD averages between 1985-1999. The exit rate and start-up JC rate are computed using BDS data.

4 Computational Strategy

Firms need to forecast $\theta'$ in order to compute the expected vacancy-filling rate $H(U,V)$. The variable $\theta$ is determined in equilibrium. While firms take this function as given, it must be consistent with the relationship generated by the model. In the stationary model without aggregate shocks there is a steady state value $\theta^*$ which can easily be determined. Including aggregate shocks creates a non-trivial computational problem, which I solve similarly to Krusell and Smith (1998). The free-entry condition is given by (11). The labor-market tightness $\theta$ is now a slow-moving state variable about which firms must generate consistent forecasts. The solution of this model is non-trivial since firms need to forecast the entire cross-sectional joint distribution of employment and productivity in order to forecast labor market tightness in the following period. In the presence of aggregate shocks, this distribution moves over time and the state-space becomes (theoretically) infinite-dimensional. Following the seminal work of Krusell and Smith (1998) an approximate solution can be found by postulating that firms track only several moments of this joint distribution. The first moments usually turns out to be a sufficient statistic. However, as Den Haan (2010) has shown, it should also be verified that the maximum forecast errors that result from the approximated law of motion are small. In the present framework firms are ultimately interested in forecasting $\theta'$, the labor market tightness next period. The perceived law of motion of $\theta$ is denoted $\theta' = \mathbb{H}(\theta, A', A)$, where $\mathbb{H}(\cdot)$ is to be determined as part of the solution of the model. Firms make their forecasts of $\theta'$ conditional on the current realizations of $\theta$ and $A$, as well as on possible future realizations $A'$. The solution algorithm first postulates an initial guess for $\mathbb{H}(\cdot)$. Next, policy functions are computed given the guess. Following a simulation, the parameters of $\mathbb{H}(\cdot)$ are updated. This procedure is repeated until the current guess and the updated version of $\mathbb{H}(\cdot)$ are sufficiently close (consistency) and until $\mathbb{H}$ tracks the evolution of $\theta$ with high accuracy. I guess a log-linear prediction rule for $\theta'$.

$$
\log \theta_t = b_0 + b_1 \log \theta_{t-1} + b_2 \log A_t + b_3 \log A_{t-1} + b_4 \cdot I_{A_t \neq A_{t-1}}
$$
The last term, $I_{A_t \neq A_{t-1}}$, is an indicator function which takes the value of one if $A_t \neq A_{t-1}$. The coefficients that minimize the stopping criterion are given by

$$\log \theta_t = -0.0087 + 0.9939 \cdot \log \theta_{t-1} + 20.996 \cdot \log A_t - 21.095 \cdot b_3 \cdot \log A_{t-1} + 0.2327 \cdot I_{A_t \neq A_{t-1}}.$$

This functional form for $\mathbb{H}(\cdot)$ generates an $R^2 = 0.9994$ and a maximum forecast error of $0.005\%$. Accuracy plots can be found in the Appendix A.2.

Note that without financing friction (i.e. no variation in $q^h$) the computational problem is much easier to solve. When the only shocks are to $a$ the model behaves very similarly to the standard HR model. In particular, the free entry condition reduces the computational burden because the future value of $\theta$ can be computed without a Krusell-Smith type algorithm for the cross-sectional distribution. The reason is that with free entry aggregate labor demand becomes perfectly elastic and for each $a$ there exists one value of $\theta$ which is consistent with equilibrium. Free-entry of new firms makes the tightness parameter $\theta$ respond 1:1 to changes in the aggregate state $a$. However, such a model generates unrealistically volatile entry rates and basically reduces the model to a function of the aggregate state $a$, with some propagation through the adjustment costs.

The simulation of the model is carried out using a non-stochastic simulation technique. The algorithm does not draw a random sequence of idiosyncratic shocks for each firm and play out the policy function for a large number of periods. Instead, my algorithm computes the exact mass of firms at each grid point jointly representing idiosyncratic productivity and employment. This solution method is applicable for both the stationary and non-stationary version of the economy. The main advantages of this approach are its speed and the fact that it eliminates sampling error. Den Haan (2010) showed that this latter source of error can become important in Krusell-Smith type solution algorithms. The details of this algorithm are laid out in Appendix A.3.

5 Quantitative Results

This section describes the numerical results. I evaluate the performance of the stationary model with respect to non-targeted moments and then discuss the results of the model with aggregate shocks.

5.1 Results of the stationary model

Table 3 showed the match of targeted moments. The employment change distribution as well as the exit rate and job creation by start-ups generated by the model are very close to their counterparts in the US data. The fit of the firm-age distributions of the

---

33 The labor market tightness ‘jumps’ with the aggregate state when the only shocks are to $a$. The true and the approximated law of motion are almost indistinguishable. A regression which ignores past realizations of $\theta$ produces an $R^2 > 0.99$ and a maximum forecast error of 0.0052%. The $R^2$ is not equal to 1 because $\theta$ influences the interest rate $\hat{R}$ which effects the number of entrants and hence the labor market tightness. Including past realizations of $\theta$ into the regression increases the $R^2$ to over 0.99999999.
5.2 Results with Aggregate Shocks

I now add aggregate shocks to the model in order to assess the business cycle properties of the model and evaluate its quantitative performance. To demonstrate the effect of shocks to aggregate productivity and the HPI, impulse response functions are generated. I also test alternative model specifications without financial frictions and without adjustment costs in order to build some intuition about the respective effects those features on the results. Finally, I show a policy experiment which allows me to back out the effects of the decrease in the HPI on the increase and persistence of unemployment during and after the Great Recession. The main results are summarized in Tables 5 and 6.

