

**Automatic identification of RegARIMA models in Large Scale  
Applications: Program TSW  
(TRAMO-SEATS for Windows)**

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Abstract

The paper presents an overview of the methodology behind program TSW and an application to a set of 500 monthly European export series. The application focuses on automatic model identification and the detection of problematic series.

TSW is a Windows interface of updated versions of programs TRAMO (Time series Regression with Arima noise, Missing values, and Outliers) and SEATS (Signal Extraction in ARIMA Time Series). The program estimates a general regression-ARIMA model, and computes forecasts and interpolators for possibly non-stationary series, with any sequence of missing observations, and in the presence of outliers. Several types of intervention or regression variables can be included. The program contains an option for automatic model identification, automatic detection and correction of several types of outliers, and for testing and estimation of Calendar-based effects.

From the model identified for the observed series, filters -that provide the MMSE estimators (and forecasts) of the trend, seasonal, calendar, transitory, and noise components in the series- are obtained using signal extraction techniques applied to ARIMA models. The program contains a part on diagnosis and on inference, and an analysis of the properties of the estimators and of the estimation and forecasting errors.

Because the programs can efficiently and reliably handle automatic applications to sets of many thousand series, they are already being used intensively in data producing agencies (perhaps the most widely used application is Seasonal Adjustment). They are also used at policy making institutions, business, and research.

The paper presents the aggregate results obtained for the full set of series when a purely automatic procedure is applied. It serves thus to illustrate the performance and reliability of TSW in routine applications, when many series need to be treated fast. The aggregate results summarize the models obtained, the outlier and calendar effects detected, the in-sample fitting and the out-of-sample forecasting performance.

Then, it is shown how problematic series -i.e., series that fail some diagnostic or that display inconvenient features- can be efficiently identified and (possibly) corrected.

## 1. INTRODUCTION

Short-term economic data (say, monthly data) are subject to short-term “noise” that, if not properly discounted, can be seriously misleading when interpreting the underlying medium- or long-term evolution of the variables. Among the different types of noise, seasonal variations play a dominant role, and seasonal adjustment of economic data has indeed a long tradition. At present, every month many thousand series are routinely adjusted by data-producing agencies.

Since the appearance of the US Bureau of the Census (USBC) X11 method, adjustment of data has been overwhelmingly done with a few filters designed a priori to have some desirable property, namely, removal of the series variation associated with seasonal frequencies. However, blind application of these a-priori designed (APD) filters presents some shortcomings. Lacking a precise definition of seasonality, one seems bound to accept as such simply the output of the filter. Yet insofar as series with no spectral peak at a seasonal frequency may well display power for that frequency, the filter would estimate a seasonal component. This component would of course be spurious.

Ultimately, the seasonal component (or trend component) that a filter estimates is an estimator of an unobserved component and is therefore contaminated by an estimation error. How large can this error be expected to be? The answer to this question is crucial for a rigorous assessment of the present evolution and, naturally, for proper enforcement of policies based on (explicit or implicit) targeting. For example, assume a monetary policy based on annual targeting of a monetary aggregate growth. Further, assume that in December, a growth of 8% is targeted for the next year, and in January the seasonally adjusted (SA) series shows a 10% growth (on annualized terms). Could the 2% difference be explained by the size of the measurement error? In other words, is the difference not significantly different from zero? If it is not, the interest rate instrument should be left unmoved; on the contrary, if the difference turns out to be significant, the interest rate should be increased. Clearly, it is important to avoid reacting to noise and not to the proper signal.

The magnitude of the estimation error in the SA series is far from trivial. As an example, simply consider one (particularly relevant) component of the estimation error: the so-called “revision” error, implied by the two-sided features of most filters used in practice. Being two-sided, for recent periods, the filter needs to be completed with forecasts of the series and, as new observations become available and replace the forecasts, the estimator will be revised until it converges to its final value. The difference between the preliminary estimator (obtained with forecasts) and the final one is the revision error. Its effect is disturbing because in practice it implies that the SA series should be revised monthly for several years.

Maravall and Pierce (1986) considered US monetary policy in the decade of the 70's, a period of monetary aggregate targeting and seasonal adjustment with X11. A range of tolerance was set for the monetary aggregate growth, and when the SA monthly observation indicated excessive (insufficient) growth, the funds rate were to be increased (decreased). If growth fell within the tolerance range, the rate was to stay unchanged. Maravall and Pierce measured how often the final and concurrent estimators of the monthly SA M1 series implied different interest rate reactions; they found that for close to 40% of the months the concurrent and final signal disagreed.

Unfortunately, computation of the standard deviation of the SA series estimation error does not have a clear answer in the context of APD filters.

In the early 80's proposals were made to use Auto Regressive Integrated Moving Average (ARIMA) models, together with Signal Extraction techniques of the Wiener-Kolmogorov type (extended to nonstationary series) to estimate signals (or unobserved components) such as the SA series or the trend. (Important contributions were made by Box, Hillmer, Tiao, Burman, Bell, Cleveland, and Pierce; reference should also be made to the early related work of Nerlove and Grether.) The ARIMA model for each series would be identified from the stochastic structure of the series, and correct specifications could be assessed with standard diagnostics. Next, a stochastic partial fractions decomposition would yield the ARIMA model for each unobserved component, and hence seasonality would be fully defined through its model. If the model specification is correct, the filter, determined from the series structure, would avoid the danger of spuriousness. Finally, the model-based structure could be exploited to provide the desired inference, and in particular, the standard deviation of the signal estimation error.

This model-based method was appealing, but it confronted some serious problems. For large scale application it required very heavy doses of time series analyst resources, because there were no automatic model identification (AMI) procedures that were reliable enough. Further, it seemed to require heavy computational resources because the identification and estimation algorithms available were relatively cumbersome. Finally, to properly handle many real series, ARIMA models had to be extended in several directions; some important ones were the capacity to detect and remove possible outliers, possible calendar effects that are non-periodic and other regression-type effects, and the capacity to handle missing observations. To be able to deal with these extensions, ARIMA models have to be replaced by the so-called Regression-ARIMA models. Of course, outlier and calendar effect detection, as well as interpolation, have to be incorporated to the complete AMI procedure.

In the decade of the 90's, Gómez and Maravall completed a Regression-ARIMA model-based methodology, and produced a pair of connected programs to enforce it. They developed an efficient and reliable automatic procedure that could be applied to very large sets of series. In the last ten years use of the programs has become widespread throughout the world, and they are at present intensively used at economic agencies and institutions, research, business, and –most relevantly- in data producing agencies, where seasonal adjustment is the most widely used application. The two programs are named TRAMO (“Time series Regression with ARIMA noise, Missing values and Outliers”) and SEATS (“Signal Extraction in ARIMA Time Series”); they are freely available, together with program TSW (a Windows version) and additional tools and documentation, at the Bank of Spain web site ([www.bde.es](http://www.bde.es)).

The paper contains an application of program TSW to a set of 500 monthly time series. The program estimates a general Regression-ARIMA model, and computes forecasts and interpolators for possibly nonstationary series, with any sequence of missing observations, and in the presence of outliers. The program contains an option for automatic model identification, automatic detection and correction of several types of outliers, and for pretesting and estimation of Calendar-based effects. Several types of intervention or regression variables can be included. Next, the program estimates and forecasts the trend-cycle, seasonal, calendar, transitory and noise components in the series, using signal extraction

techniques applied to ARIMA models. The program contains a part on diagnosis and on inference, and an analysis of the properties of the estimators and of the estimation and forecasting errors. The last part of the output is oriented towards its use in short-term economic policy and monitoring.

The program can be run in several entirely automatic modes.

The first part of the paper describes the methodology underlying program TSW. The second part discusses application of TSW to a set of 500 monthly European export and import series. The discussion centers around the basic automatic procedure (RSA = 4 is the only input; see Caporello and Maravall, 2004) and illustrates thus what could be the performance of the program in routine applications to large sets of series.

## 2. BRIEF DESCRIPTION OF TSW

The first part of the program corresponds to TRAMO; the second part corresponds to SEATS. In the first part, the program estimates and forecasts regression models with errors that follow (most often non-stationary) ARIMA processes. There may be:

- missing observations in the series,
- contamination by outliers,
- contamination by other special (deterministic) effects.

Important cases of the latter are the trading day (TD) effect, caused by the different distribution of weekdays in different months, and Easter effect (EE), which captures the moving dates of Easter.

If  $B$  is the lag operator, such that  $B x(t) = x(t - 1)$ , given the observations  $y = [y(t_1), y(t_2), \dots, y(t_m)]$ , where  $0 < t_1 < \dots < t_m$ , the model can be expressed as

$$y(t) = \sum_{i=1}^{n_{out}} \omega_i \lambda_i(B) d_i(t) + \sum_{i=1}^{n_c} \alpha_i cal_i(t) + \sum_{i=1}^{n_{reg}} \beta_i reg_i(t) + x(t), \quad (2.1)$$

where

$d_i(t)$ : dummy variable that indicates the position of the  $i$ -th outlier,

$\lambda_i(B)$ : polynomial in  $B$  reflecting the outlier dynamic pattern,

$cal_i$ : calendar-type variable,

$reg_i$ : regression or intervention variable,

$x(t)$ : ARIMA error,

$\omega_i$ : instant  $i$ -th outlier effect,

$\alpha_i$  and  $\beta_i$ : coefficients of the calendar and regression-intervention variables, respectively,

$n_{out}$ ,  $n_c$  and  $n_{reg}$ : total number of variables entering each summation term in (2.1).

In compact notation, (2.1) can be rewritten as

$$y(t) = z'(t) b + x(t), \quad (2.2)$$

where  $b$  is the vector with the  $\omega$ ,  $\alpha$  and  $\beta$  coefficients, and  $z'(t)$  is the matrix with the columns containing the variables in the three summation terms of (2.1). The component  $z'(t) b$  represents the deterministic component and accounts for the “regression” part of the model.

Let the ARIMA model for  $x(t)$  be

$$\phi(B) \delta(B) x(t) = \theta(B) a(t) , \quad (2.3)$$

where  $a(t)$  is a white-noise  $(0, V_a)$  innovation. In (2.3),  $\phi(B)$ ,  $\delta(B)$ , and  $\theta(B)$  are finite polynomials in  $B$ . The first one contains the stationary autoregressive (AR) roots,  $\delta(B)$  contains the nonstationary AR roots, and  $\theta(B)$  is an invertible moving average (MA) polynomial. Stationarity of the AR and invertibility of the MA imply that all roots of  $\phi(z)$  and  $\theta(z)$ , respectively, where  $z = B^{-1}$ , are smaller than one in modulus.

Let  $s$  denote the number of observations per year. The polynomials assume the multiplicative form

$$\delta(B) = \nabla^d \nabla_s^{d_s} ,$$

$$\phi(B) = (1 + \phi_1 B + \dots + \phi_p B^p) (1 + \phi_s B^s) ,$$

$$\theta(B) = (1 + \theta_1 B + \dots + \theta_q B^q) (1 + \theta_s B^s) ,$$

where  $\nabla = 1 - B$  and  $\nabla_s = 1 - B^s$ . This model will be referred to as the ARIMA  $(p, d, q)(p_s, d_s, q_s)_s$  model. The model consisting of (2.2) and (2.3) will be called a regression (reg)-ARIMA model.

When used automatically, the program

- tests for the log/level transformation,
- tests for the presence of calendar effects,
- detects and corrects for three types of outliers: Additive Outliers (AO), Transitory Changes (TC), and Level Shifts (LS); where AO is an isolated spike; TC is a spike that dies off gradually; and LS is a (permanent) step;
- identifies and estimates by exact maximum likelihood (EML) the reg-ARIMA model,
- interpolates missing values,
- computes forecasts of the deterministic component  $z'(t) b$  and of the ARIMA series  $x(t)$ .
- Interpolators and forecasts are obtained as MMSE estimators.

In default estimation of model (2.2),  $b$  is concentrated out of the likelihood and estimation is iterative: Conditional on  $b$ , the ARIMA parameters are estimated by EML; conditional on the ARIMA model, the  $b$  parameters are estimated by GLS.

### 3. SUMMARY OF AUTOMATIC MODEL IDENTIFICATION (AMI) PROCEDURE

#### 3.1 Default Model and Pretesting

##### *Pretest for the Log-level Specification*

The test consists of direct comparison of the BICs of the default model in levels and in logs (with a proper correction; see Gómez and Maravall, 2001b).

##### *Pretest for Trading Day and Easter Effects*

The Test is performed with regressions using the default model for the noise and, if the model is subsequently changed, the test is redone. Because underdetection is easier to handle than overdetection, by default, TRAMO has a slight bias towards underdetection.

##### *Default Model*

When a pretest for seasonality indicates that it seems to be present in the series, the default model is the so-called Airline model, given by (for monthly series)

$$\nabla \nabla_{12} x(t) = (1 + \theta_1 B) (1 + \theta_{12} B^{12}) a(t) , \quad -1 < (\theta_1, \theta_{12}) < 1 \quad (3.1)$$

Pretesting and the starting point of AMI depend heavily on this model. Three important reasons justify this choice:

- Many studies have shown it is a model appropriate for many real monthly or quarterly macroeconomic series.
- The Airline model approximates well many other models. As an example, consider the deterministic “linear trend plus seasonal dummies” model

$$x(t) = \mu_0 + \mu_1 t + \sum_{i=1}^{11} \beta_i d_i(t) + a(t) ,$$

where the  $\mu$  and  $\beta$  parameters are constants, and  $d_i(t)$  is a dummy variable with 1 when  $t$  corresponds to the  $i$ -th month of the year and 0 otherwise. For a standard series with a few hundred observations, if the deterministic model yields a good fit, the same will be true for model (3.1) with  $\theta_1$  and  $\theta_{12}$  very close to -1.

- The model provides an excellent “benchmark” model. (For  $\theta_{12} < 0$  the model yields a sensible decomposition into trend-cycle + seasonal + irregular component, and might be preferred to a slightly better-fitting model with an awkward decomposition.)

The model contains 3 parameters (namely,  $\theta_1$ ,  $\theta_{12}$ , and  $V_a$ ) that can be given an interpretation:

- \*  $\theta_1$  is related to the stability of the trend-cycle component. (Values close to -1 imply stable trends.)
- \*  $\theta_{12}$  is related to the stability of the seasonal component. (Values close to -1 imply stable seasonality.)
- \*  $V_a$  measures the variance of the one period-ahead forecast error. (This error will contain, besides the irregular component, the effect of the innovations in the trend-cycle and seasonal component.)

