Financial Vulnerability and Monetary Policy

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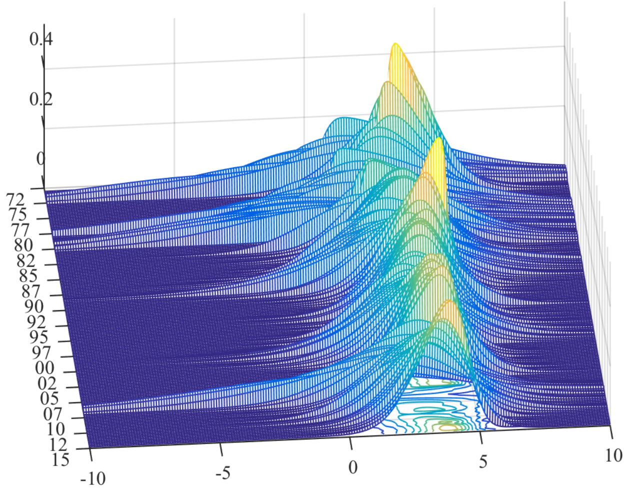
Financial vulnerability: amplification mechanisms in the financial sector

Two questions are hotly debated:

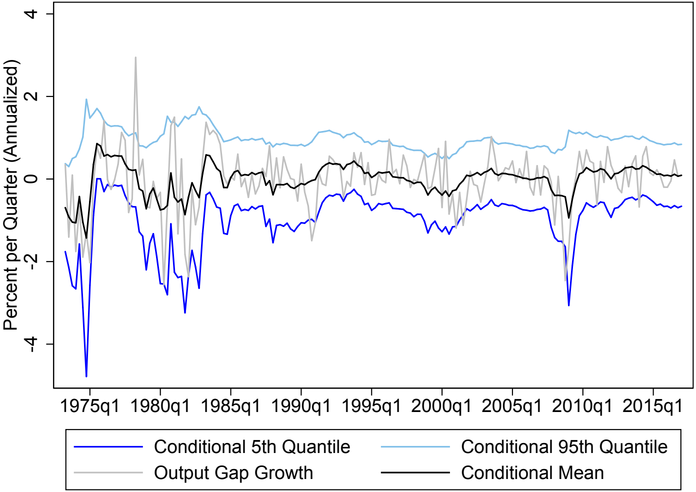
1. Does monetary policy impact the degree of financial vulnerability?
2. Should monetary policy take financial vulnerability into account?

Financial Variables Predict Tail of GDP Distribution

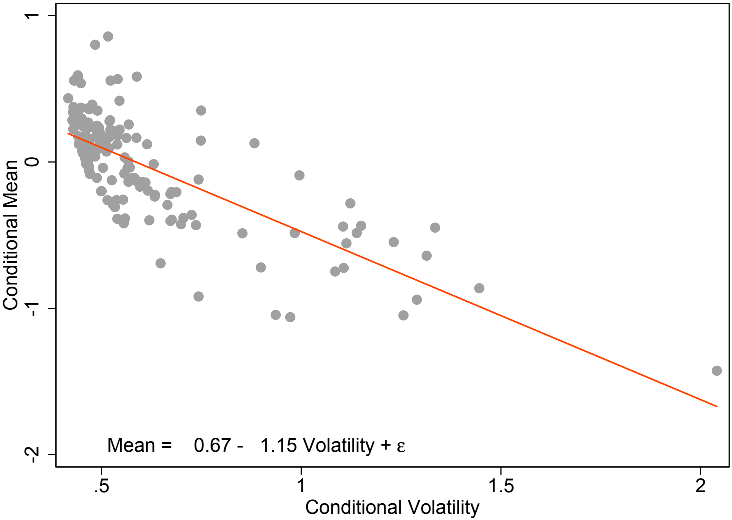
"Vulnerable Growth" by Adrian, Boyarchenko and Giannone (2016)