5.2.1 Results of the Benchmark Model

This section describes the results of the benchmark model which includes shocks to $a$ and $\phi^h$. The model is able to match the key statistics of the US labor market regarding unemployment, vacancies, and their joint movement. Those statistics are reported in the first row of Table 5. The result of the benchmark model are in the second row. Both the volatility and the autocorrelation of unemployment, vacancies, and labor market tightness are close to their counterparts in the data. However, the persistence of unemployment is overstated, while the persistence of vacancies is understated with respect to the data. The correlation between unemployment and vacancies is strongly negative, as in the data. Given that the model was not calibrated to generate these moments the close fit can be considered a success of the calibration strategy.

The second set of results focuses on the cyclical and volatility of employment in start-ups vis-a-vis incumbent firms. Two of the stylized facts presented in Section 2 were that job creation by incumbents is more strongly pro-cyclical, while job creation by start-ups is more volatile around its trend. Those facts are summarized in the first row of Table 6. The first two columns show the correlation between GDP and job creation by entrants ($E$) and incumbents ($I$). The last two rows report the standard deviation of calibrated model is shown in Table (4). The model matches the age distribution of firms well but slightly underpredicts the amount of old firms.

<table>
<thead>
<tr>
<th>Firm Age</th>
<th>Firms</th>
<th>Employment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>Start-Ups</td>
<td>11%</td>
<td>12%</td>
</tr>
<tr>
<td>Age 1-2</td>
<td>16%</td>
<td>19%</td>
</tr>
<tr>
<td>Age 3-5</td>
<td>17%</td>
<td>22%</td>
</tr>
<tr>
<td>Age 6-20</td>
<td>41%</td>
<td>40%</td>
</tr>
<tr>
<td>Age 21+</td>
<td>15%</td>
<td>8%</td>
</tr>
</tbody>
</table>

Table 4: Age-distribution of firms. Census BDS data and results from the stationary model.
the cyclical component of job creation over the trend component for the two groups. The model generates the lower pro-cyclicality of job creation by entrants with respect to incumbents. The good fit in the correlation between GDP and job creation by new firms is achieved through the effect of house prices $q^h$ on the entry process as will be explained below. Furthermore, the model replicates the higher correlation between GDP and job creation by incumbent firms. This has been an important feature of the recovery after the Great Recession.

The benchmark model can generate ‘jobless recoveries’ through the effect of house prices $q^h$ on the start-up process. Imagine a situation where both aggregate profitability and the HPI are below their unconditional means. Now both shocks start reverting back but - as we will see below - the effects on unemployment and total output of the two shocks differ significantly. Other than the shock to aggregate profitability the shock to $q^h$ exerts only very mild influence on total output. By directly impacting entry, the decrease in $q^h$ has a large effect on hiring by start-ups, and thus on unemployment. The fraction of total hiring by start-ups is overproportional to their share of total output. Therefore, if the number of entrants decreases, the effect on unemployment is larger than the effect on GDP. Incumbent firms are only indirectly affected by the HPI through an effect on $\theta$. On the other hand, shocks to $a$ have the effect that hiring - and most importantly - output by incumbent firms changes. Since the lion’s share of total output is produced by incumbent firms, an increase in $a$ after an initial negative shock has an immediate effect on output and employment. This is why a shock to $a$ alone cannot generate a jobless recovery. It requires the effect on entry - exerted by shocks to $q^h$ - to make the unemployment rate react sluggishly and uncouple it from the strong co-movement with GDP. The impulse response functions will show this in more detail.

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34 I divide the series by their respective trend in order to control for the fact that otherwise the large number of jobs created by incumbents blows up the standard deviation of the series. An alternative measure that delivers similar results is the coefficient of variation.

35 See the additional material, e.g. Figure 13 in Appendix A.1.
Table 6: Data and Model Moments. Source: BDS 1977-2011. The resulting model moments have been computed using time aggregation. Data and model moments have been computed as log deviations from mean/trend. $\rho(Y, N^E)$ and $\rho(Y, N^I)$ show the correlation between GDP and gross job creation by entrants and incumbents. The standard deviation of the cyclical over the trend component of job creation by start-ups are $(\sigma(c/t)^E)$ and $\sigma(c/t)^I$ for incumbent firms.