When the preliminary test indicates that seasonality is not present, the Airline model is replaced by the “IMA (1,1) + drift” model. In this case, AMI will nevertheless reconsider the possible presence of seasonality and the model finally identified could incorporate seasonality into its structure.

Notice that a purely regular model (i.e., with no polynomial in  $B^s$ ) may still contain stochastic seasonality due to roots of the regular AR polynomial associated with a seasonal frequency.

### 3.2 Automatic Model Identification in the Presence of Outliers

The algorithm iterates between the following two stages

- automatic outlier detection and correction,
- automatic model identification.

The first model used is the default model. At each step, the series is corrected for the outliers and other regression effects present at the time, and a new AMI is performed. If the model changes, the automatic detection and correction of outliers is performed again from the beginning.

#### (a) AUTOMATIC OUTLIER DETECTION AND CORRECTION (AODC)

1. Assume that we know the location ( $t=T$ ), but not the type of outlier. Given the ARIMA model, for  $t = T$ , we compute:  $\hat{\omega}_{AO}(T)$ ,  $\hat{\omega}_{TC}(T)$ ,  $\hat{\omega}_{LS}(T)$ , the associated t-values

$$\hat{t}_{AO}(T), \hat{t}_{TC}(T), \hat{t}_{LS}(T)$$

and  $\lambda_T = \max \{ |\hat{t}_{AO}(T)|, |\hat{t}_{TC}(T)|, |\hat{t}_{LS}(T)| \}$ . Then,  $\lambda_T > VA$  is used to test for significance, where  $VA$  is an "a priori" set critical value. (The default value of  $VA$  decreases as the number of observations increases.)

2. If we don't know the timing of the outlier, we compute  $\lambda_t$  for  $t = 1, \dots, N$ , and use

$$\lambda_T = \max_t \lambda_t = |\hat{t}_{tp}(T)|$$

If  $\lambda_T > C$ , there is an outlier of type  $tp$  ( $tp$  can be AO, TC, LS) at  $T$ .

After correcting for this outlier, the process is started again to see if there is another outlier. Outliers are removed one by one, until we obtain  $\lambda_T < C$ . Finally we proceed to joint GLS estimation of the multiple outliers in the full model. A multiple regression is performed to avoid (as much as possible) masking effects.

#### (b) AUTOMATIC ARIMA MODEL IDENTIFICATION

Suppose the series  $\{x_t\}$  follows the model (2.3). TRAMO proceeds in two steps:

First, it identifies  $\delta(B)$  (unit roots). Second, it identifies the ARMA model, i.e,  $\phi(B)$  and  $\theta(B)$ .

*Identification of the Nonstationary polynomial  $\delta(B)$*

To determine the appropriate differencing of the series, we discard unit root (UR) testing for the following reasons.

When regular and seasonal UR may be present, available tests have low power. For example, in

$$\nabla x(t) = (1 - .8B) a(t), \text{ or}$$

$$\nabla_{12}x(t) = (1 - .8B^{12}) a(t),$$

the AR unit root would most likely be rejected due to the large MA root.

Besides, in AMI with AODC, one typically tries thousands of models, where the next try depends on previous results. This implies a serious data mining problem: the size of the test is a function of prior rejections and acceptances and its true size is therefore unknown. We follow an alternative approach and decide "a priori", instead of a fictitious size, the following value:

How large the modulus of an AR root should be in order to accept it as 1 (unit root).

For AR and MA roots the criterion is different. Roughly, unit AR roots cause no problem; unit MA roots (in the model for the observed series) are avoided. By default:

For an AR root, if the modulus is  $> .95$ , it is changed to 1 and the root becomes a UR.

For an MA root, if the modulus is  $> .99$  it is changed to .99, so as to have invertibility. (With .99 no numerical problems appear.) These default values can of course be modified to reflect the analyst preference.

To identify AR unit roots we use some useful results (Tiao and Tsay, 1983, 1984) on superconvergence of unit AR roots. The following example illustrates these results.

Let the true model be the AR (3) model

$$(1 - .5B) \nabla^2 x(t) = a(t)$$

Because of superconsistency of UR estimators:

- If we estimate simply an AR(1), the first UR ( $\nabla$ ) is likely to be captured.
- If we estimate, again, an AR(1) on the previous residuals, the second UR ( $\nabla$ ) is likely to be captured.
- If, alternatively, we start by estimating an AR(2), both UR ( $\nabla^2$ ) are likely to be captured simultaneously.
- Further increases in the order of the AR, or further AR(1) fits to residuals will not point to a UR.
- The previous results extend in a straightforward manner to seasonal UR.

TRAMO uses these results. First, the model  $AR(2) AR_s(1)$  with mean,

$$(1 + \phi_1 B + \phi_2 B^2) (1 + \phi_s B^s) (x(t) - \mu) = (1 + \theta B) (1 + \theta_s B^s) a_t,$$

is estimated and UR are detected in the way described.

As already mentioned, if the modulus of an MA root is relatively large, the bias in the estimator of the AR parameter can be large, and the UR can be missed. Therefore, after detecting UR with AR fits, TRAMO uses ARMA (1,1) fits to detect UR that might not have been captured because of ignoring possibly large MA roots.

Hence, after the pure AR fit, TRAMO fits multiplicative models of the form ARMA (1,1) ARMA<sub>s</sub> (1,1) with mean

$$(1 + \phi B)(1 + \phi_s B^s)(x(t) - \mu) = (1 + \theta B)(1 + \theta_s B^s)a(t) .$$

UR are identified 1 by 1, and the residuals of the last estimated model are used for a pre-test to specify a mean or not.

*Identification of the stationary ARMA model:  $\phi(B)$  and  $\theta(B)$*

The program selects the orders (p,q), where  $p = \text{dg}\{\phi(B)\}$  and  $q = \text{dg}\{\theta(B)\}$ , corresponding to the lowest BIC (p,q), where

$$\text{BIC}(p,q) = \ln(V_a(p,q)) + (p+q) \frac{\ln(N-\delta)}{N-\delta} ,$$

$V_a$  is the residual variance associated with that particular model, and  $\delta$  the number of observations lost by differencing. It searches for models of the form

$$\phi_{p_r}(B) \phi_{p_s}(B^s) \tilde{x}_t = \theta_{q_r}(B) \theta_{q_s}(B^s) a_t , \quad (3.2)$$

where  $\tilde{x}(t) = \nabla^d \nabla^{ds} x(t)$  is the differenced series, over the range  $0 \leq p_r, q_r \leq 3$ ,  $0 \leq p_s, q_s \leq 2$  (1 if used with SEATS). This is done sequentially (for fixed regular polynomials, the seasonal ones are obtained, and viceversa).

The search favours parsimony and “balanced” models (similar AR and MA orders). Parsimony improves estimation precision and out-of-sample forecasting performance. Balanced models tend to be stable over time.

*Remark:*

When analyzing series with care, TRAMO may suggest a few models (perhaps 2 or 3) all of which could be reasonably acceptable.

When used with SEATS, looking for, among these models, the one that provides a more satisfactory decomposition may provide additional tools for the choice. For example, if seasonal adjustment is the purpose of the application, a better adjustment may be one that provides a more stable seasonal component, that is estimated with more precision and is subject to smaller revisions.

#### 4. DECOMPOSITION OF THE SERIES AND SEASONAL ADJUSTMENT

The first part of the program (associated with TRAMO) estimates the possible outliers, calendar, and regression effects, which are treated as deterministic, and hence decomposes the observed series  $y(t)$  into a deterministic and a stochastic component (the two terms  $z'(t)\hat{b}$  and  $x(t)$  of equation (2.2), respectively). The deterministic component  $z'(t)\hat{b}$  is called the “preadjustment” component, and once it is removed from  $y(t)$ , an estimate of the stochastic part is obtained. This stochastic part is assumed the output of a linear stochastic process (as in equation (2.3),) and is also referred to as the “linearized series.”

In the second part of the program (associated with SEATS) the ARIMA-model-based (AMB) methodology is used to estimate stochastic unobserved components in  $x(t)$ . The unobserved components are the

- trend cycle component,  $p(t)$ ,
- seasonal component,  $s(t)$ ,
- transitory component,  $c(t)$ ,
- irregular component,  $u(t)$ ,

and, for an additive decomposition,

$$x(t) = p(t) + c(t) + u(t) + s(t) = n(t) + s(t) , \quad (4.1)$$

where  $n(t)$  denotes the SA series. (A multiplicative decomposition is transformed into an additive one by taking logs.)

Broadly, the trend-cycle captures the spectral peak around the zero frequency, the seasonal component captures the peaks around the seasonal frequencies, the irregular component picks up white-noise variation, and the transitory component captures stationary, highly transitory, variation that differs from white noise.

From the ARIMA model for the series, the models for the components are derived. This is done by partitioning the spectrum (or pseudo-spectrum) of  $x(t)$ , as described in Maravall (1995) or Gómez and Maravall (2001); the procedure follows the approach of Burman (1980).

The models for the components also have ARIMA-type expressions. Typically, for the trend-cycle and seasonal component,

$$\nabla^D p(t) = w_p(t) , \quad D = d + d_s , \quad (4.2)$$

$$S s(t) = w_s(t) , \quad S = 1 + B + \dots + B^{s-1} , \quad (4.3)$$

where  $w_p(t)$  and  $w_s(t)$  are stationary ARMA processes. Models (4.2) and (4.3) are always balanced.

The transitory component is a stationary ARMA process and the irregular component is white noise.

The processes  $w_p(t)$ ,  $w_s(t)$ ,  $c(t)$ , and  $u(t)$  are assumed to be mutually uncorrelated.

By construction, aggregation of the models for  $p$ ,  $s$ ,  $c$ , and  $u$  yields the ARIMA model (4.3) previously identified for the series  $x(t)$ .

The model for the SA-series,  $n(t)$ , is obtained through aggregation of the models for  $p(t)$ ,  $c(t)$ , and  $u(t)$ . Its basic structure is also of the type (4.2), with  $p$  replaced by  $n$ .

It is well-known that, in (4.1), there is an infinite number of ways in which the additive white noise contained in the series can be assigned to the components. Unique identification is achieved in SEATS by imposing the “canonical condition”, whereby all additive white noise is assigned to the irregular component. In this way, the variance of the later is maximized, and the rest of the components are as stable as possible, given the stochastic features of the series.

As an example, for the Airline model (3.1), the model for the trend-cycle is an IMA (2,2), say

$$\nabla^2 p(t) = (1 + \theta_{p,1} B + \theta_{p,2} B^2) a_p(t), \quad (4.4)$$

where the MA polynomial can be factorized as

$$\theta_p(B) = (1 + \alpha_p B)(1 + B).$$

The root  $B=-1$  induces a spectral zero for the frequency  $\omega = \pi$ , associated with the canonical condition. The parameter  $\alpha_p$  often takes values that are close to -1.

The model for the SA series is also of the type (4.4) – with “ $p$ ” replaced by “ $n$ ” – and the factorization of the MA polynomial yields two real roots, typically one close to -1, the other, a relatively small number. Thus, often, the SA series will not be far from the “random walk+drift” model.

The model for the seasonal component is as in (4.3), with  $w_s(t)$  following an MA ( $s-1$ ) model. (This MA will contain a unit root for a non-seasonal and non-trend frequency.)

Finally, the irregular component is white noise and there is no transitory component.

*Remark:* On some occasions, the ARIMA model identified by TRAMO does not accept a decomposition such that all components have a non-negative spectrum for all frequencies. It is then said that the model presents a “non-admissible” decomposition. When this happens, SEATS automatically modifies the model, searching for a decomposable model that is not far from the TRAMO one. The search always converges.

The component estimator and forecast are obtained, by means of a Wiener-Kolmogorov type filter, as the MMSE estimators (under the normality assumption, equal to the conditional expectation) of the signal given the observed series.

Although its implementation does not require specification of the parametric models for the components, the model-based structure offers a representation that is useful for analytical interpretation. In more compact notation, let  $x(t)$  follow the model

$$\phi(B) x(t) = \theta(B) a(t), \quad a(t) \sim wn(0, V_a), \quad (4.5)$$

where now  $\phi(B)$  also includes the unit roots. Consider the decomposition of  $x(t)$  into “signal plus non-signal” as in

$$x(t) = s(t) + n(t).$$

Let the model for the signal  $s(t)$  be

$$\phi_s(B) s(t) = \theta_s(B) a_s(t), \quad a_s(t) \sim wn(0, V_s),$$

where  $\phi_s(B)$  will contain the roots of  $\phi(B)$  associated with the component  $s$ .

Analogously, let the non-signal,  $n(t)$ , follow the model

$$\phi_n(B) n(t) = \theta_n(B) a_n(t), \quad a_n(t) \sim wn(0, V_n),$$

where  $\phi(B) = \phi_s(B) \phi_n(B)$ . Then, if  $F = B^{-1}$  denotes the forward operator (such that  $F x(t) = x(t+1)$ ), for a doubly infinite series, the WK filter to estimate the signal is given by

$$v_s(B, F) = \frac{V_s}{V_a} \frac{\theta_s(B) \phi_n(B)}{\theta(B)} \frac{\theta_s(F) \phi_n(F)}{\theta(F)}, \quad (4.6)$$

or, equivalently, by the ACF of the stationary ARMA model

$$\theta(B) z(t) = [\theta_s(B) \phi_n(B)] b(t), \quad b(t) \sim wn(0, V_s / V_a).$$

The filter given by (4.6) is two sided, centered, symmetric and convergent, and the estimator of the signal is obtained through

$$\hat{s}(t) = v_s(B, F) x(t). \quad (4.7)$$

This expression assumes the infinite realization  $[x(-\infty), \dots, x(\infty)]$ . In practice, one deals with a finite series:  $[x(1), x(2), \dots, x(T)]$ . Given that the WK filter converges, for long-enough series the estimator of the signal for the mid-years of the sample can be considered to be equal to the estimator (4.7), to be denoted the historical estimator. Its model based structure allows us to derive formulas that can be exploited for diagnostics and inference.