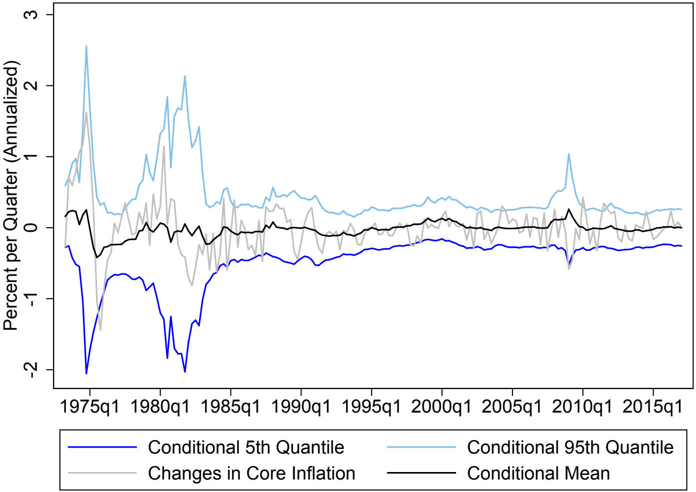
Conditional Distribution of Output Gap Growth



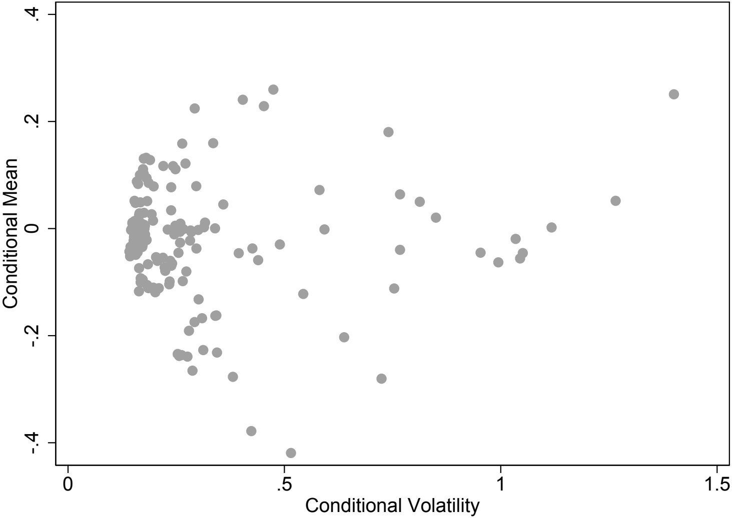
Conditional Mean-Volatility Line for Output Gap Growth



Inflation Quantiles are More Symmetric



Mean-Volatility Relation for Inflation



Modeling Financial Vulnerability: Reduced Form

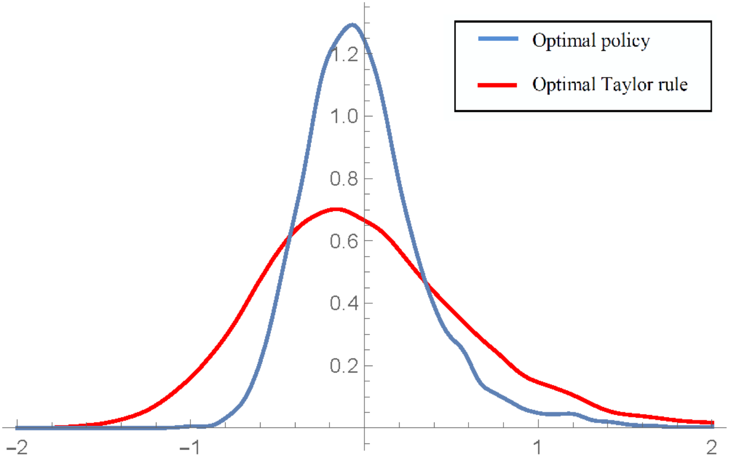
- Define GDP vulnerability V_t as

$$V_t = -\tau \mathbb{E}_t[dy_t/dt] - \mathcal{N}^{-1}(\alpha)\sqrt{\tau} Vol_t[dy_t/dt]$$

- New Keynesian IS curve with a risk premium driven by vulnerability

$$\begin{aligned}
dy_t &= \gamma^{-1}(i_t - r - \pi_t)\,dt + d(rp_t) \\
d\pi_t &= (\beta\pi_t - \kappa y_t)\,dt \\
d(rp_t) &= \eta\xi\left(V_t - s_t - \frac{\eta}{2\xi\gamma}\right)dt + \xi(V_t - s_t)dZ_t \\
ds_t &= -\rho_s(s_t - \bar{s})\,dt + \sigma_s dZ_t
\end{aligned}$$