### 5.2.2 Results of the Alternative Model Specifications

We can now compare the benchmark results to those of the model without financial frictions or without shocks to aggregate productivity. The results are summarized in the last two rows of Tables 5 and 6. Table 5 shows that the business cycle statistics of the model without the financial friction are similar to the benchmark model. The volatility of unemployment and vacancies, as well as the correlation between the two is slightly overstated. Furthermore, $\theta$ is more volatile than in the data. The fact that the model produces similar moments as the benchmark model is not very surprising given the similarity of the model without the financial friction to Cooper et al. (2007), who find similar results. The model without shocks to $a$, on the other hand, is unable to capture some of the key US business cycle statistics. In particular, the model does not generate enough variation in unemployment and vacancies. The reason is that variations in $q^h$ have a strong effect on start-ups but only an indirect effect (through labor market tightness) on incumbent firms. The movements in $\theta$ generated by changes in $q^h$ are by themselves not sufficient to generate the observed time-series volatility. Table 6 shows the model performance regarding job creation by entrants and incumbents.

### 5.2.3 Impulse Response Functions

In order to disentangle the respective effects of $\theta$ and $a$ I show several impulse response functions in Figures 7-9. Figure 7 studies a negative shock to aggregate profitability, Figure 8 shows results for a negative shock to $q^h$, and in Figure 9 both shocks occur simultaneously. For comparability between the IRFs the size of the (negative) shocks to $a$ and $q^h$ were chosen to generate the same contemporaneous increase in unemployment. The figures are all constructed in the same way: The first panel shows the effect of the shock to the exogenous state. The second panel (clockwise) shows the effects on unemployment and GDP. The third panel plots the labor market tightness $\theta$, while the last panel shows the effect on start-up activity.

I start with Figure 7 where the effects of a drop in $a$ are analyzed. The first panel shows that in period $t = 10$ aggregate profitability falls by 1.22%. This results in a
Figure 7: Impulse Response Functions for a shock to $a$. Simulation results from 10'000 repetitions of 200 periods.
contemporaneous increase of the unemployment rate by 5.8%, and a fall in GDP by 1.35%. Labor market tightness falls, both because incumbent firms post fewer vacancies and because there are fewer entrants. The last panel also shows that the mass of entrants quickly rebounds after the initial shock. The reason is that the entrants are facing a trade-off between the lower aggregate profitability and the decreased labor market tightness. The latter has the effect of making it more profitable for potential entrants to start operating. Starting in period $t = 14$ the mass of entrants is above its unconditional mean, beginning to restore the total mass of firms to its pre-recession value.

Now I turn to analyzing the implications of a negative shock to $q^h$. The first panel of Figure 8 shows that in period $t = 10$ $q^h$ decreases by 4.12%.$^{36}$ The shock generates an increase in unemployment of 5.8%. This can be seen in the second panel. The shock to $q^h$ produces a smaller decrease in GDP (0.48%) than the shock to $a$. This is because incumbent firms are only indirectly affected by the HPI shock, namely through the effect on $\theta$ which is displayed in the third panel. Labor market tightness decreases when the shock occurs and then slowly recovers. For incumbents firms and hiring entrants this implies that following the shock to $q^h$ the vacancy-filling probability $H(\theta)$ increases. This has the effect that job creation by incumbent firms increases. The last panel shows the effect on the number of start-ups. The most important difference with respect to the

$^{36}$This is a fairly large shock compared to the decrease in the HPI during the Great Recession. The average HPI growth between 2007Q1 and 2011Q1 was -1.46% per quarter, the minimum was -2.88%.
effects of a shock to $a$ is that the mass of entrants is affected both more severely and for a longer period of time. After a rebound to around 92% of its steady-state value in $t = 11$ the entry rate is only gradually moving back towards its unconditional mean. The shock to $a$ generated a tradeoff between lower profitability and lower $\theta$, which induced high entry rates after aggregate productivity had been beginning to recover. The outcome generated by the drop in $q^h$ is different in the sense that the higher entry costs outweigh the effects of the drop in $\theta$ for new entrants. This is the main takeaway from Figures 7 and 8: In the context of the model, a jobless recovery must be the result of a simultaneous shock to both $a$ and $q^h$. While the mean reversion of aggregate profitability brings GDP back to its pre-recession value, the slow recovery of the HPI has almost no output effect, but a large positive effect on the unemployment rate. Therefore, although GDP is above its recession trough, the decline in the unemployment rate is strongly underproportional to this decrease.

Figure 9 shows results for a simultaneous shock to $a$ and $q^h$. The first panel plots the two shock processes. The second panel shows that the average increase in unemployment is 10.2%, while GDP drops by 1.59%, both of which is lower than the sum of the effects of the individual shocks. Both shocks are mean reverting but the persistent $q^h$ shock keeps the unemployment rate high although GDP has practically recovered its pre-shock value (after $t = 20$ average GDP stands at 0.9978 of the pre-shock value). The effect on the number of entrants is strong. There is a sharp rebounce in the periods after the

Figure 9: Impulse Response Functions for a shock to $a$ and $q^h$. Simulation results from 1’000 repetitions of 200 periods.
5.2.4 Policy Experiment