First, from (4.5) – (4.6) it is obtained that the historical estimator is generated by the model

$$\phi_s(B) \hat{s}(t) = k_s \theta_s(B) \frac{\theta_s(F) \phi_n(F)}{\theta(F)} a_t, \quad (4.8)$$

where  $k_s = V_s / V_a$ . Therefore, the theoretical variance, autocorrelation generating function (ACF) and spectrum of the stationary transformation of  $\hat{s}(t)$  are straightforward to obtain. Using Bartlett's approximation, it is possible to compute the standard errors (SE) of the empirical estimators of the variance and autocorrelations of the stationary transformation of  $\hat{s}(t)$  –trimming some years at both ends of the series - and derive an approximate test for under/over estimation of  $s(t)$ , and for possible misspecification of the model. Under/over estimation would be revealed by variance estimates significantly below/above their theoretical values; misspecification would be revealed by significant differences between the theoretical and empirical ACFs and CCF of the components.

Further, it is straightforward to see that the theoretical covariance of  $\hat{s}(t)$  and  $\hat{n}(t)$  (the historical estimator of the non-signal) is equal to the variance of the ARMA model

$$\theta(B) z_t = \theta_s(B) \theta_n(B) b(t), \quad (4.9)$$

with  $\text{Var } b(t) = V_s V_n / V_a$ . Because  $\theta(B)$  is invertible, (4.9) is a stationary process with finite variance. It follows that the empirical contemporaneous crosscorrelation between  $\hat{s}(t)$  and  $\hat{n}(t)$  when (4.5) is non-stationary should be relatively small and positive.

The final or “historical” estimator  $\hat{s}(t)$ , obtained with a doubly infinite filter, still contains an error, equal to  $h(t) = s(t) - \hat{s}(t)$ , to be denoted “historical estimation error” (HEE). Its variance and ACF turn out to be those of the stationary ARMA process given by (4.9) (Pierce, 1979). Even for non-stationary series with theoretically infinite variance, the HEE will have finite variance and therefore the theoretical component  $s(t)$  and its estimator  $\hat{s}(t)$  will be co-integrated.

Notice that, given that in general  $\hat{n}(t)$  will be non-stationary,  $\phi_n(B)$  -and hence  $\phi_n(F)$  - will contain unit roots. According to (4.8),  $\hat{s}(t)$  will be a non-invertible series. For example, for series with seasonality, the historical estimator of the SA series and of the trend-cycle will typically contain seasonal UR in its MA polynomial. As a consequence, AR models will be inappropriate for series of historical estimators of SA series or trends.

In general, given the series  $[x(1), \dots, x(T)]$ , the MMSE estimators and forecasts of the components are obtained applying the two-sided WK filter to the series extended at both ends with forecasts and backcasts. These are computed with (4.5), the ARIMA model for the observed series. Therefore, for a finite realization, the estimator of  $s(t)$  when observations end at period T can be expressed as

$$\hat{s}(t | T) = v_s(B, F) \hat{x}(t | T) \quad (4.10)$$

where  $\hat{x}(t | T)$  denotes the extended series. By extending the series with an appropriate number of forecasts, (4.10) provides, besides in-sample estimators, MMSE forecasts of the signal, i.e.,  $\hat{s}(t | T)$  for  $t > T$ . Applying the Burman-Wilson algorithm, it is possible, however, to obtain the full effect of the doubly infinite filter with just a small number of forecasts and backcasts. (For example, 26 are needed to estimate the SA series for the most recent period in a series that follows a monthly Airline model.)

Being obtained by using forecasts, the component estimators at the end points of the series are preliminary and will suffer revisions as future data become available, until it can be assumed that the historical estimator has been reached. Depending on the model, these revisions typically last between 2 and 5 years.

The model-based framework provides (SE) of the estimators and forecasts and permits us to analyze revisions in preliminary estimators (size and speed of convergence), as well as in their rates of growth.

In what follows, the series will be assumed long enough so that the estimator of  $s$  at the central period has converged to the historical one. Under the semi-infinite realization  $[x(-\infty), \dots, x(T)]$ , the preliminary estimator of  $s(t)$  obtained at  $T$  ( $T - t = k \geq 0$ ) can be expressed as  $\hat{s}(t | t + k)$ . The revision it will suffer is given by

$$r_k(t) = \hat{s}(t) - \hat{s}(t | t + k) .$$

Notice that  $r_k(t)$  will decrease as  $k$  increases, until it can be assumed equal to 0 for large enough  $k$ .

(Specifically, assuming the filter  $v_s(B, F)$  can be truncated and replaced by a finite filter of length  $(2J + 1)$ , then for  $k > J$ ,  $r_k(t) = 0$ .)

It should be mentioned that, as seen in McElroy (2006), the semi-infinite assumption yields results that are in general close to the exact finite realization results; its adoption however simplifies significantly analytical discussion.

From (2. 8),  $\hat{s}(t)$  can be expressed as

$$\hat{s}(t) = \frac{\alpha(B)}{\phi_s(B)} a(t + k) + \frac{\beta(B)}{\theta(B)} a(t + k + 1) , \quad (4.11)$$

where  $\beta(B) / \theta(B)$  converges to zero. The polynomials  $\alpha(B)$  and  $\beta(B)$  can be easily obtained as in Bell (2007) or Maravall (1994). Taking expectations at time  $(t+k)$ , the second summation term vanishes, so that the preliminary estimator follows the model

$$\phi_s(B) \hat{s}(t | t + k) = \alpha(B) a(t + k) \quad (4.12)$$

where  $B$  and  $F$  operate on  $t$ . From (4.11) and (4.12), it follows that the revision in  $\hat{s}(t | t + k)$  follows in turn the stationary ARMA model

$$\theta(F) r(t | t + k) = \beta(F) a(t + k + 1) . \quad (4.13)$$

The estimation error contained in the estimator (or forecast)  $\hat{s}(t | t + k)$  can be expressed as

$$e(t | t + k) = s(t) - \hat{s}(t | t + k) = [s(t) - \hat{s}(t)] + [\hat{s}(t) - \hat{s}(t | t + k)] . \quad (4.14)$$

The first bracket in the r.h.s. of (4.14) is the historical estimation error,  $h(t)$ , that follows the ARMA model (4.9). The second bracket is the revision error,  $r(t | t + k)$ , that follows model (4.13). As shown in Pierce (1980), the two errors are uncorrelated so that, from

$$e(t | t + k) = h(t) + r(t | t + k) ,$$

the model for the total estimation error is easily obtained. Its variance and ACF are those of the model

$$\theta(B) e(t | t + k) = \delta(B) d(t) , \quad (4.15)$$

where the MA part is the sum of two (finite) uncorrelated MA's, i.e.

$$\delta(B) d(t) = \theta_s(B) \theta_n(B) b(t) + \beta(F) a(t + k + 1) .$$

From models (4.9), (4.13), and (4.15), inferences can be drawn concerning errors in the different types of estimators. From the variances, confidence intervals can be computed, and knowledge of the ACF permits us to compute errors in the growth (or linearized rates of growth when logs have been taken) over any desired period.

Three remarks are worth mentioning:

- (a) Preliminary estimator, historical estimator and component –i.e.,  $\hat{s}(t | t + k)$ ,  $\hat{s}(t)$ , and  $s(t)$  – share the same AR polynomial and unit roots.
- (b) The total estimation error, the error in the historical estimator, and the revision error share the same stationary AR polynomial, equal to  $\theta(B)$ , the MA polynomial of the model for the series. Given that  $\theta(B)$  is always invertible, the three errors will be stationary so that preliminary, historical estimator and component are all pairwise cointegrated.
- (c) Because the “true” component  $s(t)$  will never be observed, the error in the historical estimator is more of academic rather than practical interest. In practice, interest centers on revisions. From (4.13), the revision standard deviation, will be an indicator of how far we can expect to be from the optimal estimator that will be eventually attained, and the speed of convergence of  $\theta(B)^{-1}$  will dictate the speed of convergence of the preliminary estimator to the historical one. (Fortunately, most often, very slow convergence is associated with very stable seasonals, with

very small revisions.) This information allows us to answer questions of applied concern, such as for example:

- Is it worth it to move from a once-a-year adjustment to a concurrent one?
- For how long should the series be revised?

## 5. MIXING THE TRAMO AND SEATS RESULTS

### 5.1 FINAL ESTIMATORS

The stochastic components estimated by SEATS provide the decomposition of the linearized series that has been preadjusted by TRAMO. The deterministic effects estimated by TRAMO that form the preadjustment component can be combined with the SEATS estimators in order to obtain the final estimators of the components. By default,

- Level Shift outliers will be assigned to the final trend.
- Additive and Transitory Change outliers will be assigned to the final irregular component.
- Calendar effects (Trading Day, Easter, Leap Year, moving holidays) will go to the final seasonal component after proper centering. The mean resulting from the centering will go the trend. (They also may form a separate component.)
- Regressions and intervention variable effects can form a separate component or be assigned to the one thought appropriate. (If assigned to the seasonal component, the effects will always be centered and the resulting means added for the trend.)

### 5.2 CHANGE OF MODELS

On occasion, SEATS may change the ARIMA model selected by TRAMO, and use the modified one to decompose the series. The aggregate forecast of the series, as well the linearized series, will be those of the TRAMO model, and the components' forecast in SEATS will be modified accordingly. The most important reasons for the change of model are the following.

A. The ARIMA model of TRAMO does not accept an admissible decomposition. An admissible decomposition is one in which all the component's spectra are non-negative for all frequencies.

Examples of models that do not admit an admissible decomposition are the following.

a) The Airline model (3.1) when  $\theta_s$  is larger than -roughly- -0.20. (The particular value depends on  $\theta_1$ .) The NA model is replaced then by an Airline model with  $\theta_s = 0$ .

Given that typically  $\theta_s$  is small, the only effect of the change is to pass on to the irregular component some negative and small correlation of little relevance. NA models of this type are not often encountered.

b) The  $(110)(011)_s$  model:

$$(1 + \phi_1 B) \nabla \nabla_s x(t) = (1 + \theta_s B^s) a(t) ,$$

when  $\phi_1$  falls in the range (-0.2 - -0.4). Given that AR(1) polynomials with  $\phi_1$  in that range are practically indistinguishable from an MA(1) polynomial with  $\theta_1 \cong -\phi_1$ , SEATS replaces the NA model with the Airline model. As before, these models are not often found.

- c) For some combination of parameter values, unbalanced models of the type  $(p,d,q) (\bullet)_s$  with  $p = 0$  or  $1$  and  $q = 2$  or  $3$ , where the parameter  $\theta_3$  (or  $\theta_2$  if  $q = 2$ ) is relatively small, yield NA models, SEATS sets this parameter equal to zero. The effect will be a slight coloring of the noise, mostly irrelevant as far as the decomposition of the series is concerned. Non-admissibility of models with relatively long regular MA polynomials typically occurs when one of the MA roots is complex with a frequency associated with a period longer than 6 months and a significant modulus. For monthly data, this root is likely to induce a spectral minimum between the zero and the twice-a-year frequency.
- d) For some combination of parameter values, unbalanced models of the type  $(p,d,q) (\bullet)_s$  with  $q = 0$  or  $1$  and  $p = 2$  or  $3$  yield NA models. In this case, SEATS factorizes the AR polynomial, analyses the roots, and attempts to reallocate them so that the resulting model is decomposable. For example, a NA model  $(3,1,0) (\bullet)_s$ , for which factorization of the regular AR(3) yields a positive real root of moderate modulus and a complex root, will be replaced by a  $(2,1,1) (\bullet)_s$  model, with the AR(2) capturing the complex root, and the regular MA(1) capturing the real AR root.
- B. a) In the model  $(\bullet) (1,1,1)_s$  when the parameter  $\phi_s$  is  $> 0$ , the AR(1) polynomial in  $\mathbf{B}^s$  is not associated with seasonality, but rather with a stationary two-year cycle. The modulus of the root does not justify assigning it to the trend-cycle component, and hence it should be considered an irregular/transitory effect. SEATS transforms the model into a  $(\bullet) (0,1,1)_s$  model. the result is a  $\theta_s$  parameter closer to  $-1$  (i.e., a more stable seasonal) and a larger variance, slightly colored, irregular.
- b) The same modification is made to a model  $(\bullet) (1,1,0)_s$ , with  $\phi_s > 0$ . The seasonal AR parameter is replaced by a negative seasonal MA parameter; typically the two models are very close.
- C. Stationary seasonal structures, such as the one present in a  $(\bullet) (1,0,1)_s$  model, tend to generate unstable seasonality because of the convergence of the seasonal component to a zero mean, and because of the correlation between the seasonal AR and MA parameters. When  $\phi_s < -0.80$ , SEATS replaces the model with the model  $(\bullet) (0,1,1)_s$ . The change of the stationary seasonal structure by a non-stationary one is made in SEATS by default. An input parameter ("statseas") permits the user to maintain the stationary specification if desired.

D. The model for the series can be changed in order to avoid a nonsense decomposition. An example can be a seasonal ARMA (1,1) structure such as

$$(1 + \phi_{12} B^{12}) x(t) = (1 + \theta_{12} B^{12}) a(t) ,$$

with  $-1 < \theta_{12} < \phi_{12} < 0$ . For this structure the spectral peaks appear at non-seasonal frequencies. These “nonsense” models are very rarely encountered.

E. Pure MA models will not be decomposed by SEATS. For example, consider a model with no seasonality other than that implied by a pure MA seasonal structure (0,0,1); such as

$$x(t) = (1 + 0.5 B^{12}) a(t) . \tag{4.16}$$

The model implies a relatively small correlation lag-12 autocorrelation (0.4), that disappears after one year when seasonally adjusting, it seems sensible to associate seasonality with longer lasting periodicities, i.e., with recurrent cycles. SEATS will not adjust a series for which the model is (4.16). Representing only highly erratic short term oscillations around zero,  $x(t)$ , given by (4.16), will simply be considered a “transitory + irregular” type of component.

It should be mentioned that the AMI procedures of TRAMO (with the default settings), when confronted with alternative model specifications that appear to be equally acceptable, will lean towards IMA (1,1) structures. These structures tend to provide good short-term forecasting, closely related to the EWMA procedure, and well-behaved decompositions into unobserved components, where the trend and seasonal component can range between highly stochastic and –to all effects- deterministic behavior. Further, balanced IMA models display important limiting properties under temporal aggregation; see Tiao (1972).

In most cases, when the model found by TRAMO is replaced with an alternative model in SEATS, the TRAMO forecast and the linearized (or preadjusted) series are preserved; the change tends to provide a more stable seasonal (and perhaps trend-cycle) component, and a larger –perhaps slightly colored-irregular component. In all cases in which the specification of the model is changed, SEATS reestimates all ARMA parameters.