GDP Distributions under Optimal Policy and Taylor Rules



Overview of Microfounded Non-Linear Model

- Firms are exactly as in basic New Keynesian model
- Households are as in New Keynesian model but
  - Cannot finance firms directly
  - Trade other financial assets (stocks, bonds, desposits) with banks
- Banks
  - Finance firms
  - Trade financial assets among themselves and with households
  - Less risk averse than household
  - Has a preference (risk aversion) shock
  - Subject to Value-at-Risk constraint
- Markets are complete and Modigliani-Miller theorem holds everywhere

Financial Structure and Available Securities

- Single source of risk: Browninan motion Z_t
- Real riskless bond in zero net supply

$$dS_{0,t} = S_{0,t} R_t dt$$

- Two real stocks in positive net supply with returns

$$\frac{dS_{goods,t}}{S_{goods,t}} = (\mu_{goods,t} - R_t)dt + \sigma_{goods,t} dZ_t$$
$$\frac{dS_{bank,t}}{S_{bank,t}} = (\mu_{bank,t} - R_t)dt + \sigma_{bank,t} dZ_t$$

where $\mu_{j,t}$ are excess returns including real dividend flow $D_{j,t}$

- Complete set of Arrow-Debreu securities in zero net supply

Price of Risk and No Arbitrage

- A state price density (SPD) is a process with $Q_0 \equiv 1$ and

$$\frac{dQ_t}{Q_t} \equiv -R_t dt - \eta_t dZ_t$$

such that for all assets j

$$S_{j,t} = \frac{1}{Q_t}\mathbb{E}_t\left[\int_t^\infty Q_s D_{j,s} ds\right]$$

where η_t is the "market price of risk"

Representative Household

- Household solves

$$\max_{\{C_t, N_t, \omega_t\}_{t \geq s}} \mathbb{E}_s \left\{ \int_s^\infty e^{-\beta(t-s)} \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varsigma}}{1+\varsigma} \right) dt \right\}$$

subject to

$$d\left(P_t F_t\right) \leq W_t N_t dt - P_t C_t dt - P_t T_t dt + \omega_t d\left(P_t S_t\right)$$

$$\omega_{goods,t} = 0$$

$$\lim_{t \to \infty} \mathbb{E}_s \left[Q_t F_t\right] = 0, \quad F_s \text{ given}$$

Firms are Standard New Keynesian

- Linear production $Y_t(i) = N_t(i)$
- The FOC for intermediate good producers linearized around deterministic steady sate gives the standard New Keynesian Phillips Curve

$$d\pi_t = (\beta\pi_t - \kappa y_t)\, dt$$

The Intermediation Sector Setup

- Each "bank" solves a standard Merton portfolio choice problem augmented by a Value-at-Risk constraint and preference shocks

$$V(X_s, s) = \max_{\{\theta_t, \delta_t\}_{t \geq s}} \mathbb{E}_s \left[\int_s^\infty e^{-\beta(t-s)} e^{\zeta_t} \log(\delta_t X_t)\, dt \right]$$

$$\begin{aligned} & s.t. \\ \frac{dX_t}{X_t} &= (R_t - \delta_t + \theta_t \mu_t)\, dt + \theta_t \sigma_t dZ_t \\ VaR_{\tau,\alpha}(X_t) &\leq a_V X_t \\ d\zeta_t &= -\frac{1}{2} s_t^2 dt - s_t dZ_t \\ ds_t &= -\kappa(s_t - \bar{s}) + \sigma_s dZ_t \\ X_s & \text{ given} \end{aligned}$$

The Banks' VaR Constraint

- Let $\hat{X}_t$ be projected wealth with fixed portfolio weights from t to $t+\tau$
- $VaR_{\tau,\alpha}(X_t)$ is the α^{th} quantile of the distribution of $\hat{X}_{t+\tau}$ conditional on time-t information
- VaR limit is proportional to X_t

$$VaR_{\tau,\alpha}(X_t) \leq X_t a_V \iff g(t, \theta_t, \delta_t, R_t, s_t) \leq \log \frac{1}{1-a_V} \equiv \overline{VaR}$$

where

$$g(t, \theta_t, \delta_t, R_t, s_t) \equiv -\tau \mathbb{E}_t[d \log \hat{X}_t] - \mathcal{N}^{-1}(\alpha)\sqrt{\tau} Vol_t(d \log \hat{X}_t)$$