Tables 5 and 6 showed that the model is able to match the key properties of the US labor market as well as the cyclical and volatility of job creation by entrants and incumbent firms. The impulse response functions were meant to create some intuition about the effect of the two shocks. I now test in how far the model can replicate the relationship between the cyclical components of GDP growth and unemployment during the 'Great Recession'. To evaluate the model’s performance in this respect I feed in the observed house price index between 1990Q1 and 2013Q1 (see Figure 17). Furthermore, I pick the sequence of aggregate productivity shocks to match the cyclical component of GDP over the same period. I simulate the model for 93 periods after some initial periods for the model to reach the stationary distribution. I choose 93 periods because this corresponds to the number of quarterly observations. The results are presented in Figure 10. The co-movement of the two time series is extremely strong, particularly during the 'Great Recession', indicated by the third shaded area. The simulated data is able to explain 72.23% of the variation of the unemployment rate observed in the data. For the period starting in 2006 the simulated data can even explain 84.66% of the movement in the initial shock but no overshooting, as the dampening effect of the low $q^h$ prevails over the mean reversion in the shock to $a$.
unemployment rate. The recovery is 'jobless' because of the ongoing negative influence of the low HPI on start-up job creation. Like in the data this leads to high levels of unemployment even after the official recession end. We see that job creation by start-ups decreased prior to the beginning of the recession. The model has this feature simply because the drop in the HPI precedes the decline in aggregate productivity.\textsuperscript{37} Net job creation by incumbents begins to recover before job creation by start-ups. This is the case because at the end of the recession incumbent firms take advantage of the high vacancy filling probability due to the low $\theta$, while hiring for start-ups remains costly because of the ongoing low $q^h$ which increases the cost for setting up shop. What the model is unable to match is the time lag in the respective troughs of the HPI and job creation by start-ups. In the simulation job-creation by start-ups coincides with the trough in the HPI series, while in the data job creation by start-ups was lower in 2011 than in 2009.

In Appendix A.2 I repeat this experiment when there are only shocks to $a$ or $q^h$. Figures 24 and 25 show that although the variation in $q^h$ generates a lot of movement in the unemployment rate it is not enough to reproduce the large increase in unemployment which accompanied the recent recession.\textsuperscript{38}

5.3 Evaluation of Results

This is a rich model in which the mapping from parameters to moments is not immediately clear. I therefore show several additional Figures here to help build some intuition for the results. Figure ?? shows results for a sample simulation of the benchmark model. ... We see that the model produces bursts of entry, particularly in reaction to changes in the aggregate shock $a$, which are larger than those observed in the data. Part of this is smoothed out by time aggregation, however.

The results of two sample simulations of the model without the financial friction and without shocks to aggregate profitability are shown in Figures 11 and 12. In Figure 11 the only exogenous variation comes from changes in $a$. The first panel shows unemployment and GDP. The comovement between the two series is strong (the correlation between the two series is -0.995). For this reason the model is unable to generate jobless recoveries. An increase in unemployment can only result from a low realization of the aggregate shock $a$. However, once $a$ returns to its unconditional mean the unemployment rate reverts back to its pre-recession value almost immediately. The second panel shows the mass of entrants, which reacts strongly to changes in $a$. In fact, the procyclicality of entry is around 60\% larger than in the data. The last panel shows the true and the approximated laws of motion of $\theta$. The series are almost indistinguishable as labor market tightness $\theta$ moves virtually 1:1 with the aggregate state. Figure 12 shows a simulation of the model when $a$ is fixed at its unconditional mean. The only exogenous variation comes from movements in $q^h$. The first panel highlights that those exogenous shocks

\textsuperscript{37}The HPI showed negative growth rates as early as Q12006, while the NBER dates the beginning of the recession in Q42007.

\textsuperscript{38}The $q^h$ shock alone explains about 59.25\% and the $a$ shock alone about 56.93\% of the variation in unemployment.
Figure 11: Sample simulation when the only shocks are to aggregate profitability. The first panel shows unemployment and GDP. The second panel shows the mass of entrants, and the last panel shows the true and approximated values of $\theta$. 
Figure 12: Sample simulation when the only shocks are to HPI. The first panel shows unemployment and GDP. The second panel shows the mass of entrants, and the last panel shows the true and approximated values of $\theta$. 
cause variations in the unemployment rate, while having almost no effect on GDP. As was discussed above, this is the main feature of the model which generates jobless recoveries. The mass of entrants, plotted in the second panel is much less volatile compared to the previous case where all exogenous shocks occurred via \( a \). Finally, the true and approximated law of motion of \( \theta \) are shown in the third panel. Again, the fit is very good (see Appendix A.3 for details).

6 Conclusion

The recent recession which lasted from the end of 2007 until mid-2009 was severe in many respects. Because the unemployment rate remains far above its pre-crisis level the recovery has been described as jobless. Second, the recession was accompanied by an unprecedented fall in the value of real estate. In this paper I claim that these two facts are related. As the main channel through which house prices can exert this influence on the unemployment rates I propose the process of lending to new firms. The model captures the idea that start-ups require external financing, for which real estate is used as collateral. As the value of this collateral falls, start-up costs increase and the number of new firms declines.

The number of start-ups in the US has fallen by over 20% since 2007. Never since the beginning of the data series in 1977 have there been as few openings of new firms or as few jobs created through firm birth than in 2010 and 2011. Young firms’ below-trend job creation can account for almost all of the persistently high unemployment rate after the end of the recession.