## **PART II: THE APPLICATION**

### 2.1 THE SERIES AND THE STARTING POINT

Our interest centers on large-scale application of TSW, which is typically the case at the statistical production level (often, of seasonally adjusted series). In these applications, detailed analysis of the results for each individual series becomes prohibitive. We focus on the overall results that the program provides for the full set, having to do with fitting of the model, features of the decomposition, and out-of-sample performance. Next, we shall deal with the problem of identification of problematic series in the set, and illustrate how some of these problematic series can be handled.

In what follows, reference will be made to input parameters. The way in which they are entered, as well as their meaning, is described in Caporello and Maravall (2004). Also, for most parameters, the F1 key provides an on-line “Help” facility.

The example consist of 500 monthly exports and imports series of 15 European Union countries, spanning the period January 1995 – January/February 2005 (121/122 observations). Our main interest will be modelling and seasonal adjustment of the 500 series.

A glance at them shows a variety of different patterns, as the few examples of figures 1 to 12 illustrate.

Figure 1: Series 1

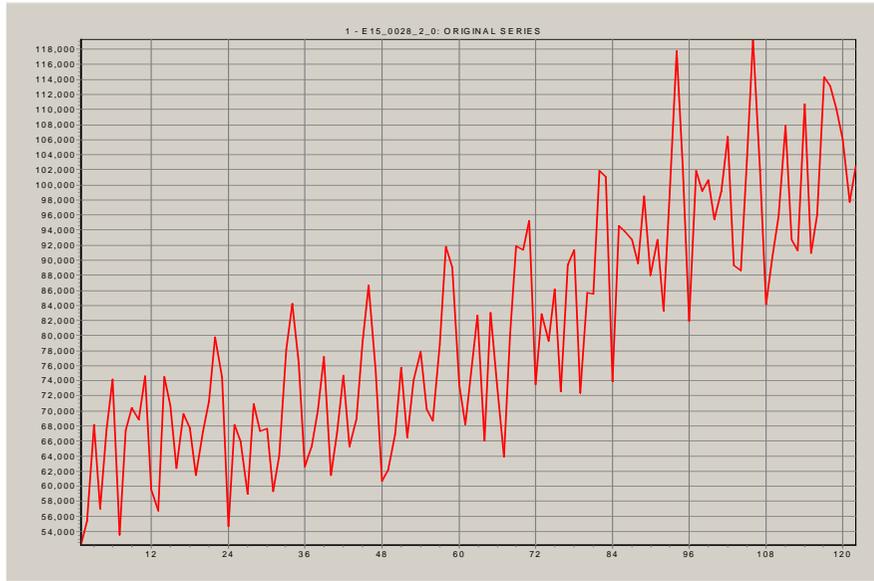


Figure 2: Series 2

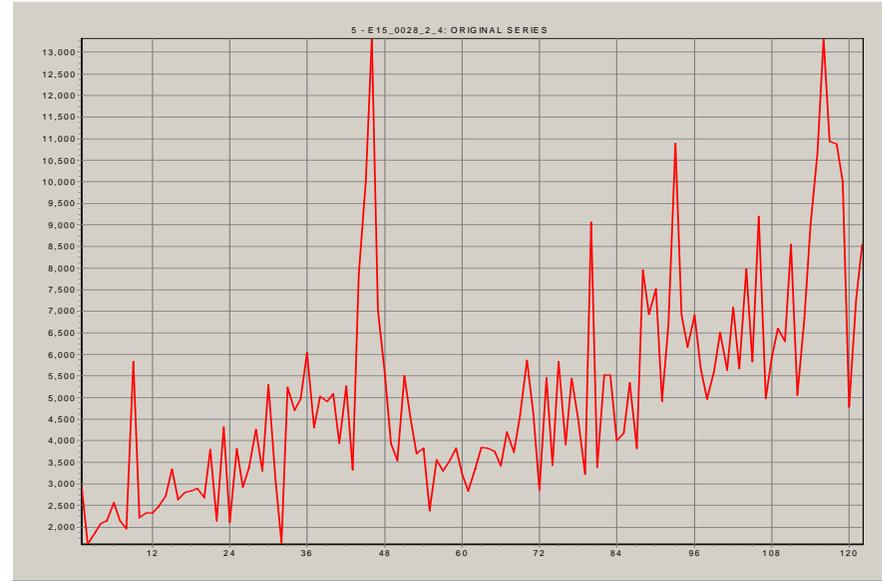


Figure 3: Series 3

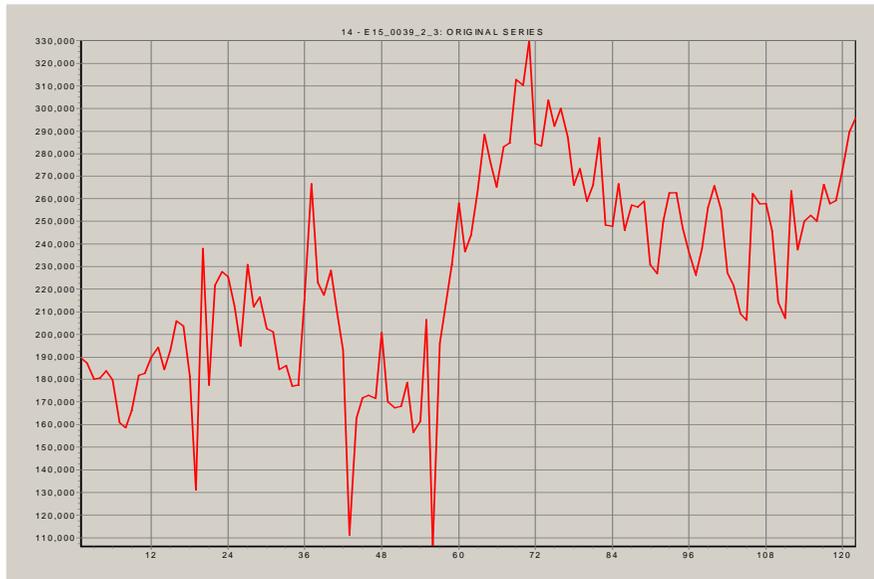


Figure 4: Series 4

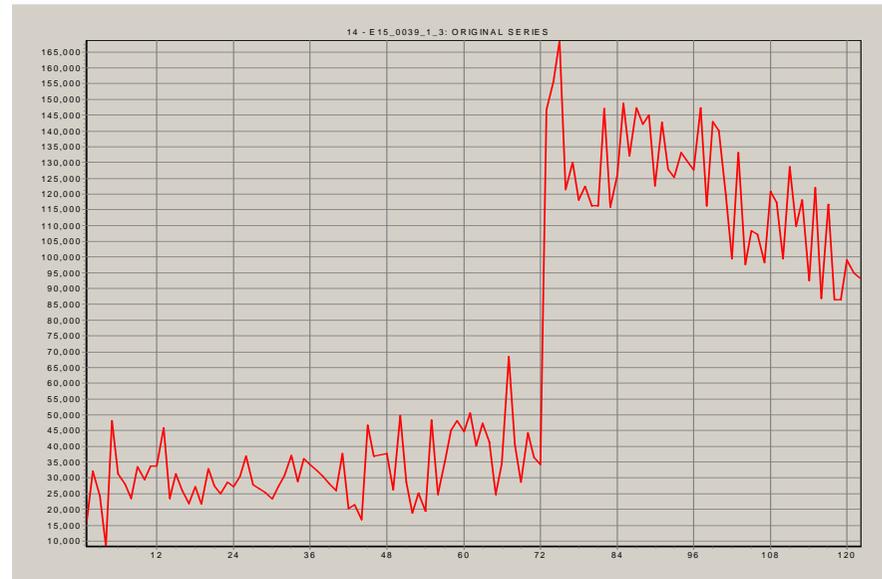


Figure 5: Series 5

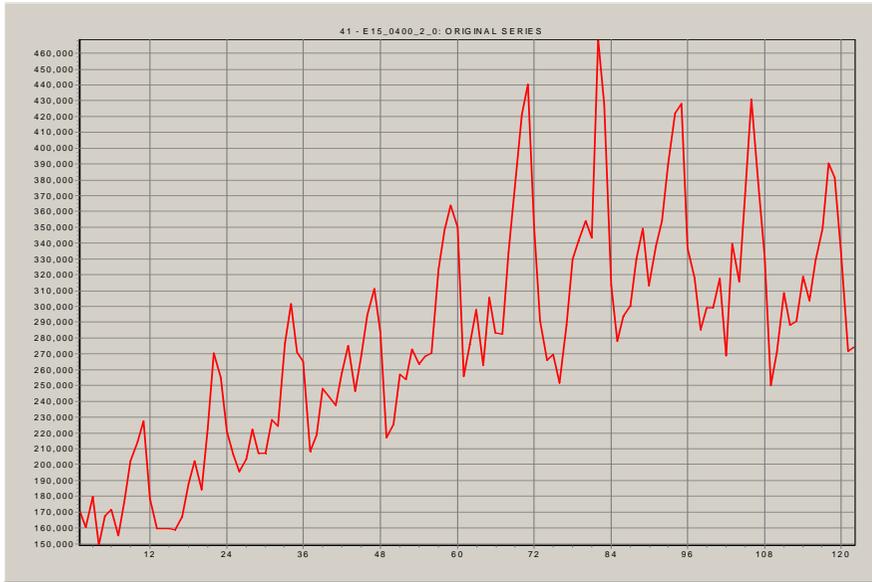


Figure 6: Series 6

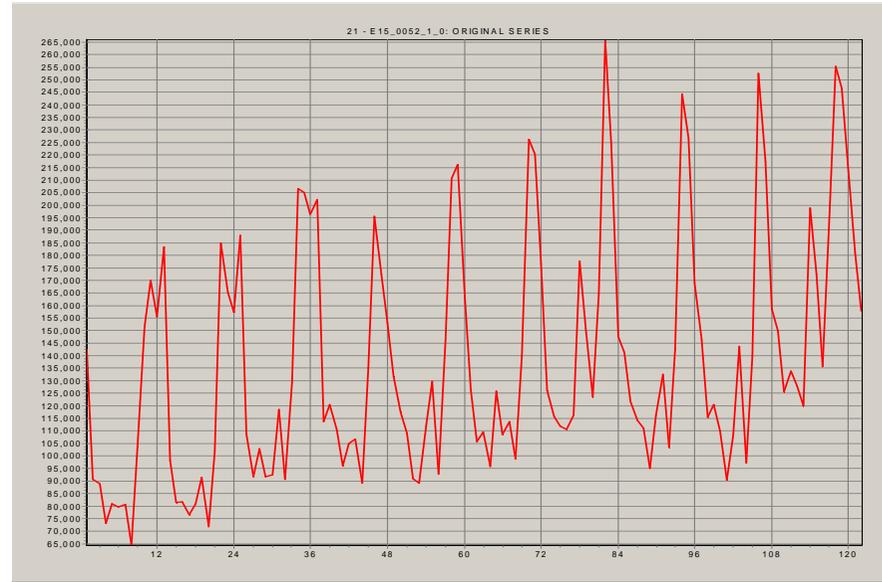


Figure 7: Series 7

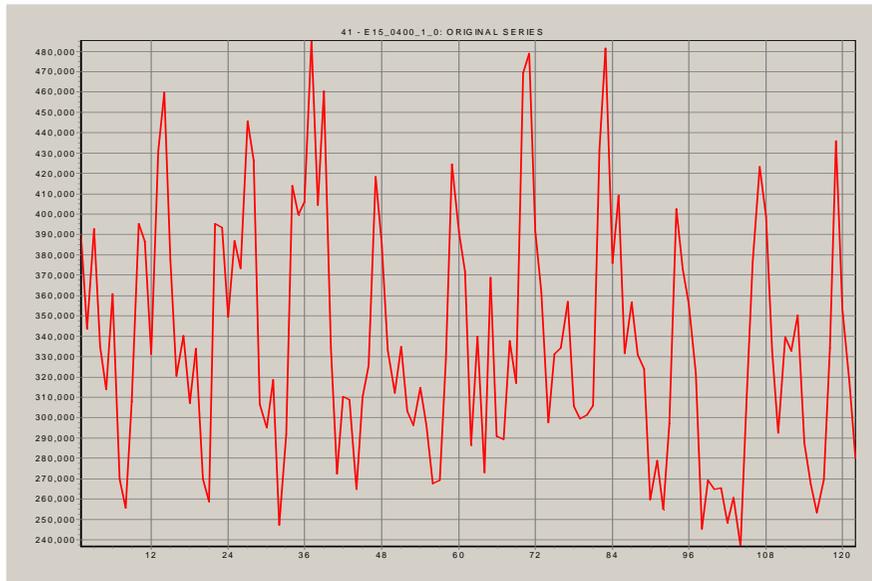


Figure 8: Series 8

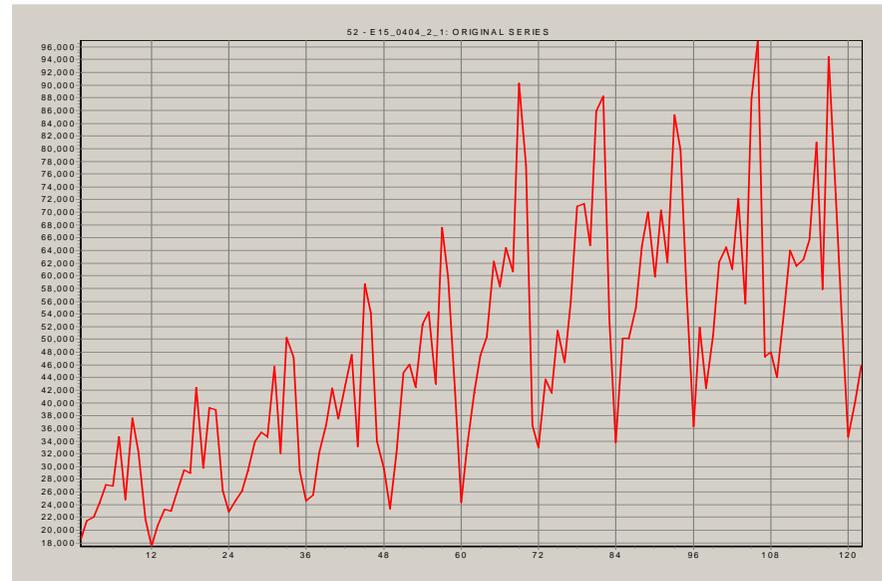


Figure 9: Series 9

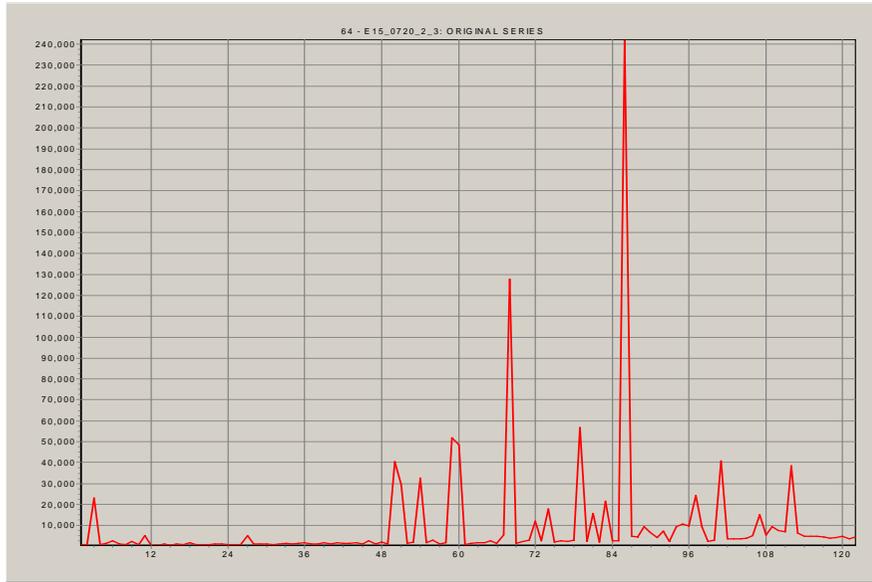


Figure 10: Series 10

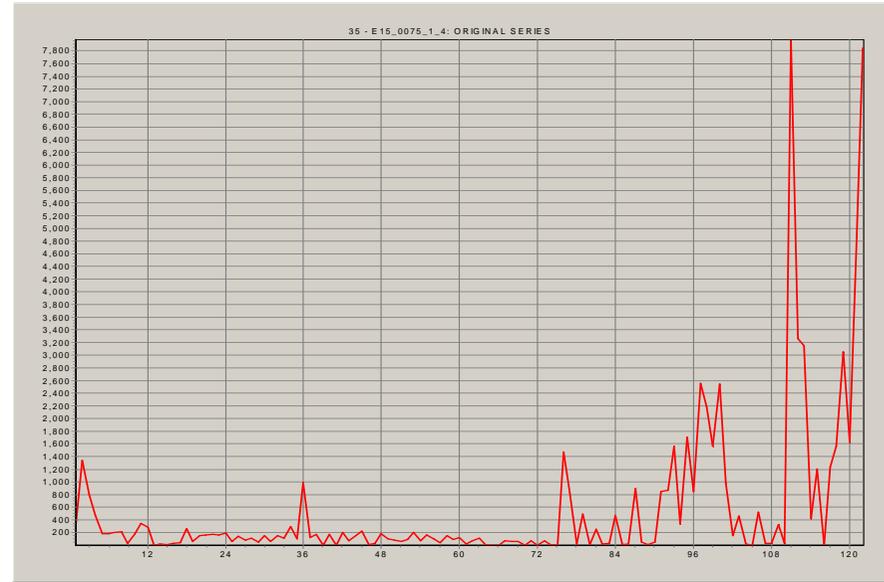


Figure 11: Series 11

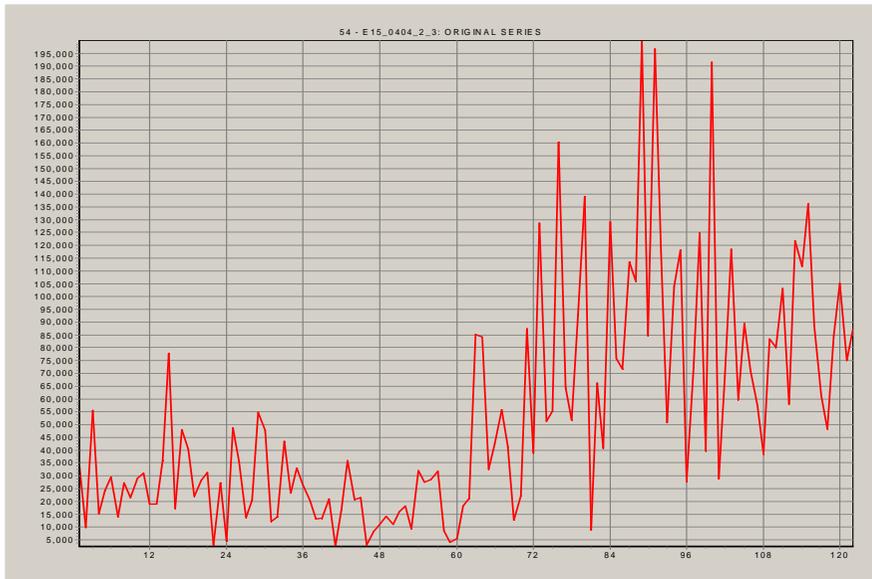
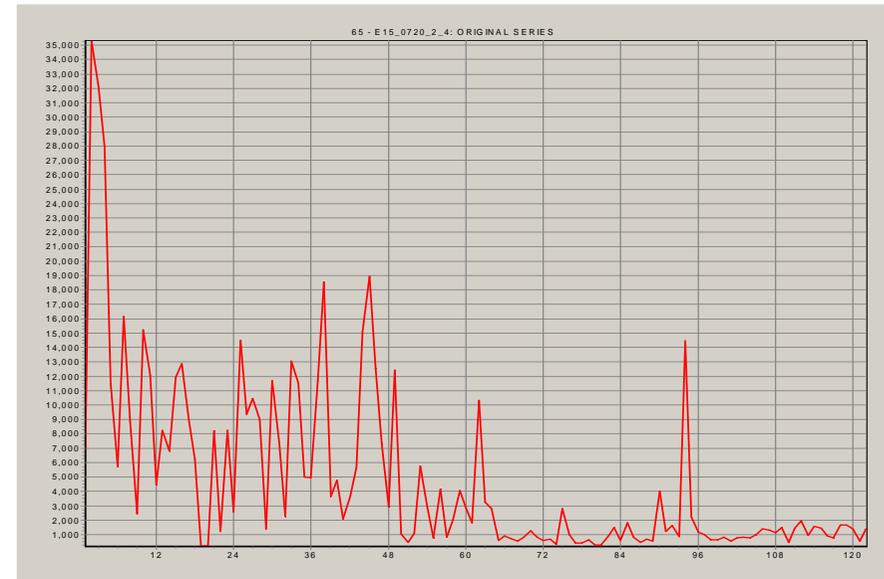


Figure 12: Series 12



Direct inspection of the figures shows that series 1 displays a standard “trend + seasonal + irregular” behavior. Series 2, 3, and 4 exhibit asymmetry possibly caused by outliers (AO, TC, and LS outliers are clearly discernable). Series 2 presents some indications of seasonality, while none is apparent for series 3, which presents instead a more cyclical behavior.

Series 5 to 8 are strongly dominated by the seasonal component, in all cases exhibiting moving (i.e., stochastic) seasonality.

Series 9 to 12 show clear instability throughout the sample period and marked heteroscedasticity. Series 9 basically consist of a sequence of outliers in the second half of the period, and series 10 is similar in essence. The two series illustrate both the uselessness of seasonal adjustment for such series, and –by looking at the last periods- the futility of forecasting them.

It should be mentioned that the 12 examples illustrate structures with different frequency of occurrence in the set of 500 series. Series 1 to 4 would represent the most dominant structures.

The recommended starting point is one of the automatic options. These options are controlled by the parameter RSA as follows.

**RSA = 0** Parameter not active.

= **1** As RSA = 3, but the default Airline model is always used.

= **2** As RSA = 4, but the default Airline model is always used.

= **3** The program tests for the log/level specification, interpolates missing observations (if any), and performs automatic model identification and outlier detection. Three types of outliers are considered: additive outliers, transitory changes and level shifts; the level of significance is set by the program and depends on the length of the series. (In our case, for series with 122 observations, the threshold t value set by default is  $VA = 3.2$  .) The full model is estimated by exact maximum likelihood, and forecasts of the series up to a two-year horizon are computed. The model is decomposed and optimal estimators and forecasts of the components are obtained, as well as their mean squared error. These components are the trend-cycle, seasonal, irregular and (perhaps) transitory component. If the model does not accept an admissible decomposition, it is replaced by a decomposable one.

= **4** As before, but a pretest is made for the presence of Trading Day, Leap Year and Easter effects, with the first effect using a one parameter specification (working / non-working days).

= **5** As RSA = 4, but the Trading Day specification uses 6 parameters (each day-of-week effect may be different).