Optimal Portfolio

The optimal portfolio is characterized by

$$
\begin{aligned}
\theta_t &= \min\{1, \max\{0, \varphi_t\}\}\, \theta_{M,t} \\
&= \gamma_t^{-1} \theta_{M,t} \\
\delta_t &= u\left(t, \min\{1, \varphi_t\}\right) f_{M,t} \\
\varphi_t \text{ such that: } \quad & g\left(t, \theta_t, \delta_t, R_t, s_t\right) = \overline{VaR}
\end{aligned}
$$

with

$$
\begin{aligned}
\theta_{M,t} &= \mu_t/\sigma_t^2 - s_t/\sigma_t \\
f_{M,t} &= \beta \\
u\left(t, z\right) &\equiv 1 + \frac{\sqrt{\tau}\left|\theta_{M,t}\sigma_t\right|}{\mathcal{N}^{-1}\left(\alpha\right)}\left(1 - z\right)
\end{aligned}
$$

Market Clearing Conditions

Intermediate goods : $(P_t(i)/P_t)^{-\varepsilon} Y_t = N_t(i)$

Final good : $C_t = Y_t$

Labor : $\left((W_t/P_t) C_t^{-\gamma}\right)^{\frac{1}{\varsigma}} = Y_t(i)$

Bank stock : $\frac{F_t \omega_{bank,t}}{S_{bank,t}} + \frac{X_t \theta_{bank,t}}{S_{bank,t}} = 1$

Goods producer stock : $\frac{X_t \theta_{goods,t}}{S_{goods,t}} = 1$

Zero net supply securities : $\frac{F_t \omega_{j,t}}{S_{j,t}} + \frac{X_t \theta_{j,t}}{S_{j,t}} = 0$ for $j \neq bank, goods$

Equilibrium Conditions

- The system of equations

$$
\begin{aligned}
dy_t &= \frac{1}{\gamma}\left(i_t - r - \pi_t + \frac{1}{2}\eta_t^2\right)dt + \frac{\eta_t}{\gamma}dZ_t \\
d\pi_t &= \left(\beta\pi_t - \kappa y_t\right)dt \\
i_t &= i(\text{state variables}) \\
R_t &= i_t - \pi_t \\
\eta_t &= \eta\left(\varphi_t, R_t, s_t\right) \\
d\varphi_t &= G_\varphi\left(\varphi_t, R_t, s_t\right)dt + S_\varphi\left(\varphi_t, R_t, s_t\right)dZ_t \\
ds_t &= -\kappa\left(s_t - \overline{s}\right) + \sigma_s dZ_t
\end{aligned}
$$

fully characterizes the equilibrium

Characterization with Output Gap Vulnerability

- To connect model with the data, introduce

$$
\begin{aligned}
V_t &\equiv -\tau \mathbb{E}_t[dy_t/dt] - \mathcal{N}^{-1}(\alpha)\sqrt{\tau} Vol_t(dy_t/dt) \\
&= -\frac{\tau}{\gamma}\left(R_t - r + \frac{1}{2}\eta_t^2\right) - \mathcal{N}^{-1}(\alpha)\frac{\sqrt{\tau}\eta_t}{\gamma}
\end{aligned}
$$

- Write $\eta_t = \eta(\varphi_t, R_t, s_t)$ in terms of φ_t, V_t, s_t

Equilibrium Conditions

- The system of equations

$$
\begin{aligned}
dy_t &= \frac{1}{\gamma}\left(i_t - r - \pi_t + \frac{1}{2}\eta(\varphi_t, V_t, s_t)^2\right) dt + \frac{\eta(\varphi_t, V_t, s_t)}{\gamma} dZ_t \\
d\pi_t &= (\beta\pi_t - \kappa y_t)\, dt \\
i_t &= i(\text{state variables}) \\
R_t &= i_t - \pi_t \\
V_t &= -\tau\mathbb{E}_t[dy_t/dt] - \mathcal{N}^{-1}(\alpha)\sqrt{\tau}\, Vol_t(dy_t/dt) \\
d\varphi_t &= G_\varphi(\varphi_t, R_t, s_t)\, dt + S_\varphi(\varphi_t, R_t, s_t)\, dZ_t \\
ds_t &= -\kappa(s_t - \overline{s}) + \sigma_s dZ_t
\end{aligned}
$$