I calibrate and compute a quantitative competitive industry model with endogenous entry and exit, firm heterogeneity, labor adjustment costs, and aggregate shocks. This model is able to match key moments of the firm distribution and employment at the micro- and macro-level. It captures the importance of new firms for employment and generates a jobless recovery. The model is able to explain over 80% of the increase and persistence in unemployment since 2007. I find that the effects of a ‘technology shock’ alone on the unemployment rate are neither strong nor persistent enough to fit the US data. I estimate that absent the deterioration of value of real estate, the increase in the unemployment rate would have been at around 40% of the actual increase. Furthermore, my mechanism generates a realistic procyclicality and time series variation in entry rates, something that previous studies have had difficulties with. Entry emerges as an important factor for the propagation of aggregate shocks. In contrast to previous studies my framework establishes a structural link between house prices, entrepreneurial activity, and the jobless recovery. This setup is suited to explain why start-up job creation began to decrease prior to the recent recession, and why - contrary to older, incumbent firms - it remains at low levels.
References


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Appendix

A.0 Data

The main dataset I use for this paper is the Business Dynamics Statistics (BDS) dataset published by the Census. This annual dataset is derived from the Longitudinal Business Database (LBD) and covers both firm size, firm age, as well as firm- and establishment level data. A unique feature of the BDS is its longitudinal source data that permit tracking establishments and firms over time. A strength of data is its robustness to ownership changes because the age of a firm is determined by the age of its oldest establishment.

I complement the analysis by considering alternative data sources obtained from the Bureau of Labor Statistics (BLS). Virtually all of my qualitative results can also be obtained with the 'Business Employment Dynamics' (BED) series by the BLS. The BED is derived from a quarterly census of all establishments under state unemployment insurance programs, representing about 98 percent of employment on nonfarm payrolls. The data frequency is quarterly. It includes data on firm age and firm size. A caveat is the limited comparability between the age and size series as the age data is based upon establishment-level data, while the size class tabulations use firm-level data instead. For this reason I present most of the trends using the BDS data.

Another source released by the BLS is the Current Employment Statistics (CES) program. This is a monthly survey of about 145'000 firms and government agencies, representing roughly 557'000 establishments. Despite its high frequency the survey nature of the CES and its limited representation of the US economy make this data source less useful for the purpose of the present paper.

The series for house prices come from the Federal Housing Finance Agency (FHFA), which provides national and state-level house price indexes from 1991 onwards. The unemployment rate was obtained from the BLS. The quarterly series of state-level personal income was obtained from the Bureau of Economic Analysis (BEA).

Table 7: Summary Statistics for Variables used in Regression

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<th>min</th>
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<th>p50</th>
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<td>-4.4e+04</td>
<td>-1.2e+03</td>
<td>-88.366</td>
<td>975.876</td>
<td>8.1e+04</td>
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<td>-2.381</td>
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<td>1.509</td>
<td>48.872</td>
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<td>ue</td>
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<td></td>
</tr>
</tbody>
</table>

A.1 Additional Material to support the Stylized Facts

This Appendix includes figures referenced to in the main text. It is intended to give further evidence for the stylized facts presented in the main text.
The Importance of Start-Ups  

Figure 13 plots an updated version of a graph used in Coles and Kelishomi (2011). It shows net job creation by start-ups and incumbent firms. Net job creation by incumbent firms is typically negative. This is related to the life cycle of a typical firm. Figure 14 shows gross job creation and destruction between 1977-2011. The left panel shows the raw data, while the right panel shows HP-filtered data. In both cases we see that while job destruction spiked during the 2007-09 recession, the spike was less pronounced than during the 2001 recession. Furthermore the graphs show that compared to all previous recessions, there has been an unusually sharp decline in job creation rates. Figure 15 compares the cyclicality of employment in entrants and incumbents. The standard deviation of the plotted series are 0.10 for start-ups and 0.02 for incumbent firms.
Figure 15: Cyclicality of job creation. Start-ups vs. employment in incumbent firms (dashed line). I HP-filter the annual data with $\lambda = 100$. Plotted is the cyclical component over the trend component. Recession dates are indicated as the shaded areas. Source: Census, BDS
Figure 16: Comparing Recession Episodes: GDP, Unemployment, number of start-ups, and job destruction. GDP and unemployment are quarterly series, start-ups and job destruction are annual. All series are HP-filtered with $\lambda = 100$ for annual and $\lambda = 1600$ for quarterly data. The x-axis shows periods since the respective pre-recession peak, i.e. last period before the official NBER recession date. Unemployment data comes from the BLS and matches the period of Census data publication. For the annual series I treat the 1980 and 1981/2 recession as a single episode.
Figure 17: Cash Shiller Home Price Index. HP-filter $\lambda = 1600$. The x-axis shows quarters since the respective pre-recession quarter (based on NBER classification). Inflation-adjusted, not seasonally adjusted. Source: Standard&Poor’s. Own computations

Comparing different Recession episodes Other studies, e.g. Sanchez and Liborio (2012) have used alternative data sources such as the Business Employment Dynamics (BED) from the BLS to show the decline in startup activity.