*Note: The automatic option assumes a particular set of values for all parameters (see Caporello and Maravall, 2004) that can be modified by the user. To do this, after setting the RSA parameter, one simply needs to enter the modified parameters. AMI can be thus personalized.*

Considering that the export and import series may well be affected by trading day, and that the series are of moderate length, the option RSA = 4 seems the most appropriate starting point. Entering ITER =

2 (same input for all series) and RSA = 4, execution time of the 500 series –in a standard 2007 portable PC- takes scarcely 30 seconds.

## 2.2 AUTOMATIC MODEL IDENTIFICATION AGGREGATE RESULTS

### 2.2.1 In Sample Fit

Clicking in the icon “Out\_Matrix”, a set of matrices summarizing the results is provided. Selecting “Model Summary”, the aggregate summary results are displayed in five tables.

TSW : 30-Jun-2009 Release : 141

DATE : 2009-07-11 16:59:36

Input Parameters :

mq=12 rsa= 4

Input File : eur15\_export\_tsw.xls,eur15\_import\_tsw.xls

SERIES IN FILE : 500

SERIES PROCESSED : 500

### SUMMARY RESULTS

TABLE 1 : **GENERAL FEATURES**

	# of series	%
Levels	36	7.20
Logs	464	92.80
Regular Difference	449	89.80
Seasonal Difference	376	75.20
Stationary	21	4.20
Non Stationary	479	95.80
Purely Regular Model	78	15.60
Nz Too Small for complete AMI	0	0.00
Airline Model (Default)	254	50.80

TABLE 2: **DIFFERENCES**

# of series with	D = 0	D = 1	D = 2	Total
BD = 0	21 ( 4,20%)	103 ( 20,60%)	0 ( 0,00%)	124 ( 24,80%)
BD = 1	30 ( 6,00%)	345 ( 69,00%)	1 ( 0,20%)	376 ( 75,20%)
Total	51 ( 10,20%)	448 ( 89,60%)	1 ( 0,20%)	500 (100,00%)

TABLE 3 : **ARMA PARAMETERS**

% of series with	P	Q	BP	BQ	Total
0	71.20	28.00	89.20	22.20	
1	17.80	68.60	10.80	77.80	
2	8.00	2.40	0.00	0.00	
3	3.00	1.00	0.00	0.00	
Total > 0	28.80	72.00	10.80	77.80	
Average # of param. per series	0.43	0.76	0.11	0.78	2.08

TABLE 4 : **MISSING VALUES AND REGRESSION**

A) <b>Outliers</b>					
	MO	AO	TC	LS	Tot
% of series with	0.20	43.80	30.20	28.00	65.40
average # per series	4.00	0.82	0.43	0.39	1.63
maximum # per series	4	21	4	4	24
minimum # per series	0	0	0	0	0
B) <b>Calendar Var.</b>					
	TD	EE	Tot		
% of series with	76.20	17.20	77.00		

It is seen that close to 93% of the series are modeled in the logs and that 4% are found stationary. For 16% of them the model has no multiplicative seasonal structure. The default Airline model is found appropriate for approximately 50% of the series.

Concerning unit roots, the transformation  $\nabla\nabla_{12}$  is chosen for close to 70% of the series; for 21% of them only  $\nabla$  is needed, and for 6% only  $\nabla_{12}$  is needed. A slightly smaller percentage requires no differencing, and for one series the transformation  $\nabla^2\nabla_{12}$  seems appropriate. As for the ARMA parameters, the average number per series is 2.08 (minimum is 0, maximum is 5,) implying thus highly parsimonious models. The models are dominated by IMA(1,1) regular and seasonal structures; 30% of the series require regular AR structures, half of them with  $p > 1$ , and a stationary AR seasonal structure is needed for about 11% of the series.

Slightly more than 1/3 of the series do not need outlier correction, and the average number of outliers per series is 1.6 (or one outlier every 76 months), a relatively low frequency. The AO type is the most dominant, with the rest evenly distributed between LS and TC outliers. As for calendar effects, TD is detected in over 75% of the series; EE is far less significant (roughly, 17%).

Finally, the results of several (approximate) tests are given. Q is the Ljung-Box test for residual autocorrelation (in our case,  $\chi^2$  with approximately 22 df), N is the Behra-Jarque test for Normality of the residuals ( $\chi^2$  with 2 df), SK and Kur are t-test for skewness and kurtosis in the residuals, QS is the (modified) Pierce test for residual seasonality ( $\chi^2$  with 2 df), Q2 is the McLeod and Li test for linearity in the residuals (in our case,  $\chi^2$  with, approximately 24 df) and Runs is a t- test for randomness in the signs of the residuals.

Given that forecasting and seasonal adjustment are our main concerns, the most relevant of the previous diagnostics are Q (the model has not fully captured the linear dynamics,) SK (asymmetry may notably affect point estimation,) and QS (the model has not fully captured the seasonal dynamics). At the other end, Q2, which detects GARCH and Bilinear type of structures, is of little relevance because for symmetric distributions it is likely to have a small effect on point estimation of the regARIMA model parameters; large values of Q2 will mostly indicate that the SE of the component estimators and of the series forecasts are not constant in time.