fully characterizes the equilibrium

"Second Order" Linear Approximation

- Linearize drift and volatility

$$
\begin{aligned}
dy_t &= \left(\frac{1}{\gamma}(R_t - r) + \hat{\eta}\xi\left(V_t - \varphi_t - s_t - \frac{\hat{\eta}}{2\xi\gamma}\right)\right)dt + \\
&\quad + \xi\,(V_t - \varphi_t - s_t)dZ_t \\
d\pi_t &= (\beta\pi_t - \kappa y_t)\,dt \\
i_t &= i(\text{state variables}) \\
R_t &= i_t - \pi_t \\
V_t &= -\tau\mathbb{E}_t[dy_t/dt] - \mathcal{N}^{-1}(\alpha)\sqrt{\tau}\,Vol_t(dy_t/dt) \\
d\varphi_t &= (\Upsilon_0 + \Upsilon_r R_t + \Upsilon_\varphi \varphi_t + \Upsilon_s s_t)\,dt + \\
&\quad + (\Psi_0 + \Psi_r R_t + \Psi_\varphi \varphi_t + \Psi_s s_t)\,dZ_t \\
ds_t &= -\kappa\,(s_t - \bar{s}) + \sigma_s dZ_t
\end{aligned}
$$

Optimal Monetary Policy

- Focus on simpler case with no direct impact of monetary policy on φ: Mechanism is through general equilibrium (prices of risk) only
- Abstract from Phillips Curve
  - Either fixed prices or long-run equilibrium
  - Straightforward to incorporate into analysis
- General case still linear-quadratic, can be solved in closed form

Optimal Monetary Policy

- Central bank solves

$$L = \min_{\{R_s\}_{s=t}^{\infty}} \mathbb{E}_t \int_t^{\infty} e^{-s\beta} y_s^2 ds$$

subject to

$$dy_t = \left(\frac{1}{\gamma} (R_t - r) + \hat{\eta}\xi \left(V_t - s_t - \frac{\hat{\eta}}{2\xi\gamma} \right) \right) dt + \xi (V_t - s_t) dZ_t$$

$$V_t = -\tau \mathbb{E}_t[dy_t/dt] - \mathcal{N}^{-1}(\alpha)\sqrt{\tau} Vol_t(dy_t/dt)$$

$$ds_t = -\kappa (s_t - \bar{s}) + \sigma_s dZ_t$$

Optimal Monetary Policy

- Plugging

$$\mathbb{E}_t[dy_t/dt] = \frac{1}{\gamma}(R_t - r) + \hat{\eta}\xi\left(V_t - s_t - \frac{\hat{\eta}}{2\xi\gamma})\right)$$

$$Vol_t(dy_t/dt) = \xi(V_t - s_t)$$

into

$$V_t = -\tau\mathbb{E}_t[dy_t/dt] - \mathcal{N}^{-1}(\alpha)\sqrt{\tau}Vol_t(dy_t/dt)$$

we see that V_t and R_t are one-to-one

Output Gap Mean-Volatility Tradeoff

- R_t and V_t are one-to-one, so think of V_t as central bank's choice
- Eliminating R_t, dynamics of the economy are

$$dy_t \quad = \quad \xi \left(M \times V_t + \frac{\mathcal{N}^{-1}(\alpha)}{\sqrt{\tau}} s_t \right) dt + \xi \left(V_t - s_t \right) dZ_t$$