Indicators for Credit Supply, Interest Rates, and Home Equity Extraction On the one hand, the lending environment has become tighter during the last recession. Many studies point to the idea that the decrease in credit supply is the result of illiquid funding markets faced by commercial banks and a reassessment of bank lending practices and business strategies (see literature review). Banks whose balance sheets have been more severely affected by increased loan defaults may either have insufficient capital to make additional loans, or may choose to conserve capital instead of making loans to entrepreneurs (United States Congressional Oversight Panel, 2011, 2010). Other than during previous post-WWII recessions the percentage of institutions reporting negative quarterly net income increased to over 30% in 2009. 49 According to the Federal Reserve’s ‘Senior Loan Officer Opinion Survey on Bank Lending Practices’ by the end of 2008, 69.2% of banks reported that they had tightened credit standards, especially for firms with annual sales less than $50 million (80%). Results are shown in Figure 20.

49Based on FDIC data. The average number of institutions with negative quarterly income between 1990 and 2006 was 8.39%. During 2001 and 2002 the highest percentage was 14.87%. See also Figure 19 where the increase in interest rates was much less pronounced during 2001 than 2008.
Figure 18: Domestic Commercial and Industrial Loans to U.S. Addressees. The blue solid line is C&I loans under $1 Million (in Millions of $). The orange dotted line is all C&I loans (in Millions of $). The yellow dash-dotted line is the number C&I loans under $1 Million. Source: FDIC

Figure 19: Commercial and Industrial Loan Rates Spreads over intended federal funds rate, by loan size and risk (E2). Source: Federal Reserve
Figure 20: Results from 'Senior Loan Officer Opinion Survey on Bank Lending Practices'. The blue line plots the net percentage of banks reporting tightening standards for C&I loans to firms with annual sales of less than $50 million. The orange line plots the net percentage of banks reporting stronger demand for C&I loans from those same firms. Source: Federal Reserve.

Figure 21: Used and Unused Home Equity Lines. Source: FDIC
Decomposing Changes in the Unemployment Rate Following the methodology developed in Elsby et al. (2009) I use data from the Bureau of Labor Statistics (BLS) and decompose changes in the unemployment rate into changes due to variations in the inflow rate and changes due to variations in the outflow rate of unemployment. The data shows that the increase in the unemployment rate was mainly due to decreases in the outflow from unemployment, i.e. lower hiring. Using the formula for the evolution of the steady state unemployment level we can write \( u_t = \frac{s_t}{s_t + f_t} \), where \( s_t \) and \( f_t \) describe the unemployment inflow and outflow hazard rates. Log differentiation of this expression then yields \( d \log u_t \approx (1 - u_t)[d \log s_t - d \log f_t] \). See Elsby et al. (2009) for further details. An increased entry hazard would speak for higher rates of job destruction through layoffs and quits, while a decreased exit probability is related to stalling job creation and/or decreased efficiency of the matching process. While early papers such as Darby et al. (1986) suggested that increases in unemployment during recessions are mainly due to increasing number of inflows, the more recent literature has taken the opposite stand. Hall (2005a), Hall (2005b), and Shimer (2012) have made the claim that modern recessions do not share this feature and are characterized by acyclical inflow rates. I use the Q2 2013 Current Population Survey (CPS) by the Bureau of Labor Statistics (BLS). The left panel of Figure 22 plots the log variation in the inflow \( s \) and outflow rates \( f \). While the inflow rate increased at the onset of the recent recession, its cyclicality is dwarfed by that of the decrease in the outflow rate. The right panel of the same figure plots the changes in the decomposition of the unemployment rate and leads to the same conclusion: the decreases in the unemployment exit hazard has been the major contributing factor to the continuingly high unemployment rate we observe today. This result strengthens the conclusion summarized in Stylized Fact 1.
<table>
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<td>1.54%</td>
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<td>Establishments</td>
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<td>Number of Start-ups</td>
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<td>0.01%</td>
</tr>
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<td>Startup Employment</td>
<td>69.36%</td>
<td>20.90%</td>
<td>8.26%</td>
<td>1.47%</td>
</tr>
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</table>

Table 8: Size- and Employment Distributions. Source: Census/BDS. Employment is calculated using the DHS-denominator.

<table>
<thead>
<tr>
<th></th>
<th>Age 0</th>
<th>Age 1</th>
<th>Age 2</th>
<th>Age 3</th>
<th>Age 4</th>
<th>Age 5</th>
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</thead>
<tbody>
<tr>
<td>Firms</td>
<td>11.09%</td>
<td>8.54%</td>
<td>7.22%</td>
<td>6.29%</td>
<td>5.55%</td>
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<tr>
<td>Employment</td>
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<td>2.68%</td>
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<td></td>
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<tr>
<td>Firms</td>
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<td>8.16%</td>
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<tr>
<td>Employment</td>
<td>10.36%</td>
<td>8.89%</td>
<td>8.14%</td>
<td>7.94%</td>
<td>47.87%</td>
<td></td>
</tr>
</tbody>
</table>

Table 9: Firm- and Employment distributions by age. Source: Census, BDS.