TABLE 5: **SUMMARY STATISTICS**

	Mean	SD	Max	Min	Approx 1% CV	Beyond 1% CV	% of series that pass the test (99%)
<b>Length</b>	<b>121.76</b>	0.43	122	121			
<b># of ARMA param. per serie</b>	<b>2.08</b>	0.76	5	0			
<b># of outliers per serie</b>	<b>1.63</b>	2.14	24	0			
<b>Q</b>	22.70	6.84	56.79	5.76	40.29	0.20	<b>99.80</b>
<b>N</b>	1.45	1.50	10.54	0.01	9.21	0.20	<b>99.80</b>
<b>SK</b>	0.08	0.95	2.79	-2.68	2.58	0.60	<b>99.40</b>
<b>Kur</b>	-0.09	0.74	2.78	-1.72	2.58	0.20	<b>99.80</b>
<b>QS</b>			9.51	0	9.21	0.40	<b>99.60</b>
<b>Q2</b>	23.26	8.26	64.66	7.43	42.98	2.80	<b>97.20</b>
<b>Runs</b>	-0.07	0.88	2.38	-3.13	2.58	0.20	<b>99.80</b>

(Note: 1 series  $\equiv$  0.20% of the series in the set)

Considering that, at the 1% level, 5 out of the 500 series could be expected to fail each test, the aggregate results are excellent. Only 1 series fails to pass the residual autocorrelation Q test, and the same is true for the Normality N, the kurtosis Kur, and the Runs test; 2 series fail to pass the residual seasonality QS test; 3 series fail the skewness SK test; and 14 fail the Q2 test.

Therefore, excluding Q2, the number of rejections is smaller than the test size for all tests. Judging from these results, the automatic procedure has been extremely successful in capturing the dynamics of the series by means of a parsimonious regARIMA model. Where there to have been more outliers, one would be tempted to allow for a few more rejections by moderately reducing the number of outliers (i.e., by increasing VA, the threshold detection level.)

Figures 13 to 20 summarize the previous results. Figure 13 displays the percentage of series that have a certain number of outliers (AO, TC, and LS in different colours). Figure 14 presents the number of ARMA parameters per series. The next seven figures compare the histograms and asymptotic distributions for each of the 7 (approximate) tests performed on the residuals: Autocorrelation, Normality, Skewness, Kurtosis, non-Linearity, Randomness in signs, and residual seasonality. Considering the critical values given in Table 5 (5<sup>th</sup> row), few anomalies are detected. Still, some problematic series can be pointed out, such as the one with a Q close to 57, the one with an N and a QS close to 10, or the one with 24 outliers.