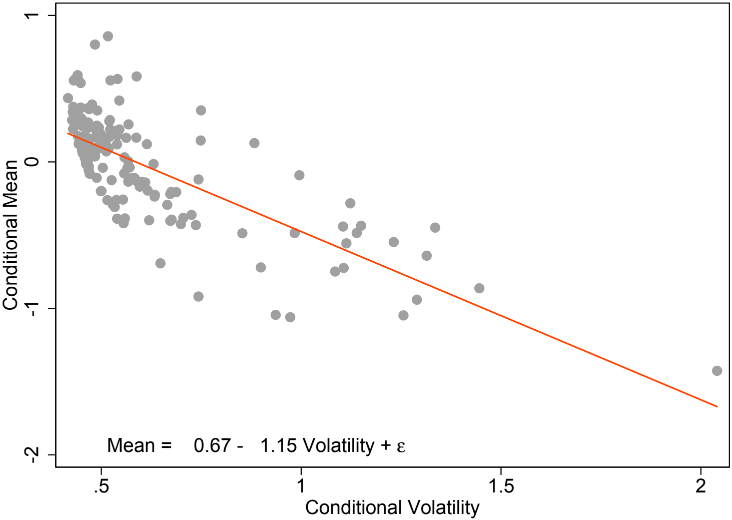
where

$$M \equiv -\frac{1 + \mathcal{N}^{-1}(\alpha)\sqrt{\tau}\xi}{\tau\xi}$$

is the slope of the mean-volatility line for the output gap

$$\mathbb{E}_t \left[dy_t / dt \right] \quad = \quad M \times Vol_t \left(dy_t / dt \right) - \frac{1}{\tau} s_t$$

Conditional Mean-Volatility Line for Output Gap Growth



Tradeoff for Monetary Policy

- Mean variance tradeoff since $M < 0$

$$dy_t = \xi \left(M \times V_t + \frac{\mathcal{N}^{-1}(\alpha)}{\sqrt{\tau}} s_t \right) dt + \xi \left(V_t - s_t \right) dZ_t$$

- Changes in V_t move the economy along the mean-vol line
- Shocks s_t shift the line up and down
- Because $M < 0$, we have $\partial V_t / \partial R_t < 0$: Tighter policy reduces vulnerability
- Divine coincidence broken by financial vulnerability

The Optimal Monetary Policy

- Re-introduce Phillips Curve
- Augmented Taylor

$$i_t = \phi_0 + \phi_\pi \pi_t + \phi_y y_t + \phi_v V_t$$

- Can be expressed as flexible inflation targeting

$$\pi_t = \psi_0 + \psi_y y_t + \psi_v V_t + \psi_s s_t$$

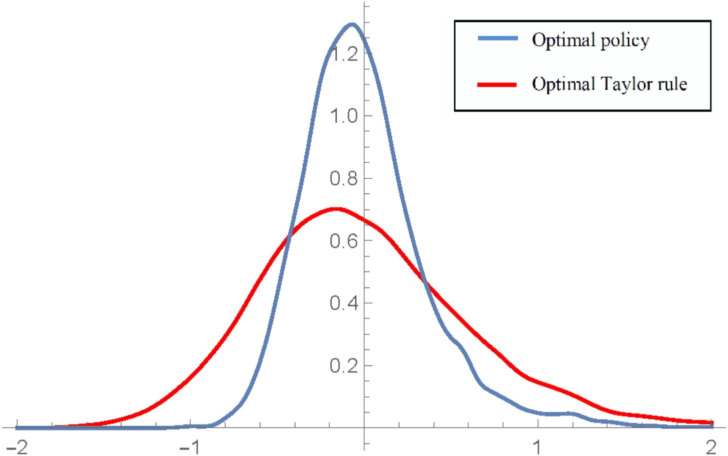
- Coefficients ϕ and ψ are a function of structural parameters that govern GDP vulnerability

Calibration

- Calibration comes directly from a regression of the conditional mean on the conditional vol of the output gap
- Pick $\beta = 0.01, \alpha = 5\%, \tau = 1$ and match intercept, slope, standard deviation and AR(1) coefficient of residuals to get

$$\begin{aligned} \xi &= 0.36 \\ \bar{s} &= -0.67 \\ \sigma_s &= 0.61 \\ \kappa &= 2.14 \end{aligned}$$

Welfare Gains



- Our calibration gives answers different from Svensson, Curdia and Woodford
- Empirically
  - Look at different data – risk premia in financial sector
  - Forecast downside risk, not just conditional means
- Theoretically
  - Different amplification mechanism brought about by VaR constraint
  - Not just about crises and tails; tradeoff is always present

Conclusion

- The NK model can be augmented by
  - A financial sector that intermediates subject to a Value-at-Risk constraint
  - Shocks to financial sector
- The “second order” linearization approximation
  - Matches the stylized fact that conditional upper GDP quantiles are constant, while lower GDP quantiles move with financial conditions
  - Mathematically tractable
- Optimal monetary policy always depends on vulnerability
  - Optimal monetary policy conditions on vulnerability
  - Vulenrability responds to monetary policy
  - Magnitudes are potentially large quantitatively