The Size-Age Distribution of Firms and Establishments

A.2 Model Properties

This Appendix includes proofs, derivations, and details about the properties and fit of the model.

Policy Function for Employment The policy function for employment is shown in Figure 23.

Proof of Proposition 2

Proof. Part 1: An agent will always choose to collateralize the highest possible fraction of the loan. Denote this fraction as $\mu$. The entrepreneur chooses this fraction to minimize his interest payments $R = 1 - \mu + \bar{R} (1 - \mu)$. The minimization problem reads $\min_{0 \leq \mu \leq 1} c_e - c_e \cdot 1 - \mu - c_e \cdot \bar{R} (1 - \mu)$ subject to the collateral constraint $\mu \cdot c_e \leq q^h$ and $0 \leq \mu \leq 1$. The collateral constraint says that the value of the secured fraction of the loan, $\mu \cdot c_e$, cannot exceed the value of the collateral. The resulting corner solution is $\mu = \min\{\frac{q^h}{c_e}, 1\}$. If $\frac{q^h}{c_e} \geq 1$ then $\mu = 1$ and $\bar{R} = 1$. If $\frac{q^h}{c_e} < 1$ we have $\mu = \frac{q^h}{c_e}$ and $\bar{R} = \frac{q^h}{c_e} + \bar{R} (1 - \frac{q^h}{c_e})$.

Part 2: In a given period the bank lends an uncollateralized amount $x$ to a mass $M_t$ of ex-ante identical entering entrepreneurs. A fraction $F(\bar{\varepsilon}^x) = \int_0^{\bar{\varepsilon}^x} d\nu$ of the $M_t$ entrants will receive an initial productivity draw below the exit threshold $\bar{\varepsilon}^x$ and hence default on the loan. The remaining fraction $1 - F(\bar{\varepsilon}^x) = \int_{\bar{\varepsilon}^x}^{\infty} d\nu$ will receive a draw above $\bar{\varepsilon}^x$ and
repay the initial loan times the non-default interest rate $\hat{R}$. The zero-profit condition of the bank implies $M_t x - \hat{R} \cdot M_t x \int_{\bar{\epsilon} x}^{\infty} \, d\nu = 0$ or $\hat{R} = (\int_{\bar{\epsilon} x}^{\infty} \, d\nu)^{-1}$. Clearly, $\frac{\partial}{\partial \bar{\epsilon} x} \int_{\bar{\epsilon} x}^{\infty} \, d\nu < 0$, so it follows that $\frac{\partial \hat{R}}{\partial \bar{\epsilon} x} > 0$.

**Proof of Corollary**

**Proof.** We have

\[
\begin{cases}
\frac{\partial \hat{R}}{\partial q^h} = \frac{1}{c_e} - \frac{1}{c_e} \cdot \hat{R} \leq 0 & \text{if } q^h < c_e \\
\frac{\partial \hat{R}}{\partial q^h} = 0 & \text{if } q^h \geq c_e.
\end{cases}
\]

Furthermore

\[
\begin{cases}
\frac{\partial \hat{R}}{\partial \hat{R}} = 0 & \text{if } q^h \geq c_e \text{ or } \bar{\epsilon} x = 0 \\
\frac{\partial \hat{R}}{\partial \hat{R}} = -\left(\frac{c_e - q^h}{c_e}\right) < 0 & \text{else}
\end{cases}
\]

From Proposition 3.2 $\frac{\partial x}{\partial a} < 0$ and $\frac{\partial x}{\partial \theta} > 0$. Since $\frac{\hat{R}}{\bar{\epsilon} x} > 0$ we obtain the results stated in the corollary.

Figure 23: Employment Policy Function for given values of $a$, $\theta$, and $e_{-1}$. 

55
Figure 24: Cyclical component of the unemployment rate. Data vs. simulation using estimated processes only for $q^h$ between 1990 and 2011. Shaded areas correspond to NBER recession dates.
Figure 25: Cyclical component of the unemployment rate. Data vs. simulation using estimated processes only for a between 1990 and 2011. Shaded areas correspond to NBER recession dates.
A.2.1 Additional Model Simulations

A.2.2 Accuracy of the Solution

Figure ?? shows an accuracy plot which compares the actual values of $\theta$ from a simulation of the model with the model forecast based on $H$. Importantly, the latter series does not include the actual $\theta$ as an input, but makes forecasts based on the last-period prediction. This means that errors are allowed to accumulate over time. Figure ?? shows that the two lines are almost indistinguishable. The average percentage difference is 0.002%. The maximum percentage difference is 0.005%. This suggests that $H$ is successful in tracking the simulated dynamics of $\theta$.

A.3 Computational Strategy

For the solution of the model I use a non-stochastic grid method. While this method requires finer grids for firm-specific labor and productivity, it has the great advantage of eliminating sampling error. As Den Haan (2010) shows, sampling error can lead to severe distortions in the model’s results. This is all the more important in my setup, as the mass of entering firms can be small relative to the mass of incumbents. Therefore sampling uncertainty may bias the results even though the overall number of firms is large.