Figure 13: % of series with a certain number of outliers

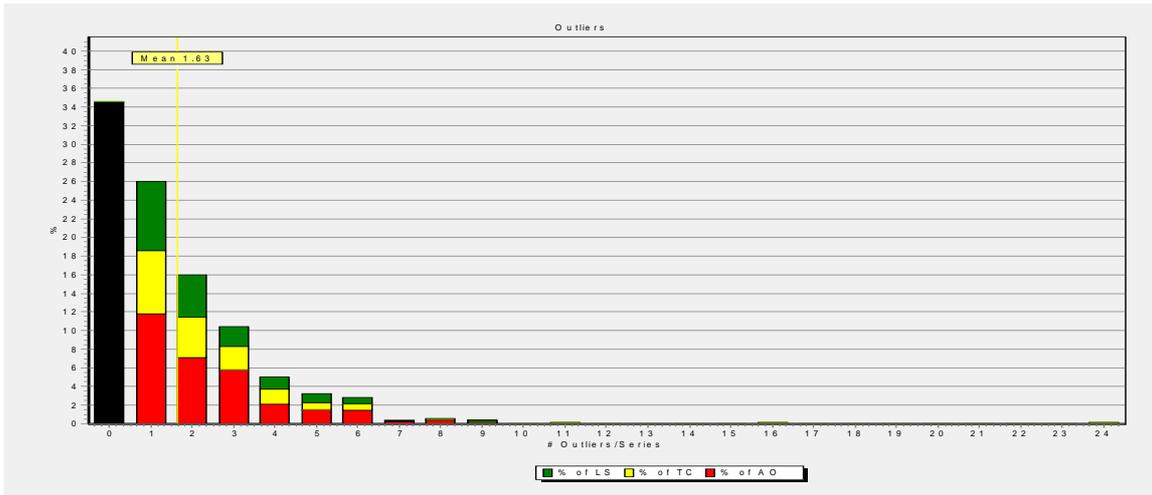


Figure 14: % of series with a certain number of ARMA parameters

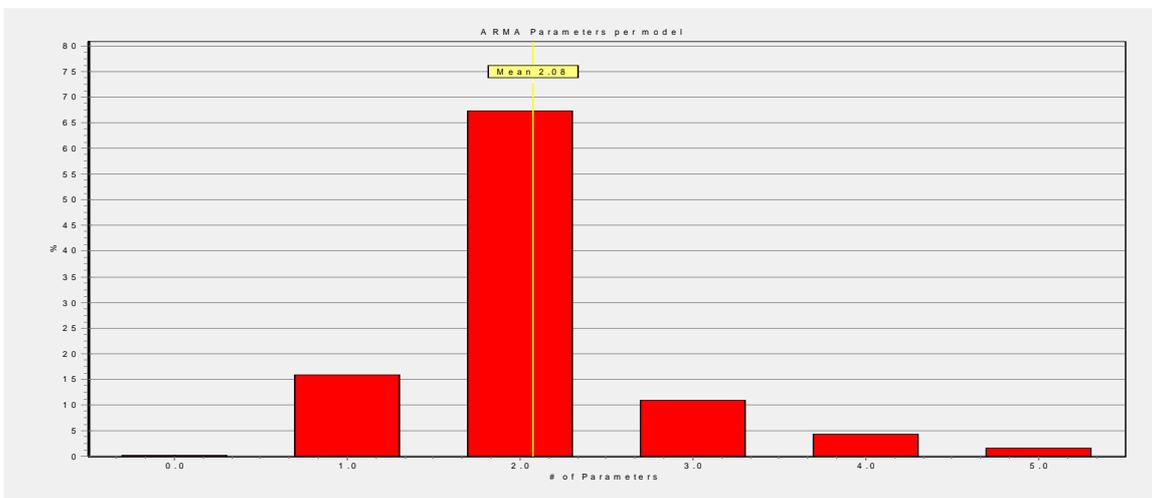


Figure 15: Q-test

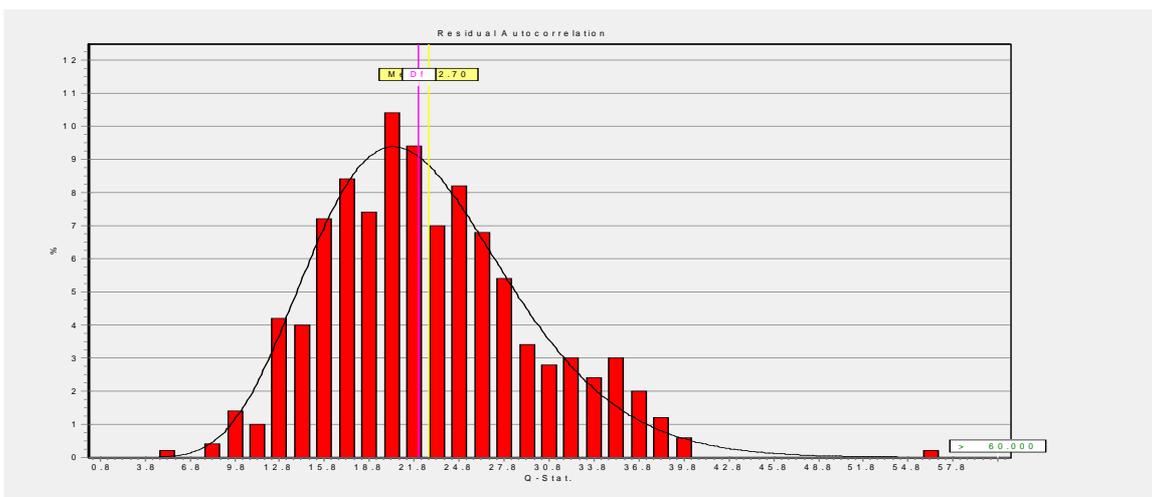


Figure 16: N-test

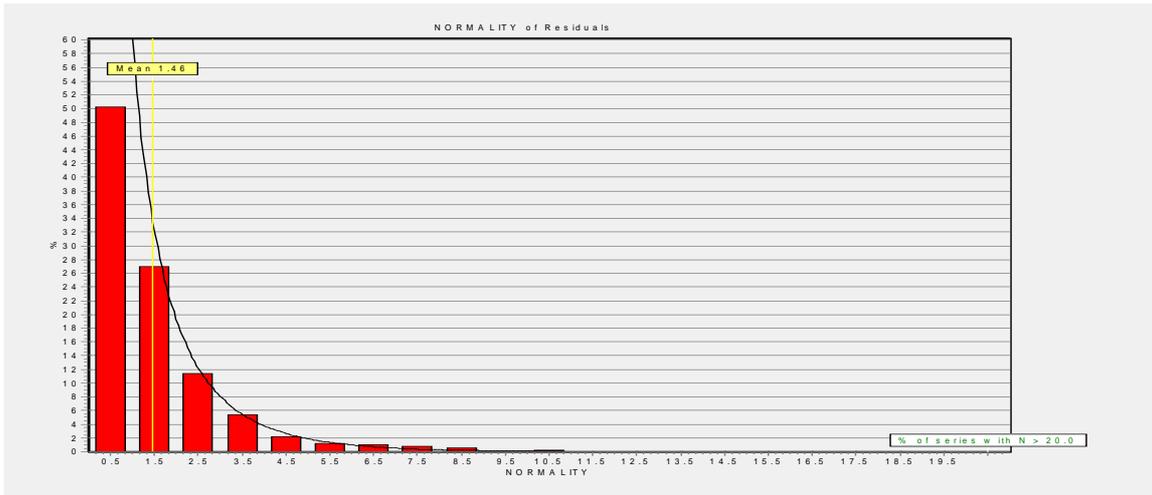


Figure 17: Sk-test

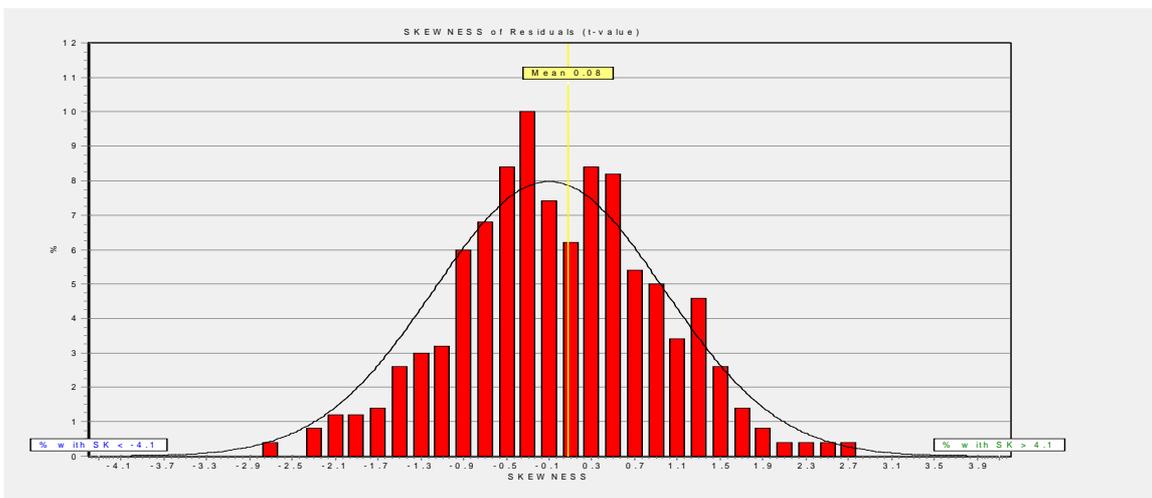


Figure 18: Kur-test

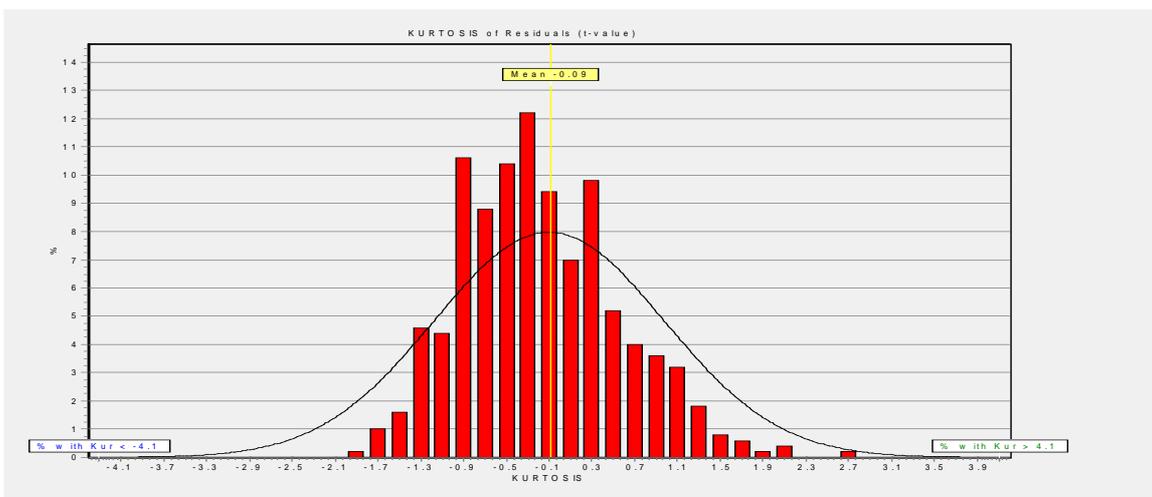


Figure 19: Q2-test

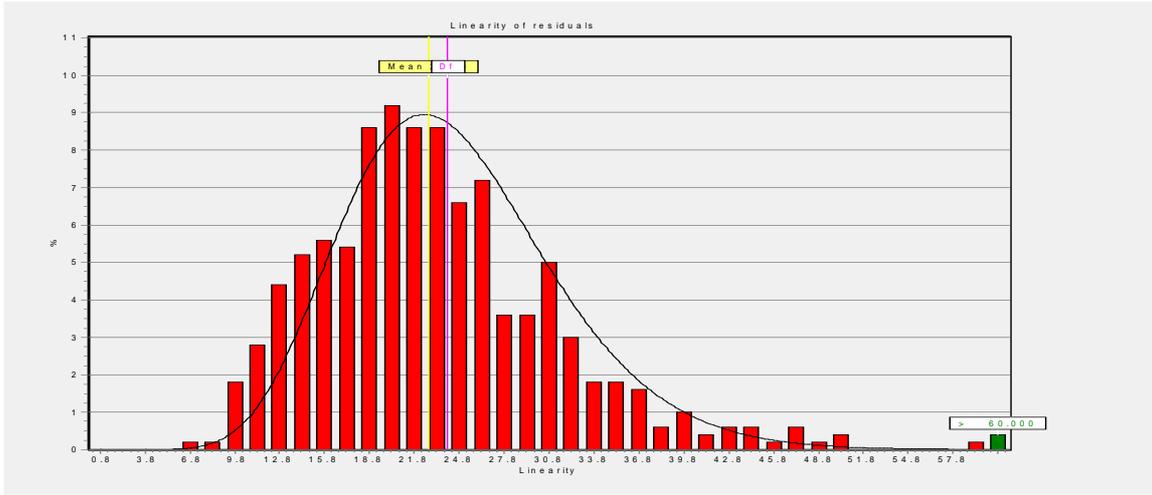
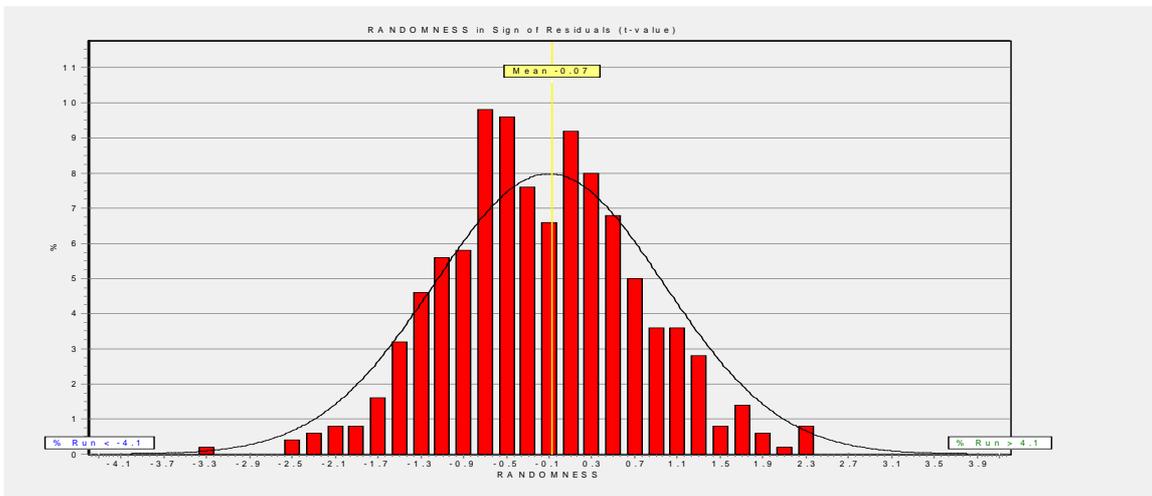
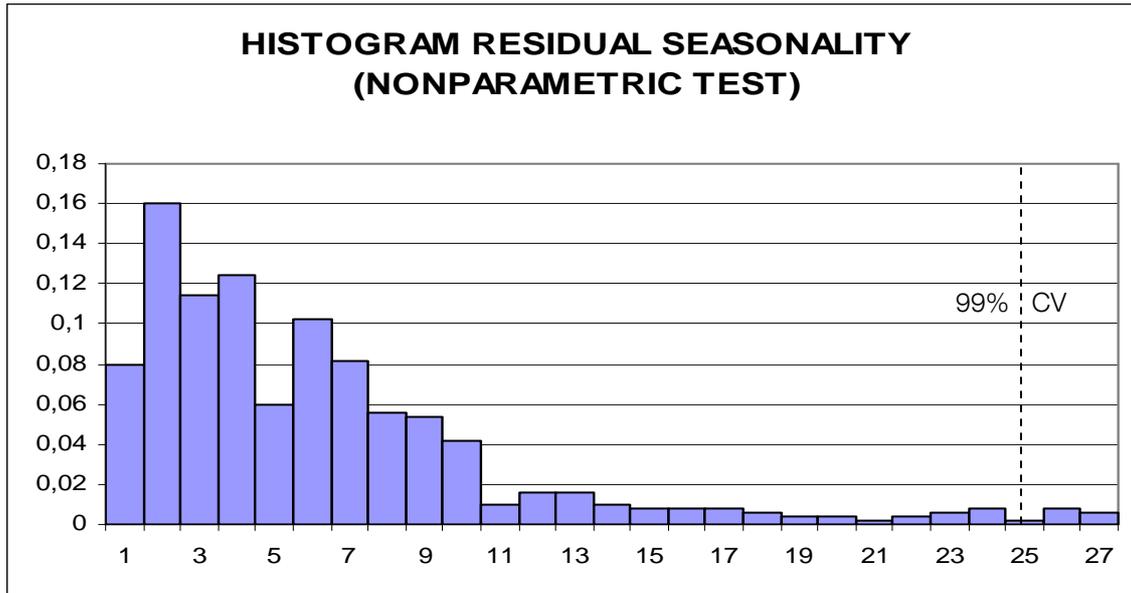


Figure 20: Runs-test



Two additional tests for in-sample fit have been added. One is a nonparametric test for the presence of seasonality which is applied to the model residuals (see Kendall and Ord, 1991 who attribute the test to M. Friedman). At the 99% significance level, out of the 500 series, only 5 (or 1%) display (borderline) evidence of residual seasonality, in line with the test size. (See Figure 21; the 99% critical value is about 25.) The other test looks at the residuals for the first and second half of the series, and tests for the equality of the two means and variances. Again, at the 99% significance level, 7 series fail the test, close to the 5 that could be expected.

Figure 21



As a final remark, the “modal” model can be described as the Airline model, given by (1.4), for the logs and with no mean, with one or perhaps two outliers (one of them an AO), and TD effect.

Besides the parameters associated with the outliers, the model contains 4 parameters. In terms of (1.1) and (1.4),  $\theta_1$  controls the stability of the trend,  $\theta_{12}$  controls the stability of the seasonal component,  $\alpha$  captures the magnitude of the TD effect, and  $\sigma_a$  reflects the overall forecasting accuracy. For the sample size considered this seems a sensible and parsimonious parametrization.

### 2.2.2 Out-of-Sample Forecasts

It is well-known that it is easier to get a good in-sample fit than a good out-of-sample forecasting. A fast and simple forecasting check can be done by running the TERROR application of TSW with  $k_1 = 3$  and  $k_2 = 4$  as thresholds for the t-values of the forecast errors (see Caporello and Maravall, 2003). This implies applying the automatic procedure (RSA = 3 with pretests for TD and EE) to all series without considering the last observation. The out-of-sample one-period-ahead forecasts are computed as well as their associated SE. Let the forecast for the last period (T) made at period (T-1) be  $y(T|T-1)$  and denote by  $\sigma(T|T-1)$  the associated SE. The standardized forecast error is given by  $\varepsilon(T|T-1) = [y(T) - y(T|T-1)] / \sigma(T|T-1)$ . The program provides the series for which  $|\varepsilon| > k_2$  and the series for which  $k_1 < |\varepsilon| < k_2$ . These are given in the following table, which contains the two groups ( $4 > |\varepsilon| > 3$ ) and ( $|\varepsilon| > 4$ .)

Table 6: Terror TSW Series List

DATE : 2009-07-12 16:30:57

Input Parameters :

mq=12 itrad=-2 least=-1 terror=1 sens=3 k1= 3.000 k2= 4.000

SERIES TITLE	Date	New Value (log)	Forecast (log)	Diff.	StdDev	T-Value
15 - E15_0039_2_4	02-2005	9.519554	9.035841	0.4837132	0.1348018	3.588328
95 - E15_1051_2_4	01-2005	9.700495	10.33736	-0.636864	0.2097786	-3.035887
143 - E15_1120_2_2	01-2005	12.50885	12.26593	0.2429178	0.0505890	4.801796
146 - E15_1120_2_5	01-2005	7.492492	7.330286	0.1622053	0.0427412	3.795062
147 - E15_1120_2_6	01-2005	7.935219	7.802855	0.1323640	0.0408977	3.236465
238 - E15_5400_2_7	02-2005	-	-	-1599.320	492.0600	-3.250253
44 - E15_0400_1_3	02-2005	11.94370	12.64883	-0.705130	0.1791757	-3.935412
78 - E15_0732_1_7	02-2005	8.175355	8.322451	-0.147097	0.0449377	-3.273347
143 - E15_1120_1_2	01-2005	13.03579	12.86020	0.1755931	0.0538027	3.263649
144 - E15_1120_1_3	01-2005	12.85950	13.34695	-0.487447	0.1437670	-3.390535
153 - E15_1130_1_2	01-2005	13.11993	12.95768	0.1622437	0.0506092	3.205815
154 - E15_1130_1_3	01-2005	12.89644	13.33765	-0.441208	0.1374026	-3.211058
179 - E15_1415_1_8	02-2005	6.733289	6.874406	-0.141117	0.0463187	-3.046666
182 - E15_1811_1_1	01-2005	12.78063	13.00449	-0.223867	0.0634605	-3.527659

#### Summary Statistics

500 Series were tested.

13 Releases exceeded the critical value  $|t| = 3$ .

1 Releases exceeded the critical value  $|t| = 4$ .

0 Series produced a Run-Time EXCEPTION.

0 Series did not match TERROR memory constraints.

486 Series passed the test:  $|t| < 3$ .

If the 500 values of  $\varepsilon$  were sampled from a  $N(0,1)$  distribution, the most likely number of  $\varepsilon$ 's in the ranges  $(3 < |\varepsilon| < 4)$  and  $(|\varepsilon| > 4)$  would be 1.3 and 0, respectively. Given that the  $|t|$  value set by default for an outlier (parameter VA) in series with 122 observations is 3.2, one would expect about 7 outliers among the last 500 observations. At time T-1, these outliers would be detected as forecast errors with  $|t| > 3$ . One would thus expect that, if the 500  $\varepsilon$ 's were drawn from independent  $N(0,1)$  distributions, after allowing for outliers, 8 (or 9) series would fail the test  $|\varepsilon| < 3$ .

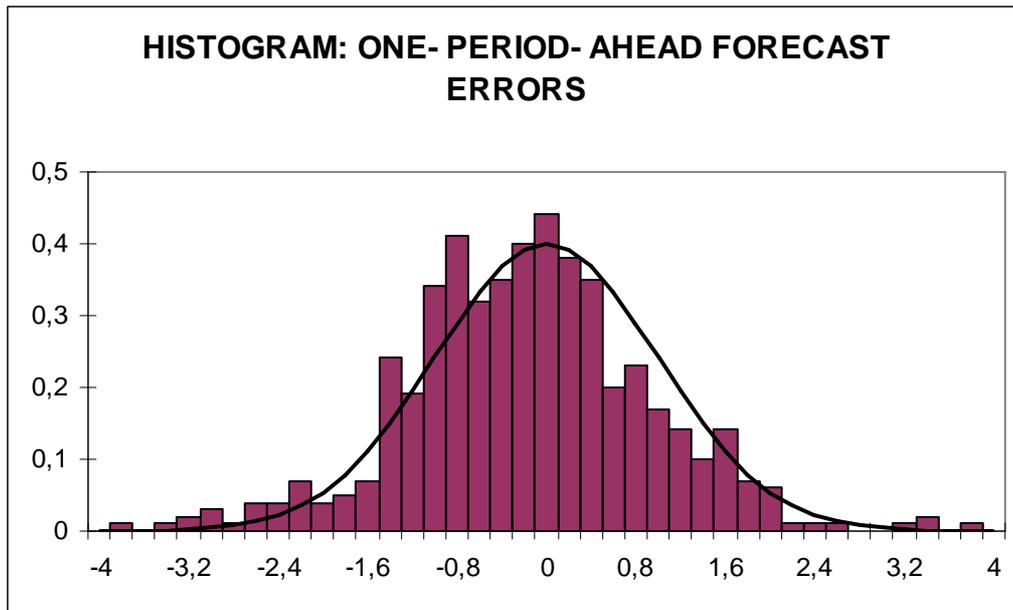
Of the 500 forecast errors obtained, 10 have fallen in the range  $(3 < |\varepsilon| < 4)$  and none in the range  $(|\varepsilon| > 4)$ .

Although one should bear in mind that the independence assumption cannot be fully expected to apply to the set of series considered, comparing the number of test failures (14) to what could be its expected value (8 or 9) in a set of 500 series, the out-of-sample forecasting performance seems comforting.

Still, the previous table indicates that, at least, series 15, 143 and 146 in the exports group, and series 44 and 182 in the imports group, for which the forecast error  $|t|$  value is above 3.5, could be considered problematic. (To identify an export or import series, let the last two digits in the series title be I\_J. When I = 2, it is an export series; I = 1 indicates imports.)

The histogram of the 500 standardized one – period – ahead forecast error is displayed in Figure 22. Despite a slight negative bias, possibly due to series crosscorrelations, the histogram conforms reasonably well to the asymptotic  $N(0,1)$  distribution.

Figure 22



### 2.2.3 A Remark on Extrapolation of the AMI results

From the results of the last two sections, the performance of the automatic procedure, both in terms of fitting and short-term forecasting, seems excellent. In our experience, however, albeit not misleading, the results are somewhat above average, as we proceed to illustrate. The set of 500 series was extracted from a complete set of 6773 foreign trade indicators from Eurostat. The length of the series varies between 72 and 170 monthly observations. Redoing the exercise of Section 2.2.1 for the full set, the percentage of series that passed the seven tests at the 9% level were:

- 99.3% for Q
- 98.1% for N
- 98.5% for Sk
- 98.6% for Kur
- 99.7% for QS
- 96.9% for Q2
- 99.7% for Runs.

For three of the test (Q, QS, and Runs) the 99% level is exceeded. Three more (N, Sk, and Kur) exceed the 98% level, and for Q2 the level is close to 97%.

Therefore, the fitting diagnostics for the full set are close to those obtained for the set of 500. the out-of-sample forecast exercise –performed now for the 6773 series- is less satisfactory. The proportion of series for which the standardized forecast error is larger than 3 in absolute value becomes 0.06, above the 0.03 proportion obtained for the set of 500 series.

Altogether, the results for the AMI of the 500 series are above average, but not in an alarming manner. It will often be the case that, when modelling large sets of monthly (or quarterly) series, the automatic procedure will yield a good model for, roughly, 85-95% of the series in the set.