Before beginning the simulation I create fine grids for $n$ and $\epsilon$. Denote the number of grid points by $\#_n$ and $\#_\epsilon$, respectively. I specify an initial distribution over the points $[n_i, \epsilon_j]$, where $i \in [1, 2, \ldots, \#_n]$ and $j \in [1, 2, \ldots, \#_\epsilon]$. This determines the mass of firms with employment $n_i$ and productivity $\epsilon_j$. The simulation then follows this iterative process:

1. At each grid point incumbent firms decide whether to continue operation or exit. The decision is based on equation (9) above.

2. New firms enter based on equation (11).

3. The aggregate productivity state realizes according to its law of motion specified in (13).

4. The idiosyncratic productivity state realizes. This implies distributing the mass at each point $[n_i, \epsilon_j]$ to a new point $[n_i, \epsilon_k]$, where $k \in [1, 2, \ldots, \#_\epsilon]$, according to the law of motion specified in (14).

5. Apply the employment policy function. This involves distributing the mass at each point $[n_i, \epsilon_k]$ to $[n_i', \epsilon_k]$, where $n_i'$ is given by the firm’s policy rule resulting from the maximization of (4).

6. Go back to step 1.
The simulation algorithm takes as given the policy functions for employment (hires, fires, and inaction) \( \phi_e \), and exit, as well as the laws of motion of all exogenous states, \( \pi_e \) and \( \pi_A \). To find a solution for a given aggregate state \( A \), it iterates on a distribution over employment and idiosyncratic productivity, \( \lambda(e, \epsilon) \) and finds its fixed point, where

\[
\lambda_{t+1}(\bar{e}_l, \bar{\epsilon}_m) = \sum_{i=1}^{M} \sum_{j=1}^{N} \Pr(\phi_e(\bar{e}_i, \bar{\epsilon}_j) = \bar{e}_l | \bar{e}_t = \bar{e}_i, \epsilon_t = \bar{\epsilon}_j) \pi_{jm} \lambda_t(\bar{e}_i, \bar{\epsilon}_j).
\]

The distribution \( \lambda \) has dimensionality \( (#_e \cdot #_\epsilon \times 1) \), where \( #_e \) and \( #_\epsilon \) respectively refer to the number of grid points for employment and the idiosyncratic shock. In practise the law of motion is set up by combining the policy functions and the law of motion for the idiosyncratic state into a large transition matrix \( \Gamma \), which has dimensionality \( (#_e \cdot #_\epsilon \times #_e \cdot #_\epsilon) \). This transition matrix \( \Gamma \) may vary for incumbents and entering firms, since entrants are allowed to have a different initial transition matrix for the idiosyncratic shock. The non-zeros in the row associated with \( \bar{e}_i, \bar{\epsilon}_j \) are then defined as

\[
\Gamma((i-1) \cdot #_e + j, (\phi_e(i, j) - 1) \cdot #_\epsilon + 1 : \phi_e(i, j) \cdot #_e) = \pi_e(i, :) \cdot (1 - \phi_x(i, j)).
\]

Then we can rewrite the law of motion for \( \lambda \) as

\[
\tilde{\lambda}_1 = \tilde{\lambda}_0 \Gamma,
\]

and the solution can be found by iteration or solving \( \tilde{\lambda} = \tilde{\lambda} \Gamma \), where \( \tilde{\lambda} \) is the eigenvector of \( \Gamma \) that is associated with its unitary eigenvalue.

In the presence of an aggregate shock the algorithm can obviously not be used to compute a stationary distribution. But the same logic applies and a distribution \( \lambda \), which then has dimensionality \( (#_e \cdot #_\epsilon \cdot #_A \times 1) \) and a transition matrix \( \Gamma \) which then has dimensionality \( (#_e \cdot #_\epsilon \cdot #_A \times #_e \cdot #_\epsilon \cdot #_A) \) can be set up. The simulation then consists of drawing a random sequence of realizations of the aggregate shock and computing \( \lambda_1 = \lambda_0 \Gamma \). The code is available upon request.

### A.4 Extensions (in progress)

**Introducing Financial constraints for all firms** I introduce a working-capital assumption into the model. Firms have to pay a fraction \( \lambda \) of their period expenses at the beginning of the period. Those expenses include the wage bill \( w \cdot e \) and adjustment costs, including the fixed cost. To finance those costs, firms borrow using their housing value \( q^h \) as collateral just like entrants in the benchmark model. For any uncollateralized fraction of the loan the firm has to pay the higher interest rate \( \hat{R} \geq 1 \). At the end of the period, once profits are realized, the entrepreneur pays back the loan to the bank. I assume that an entrepreneur’s realization of \( \epsilon \) is perfectly observable by the bank. This modification essentially makes \( q^h \) a state variable of the entrepreneur’s problem.

Results in progress...
Alternative wage setting: applying Stole and Zwiebel (1996) To apply the Stole and Zwiebel (1996) framework I assume that the agent’s utility function be linear, \( Z(c) = c \). As in Elsby and Michaels (2013) this is done to obtain a closed form solution for the problem.

Details to follow.