## 2.3 SUMMARY RESULTS FOR THE INDIVIDUAL SERIES

Besides the tables with the arrays output by the program and a selection of the graphs, when more than one series is treated the following files are produced.

### **Matrices with Summary Results for Each Series**

By clicking on the “Out\_Matrix” icon of the main window, a detailed summary of the results obtained for each series is presented. This summary comprises several tables (“matrices”) with different information; each row represents the result for a particular series. The tables and their column entries are as follows.

#### a) **TRAMO Matrices.**

##### Model fit.

NZ: number of observations.

Lam: 0 when logs; 1 when levels.

Mean: 0 when mean is 0; 1 when  $\neq 0$ .

(p, d, q): orders of regular part of ARIMA model.

(bp, bd, bq): orders of seasonal part of ARIMA model.

SE (res): Standard error of white-noise residuals.

BIC: Bayesian Information Criterion.

The next columns of the matrix contain Residual Diagnostics:

Q(24): Ljung-Box test for residual autocorrelation, computed with 24 autocorrelations, asymptotically distributed (a.d.) as  $\chi^2([24 - (p + q + bp + bq)] \text{ d.f.})$  .

N - test: Behra-Jarque test for Normality of the residuals, a.d. as  $\chi^2(2 \text{ d.f.})$  .

SK (t): under  $[H_0 : \text{skewness (residuals)} = 0]$ , a.d. as a  $N(0,1)$ .

KUR (t): under  $[H_0 : \text{kurtosis (residuals)} = 3]$ , a.d. as a  $N(3,1)$ .

QS: (Modified) Pierce test for the presence of seasonality in the residual autocorrelation, a.d. approximately as  $\chi^2(2 \text{ d.f.})$  .

Q2: McLeod and Li test on linearity of the process versus bilinear or ARCH-type structures, computed with 24 autocorrelations, and a.d. as  $\chi^2(24 \text{ d.f.})$  .

RUNS: the t-value associated with  $[H_0 : \text{signs of the residuals are random}]$ ; a.d. as  $N(0,1)$ . Approximated critical values (CV) at the 95% level, for the 7 test, are given in the last row of the matrix.

##### Note on the QS Test

Pierce’s approximation did not take into account the signs of the seasonal autocorrelations. Thus, as an example, for monthly residuals  $\rho_{12}$  and  $-\rho_{12}$ , and  $\rho_{24}$  and  $-\rho_{24}$  would yield the same test statistics. However, failure of the test would properly indicate residual seasonality

only when the signs of the seasonal autocorrelations are appropriate. (A strong negative value of  $\rho_{12}$  may indicate a two-year periodic effect.) The modified Pierce test takes the signs into consideration. When they do not imply a seasonal effect, the statistics QS is made zero.

#### ARMA parameters.

PHI: Regular AR parameters.

BPHI: Seasonal AR parameter.

TH: Regular MA parameters.

BTH: Seasonal MA parameter.

All are given with the associated t-value. The polynomial is always represented with "+" sign, as in  $(1 + \phi_1B + \dots)$

#### Roots.

For each series model all AR and MA polynomials are factorized. The modulus and period associated with each root are given. (The sign "-" denotes the period for a zero frequency i.e., an  $\infty$  period root.)

#### Deterministic effects.

TD: Presence of Trading Day effect. (Number of parameters in the TD specification; LY effect is also included.)

EE: Presence of Easter effect.

#OUT: Total number of outliers.

AO: number of Additive Outliers.

TC: number of Transitory Change outliers.

LS: number of Level Shift outliers.

REG: additional regression variables.

MO: number of Missing Observations.

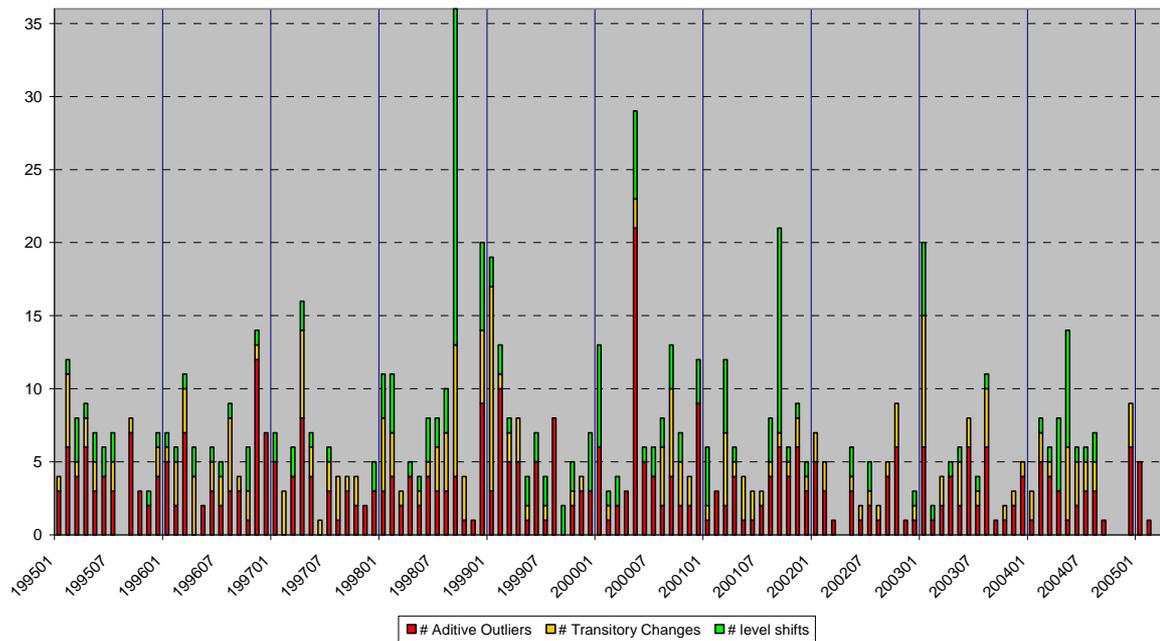
MEAN(t): Mean of the differenced series (and associated t-value)

#### Outliers.

For each series and each detected and corrected outlier, the type, date, and t-value are given.

The macro **GroupOutliers** (available at the Bank of Spain web site,) facilitates analysis of the outliers by classifying them according to different criteria, namely: by date; by cardinal position (number  $I = 1, 2, \dots, 500$ ) of the series in the set, by type of outlier (AO, TC, LS), and by absolute value of the associated t statistics ( $|t|$ ). As an example, Figure 11 exhibits the distribution of outliers over the sample period. One then may identify, for example, groups of series that share outliers or periods with an abnormal concentration of them.

Figure 23: Distribution of outliers (by Date)



In our example, there is some moderate amount of outlier sharing; the most shared outlier appears in 36 series, mostly as an LS outlier. The central years of the sample period exhibit a slightly larger frequency of outliers, while the last years contain slightly fewer outliers.

#### Calendar Effect.

The coefficients (and associated t values) of the TD, Leap Year, and EE variables are given.

Besides the aggregate Model Summary of Section 2.1.1, two more tables are produced: one ("Regressions") with the coefficients of the intervention and regression variables entered by the user and the t-values thereof; the other ("Input") with the input parameters for each series. In our example there are no such regression variables, and all series share the same input (RSA = 4, an entirely automatic procedure).

#### b) **SEATS Matrices.**

##### General.

Preadj.: Y if preadjusted with TRAMO; N otherwise.

Model changed: Y if SEATS has modified the model of TRAMO; N otherwise.

Approx. to NA: Y if the model has been changed because it did not have an admissible decomposition; N otherwise.

m, p, d, q, bp, db, bq: mean and orders of the model used by SEATS.

SD ( $a_t$ ): Standard error of the extended residual computed by SEATS.

Spectral Factorization: 0 if the decomposition of the ARIMA model has been successful; E otherwise.

Check on ACF: Check that the empirical ACF of the estimate obtained is in agreement with the theoretical ACF of the MMSE estimator. E: not in agreement; 0 otherwise.

Check on CCF: As the previous one, but for the theoretical and empirical cross-correlation functions between the stationary transformation of the components' estimators.

CMT: Y if the component (trend-cycle, seasonal, irregular and transitory component) estimated by SEATS is modified by some of the deterministic effects estimated by TRAMO; N otherwise.

*Note on TRAMO and SEATS residuals: The number of TRAMO residuals (output by the Kalman filter) are equal to the length of the series minus the order of differencing, minus the degrees of freedom lost in parameter estimation, and minus the missing observations. These are the residuals that should be expected to be white noise.*

*The SEATS residuals are obtained by filtering the linearized interpolated series with the ARIMA model, and residuals for the first periods lost by differencing are estimated by ML. These "extended" residuals cannot be expected to strictly be white noise, yet they should not –and in general will not– be far from the white-noise residuals of TRAMO. Their advantage is that, in this way and in general, a residual can be assigned to each period of the series length.*

#### Parameters I (Standard Errors).

SD (innov): Standard deviation of the innovations for the trend-cycle (T-C), seasonal, irregular, and transitory components, and of the SA series.

SE Est.: Standard error of the concurrent estimator of the component (SA series and T-C).

SE Rev.: Standard error of the revision in the concurrent estimator of the SA series and T-C component.

*Note: The previous SD and SE are expressed in units of the series (additive decompositions) or in units of the logs (multiplicative decomposition). In the latter case the units are (approximately) fractions of the levels.*

SE Rates of growth: For multiplicative decomposition, standard error of the concurrent estimator of the following two rates of growth (measured with the SA series and the T-C component):

- SE T11: last month rate of growth.
- SE T1 Mq: annual rate of growth, centered in the last observed month and completed with forecasts.

Both are expressed in percent points.

For additive decompositions, "rate of growth" should be replaced simply by "growth", expressed in units of the series.

#### Parameters II (Additional features).

Convergence: % reduction in the variance of the revision in the concurrent estimator of the SA series and of the T-C component

- after 1 more year of observations;

- after 5 more years of observations.

Signif. Season. (95%): Number of months per year with a significant seasonal component

- for historical estimation;
- for last year (preliminary) estimation,
- for next year of forecasts.

DAA: Difference in absolute averages between the annual means of the series and of the SA series and T-C component. It measures the possible bias effect induced by the log transformation. Expressed as fractions of 1% of the series level.

The matrices can be saved as Excel files.

The next tables present the first 25 rows of the six TRAMO and three SEATS matrices listed above.

Table 7: Model fit

n	TITLE	Nz	Lam	Mean	P	D	Q	BP	BD	BQ	SE(res)	BIC	Q-val	N-test	SK(t)	KUR(t)	QS	Q2	RUNS
1	"1 - E15_0028_2_0"	122	0	0	0	1	1	0	1	1	0.0603814	-5.47938	24.36	0.608	0.768	-0.13	0.607	24.03	-0.19
2	"2 - E15_0028_2_1"	122	0	1	0	1	1	0	1	1	0.1044144	-4.31715	36.40	5.29	2.09	0.966	0.135	13.38	-0.98
3	"3 - E15_0028_2_2"	122	0	0	0	1	1	0	1	1	0.0784057	-4.92348	27.58	0.627	0.588	-0.53	1.67	31.62	-0.59
4	"4 - E15_0028_2_3"	122	0	0	0	1	1	0	0	0	0.2506651	-2.67348	16.13	0.721	-0.42	-0.74	0.	22.46	1.29
5	"5 - E15_0028_2_4"	122	0	0	1	1	1	0	1	1	0.2544208	-2.56929	14.42	1.54	0.074	1.24	0.407	22.68	-0.78
6	"6 - E15_0028_2_5"	122	0	0	0	1	1	0	1	1	0.0493655	-5.84876	28.43	0.031	0.162	-0.07	0.	62.25	-1.37
7	"7 - E15_0028_2_6"	122	0	0	0	1	1	0	1	1	0.0673705	-5.19348	20.76	0.194	-0.15	-0.41	6.07	23.02	0.
8	"8 - E15_0028_2_7"	122	0	0	0	1	3	0	1	1	0.0703015	-4.91008	12.57	1.11	0.985	-0.38	0.	31.06	1.40
9	"9 - E15_0028_2_8"	122	0	0	0	1	1	0	1	1	0.0446324	-5.95050	17.38	1.28	0.466	-1.03	2.30	25.44	0.
10	"10 - E15_0028_2_TTT"	122	0	0	2	1	0	0	1	1	0.0508438	-5.68990	20.35	1.73	1.03	-0.82	0.	11.61	-0.99
11	"11 - E15_0039_2_0"	122	0	0	0	1	1	0	1	1	0.0449013	-6.10537	21.82	2.67	-1.48	-0.68	1.67	29.51	0.
12	"12 - E15_0039_2_1"	122	0	0	0	1	1	0	1	1	0.0654843	-5.21699	17.37	0.110	-0.33	0.068	0.186	16.72	-1.58
13	"13 - E15_0039_2_2"	122	0	1	3	0	0	0	1	1	0.0577243	-5.47078	20.03	0.124	0.149	0.320	1.47	33.23	0.777
14	"14 - E15_0039_2_3"	122	0	0	0	1	1	0	1	1	0.0714158	-5.01038	24.97	1.41	-0.80	0.874	0.	7.429	-0.40
15	"15 - E15_0039_2_4"	122	0	0	0	1	1	0	1	1	0.1428708	-3.79042	15.00	1.84	1.30	0.388	0.	20.68	-0.97
16	"16 - E15_0039_2_5"	122	0	1	0	1	1	1	0	0	0.0849773	-4.77477	25.85	3.34	-1.82	-0.09	0.	24.81	1.85
17	"17 - E15_0039_2_6"	122	0	0	0	1	1	0	1	1	0.0557434	-5.60575	25.70	3.69	-1.79	0.695	0.	25.79	0.976
18	"18 - E15_0039_2_7"	122	0	0	0	1	1	0	1	1	0.0646807	-5.37537	15.73	1.08	1.03	-0.17	0.039	22.64	0.193
19	"19 - E15_0039_2_8"	122	1	1	2	0	0	0	1	1	46.42101	7.90887	26.56	2.38	1.52	0.274	2.76	23.59	-0.20
20	"20 - E15_0039_2_TTT"	122	0	0	0	1	1	0	1	1	0.0434309	-6.17196	21.37	0.758	0.785	-0.38	0.920	25.75	-0.19
21	"21 - E15_0052_2_0"	122	0	0	0	1	1	0	1	1	0.2076089	-2.97596	31.15	3.48	1.84	0.342	1.69	18.79	-0.39
22	"22 - E15_0052_2_1"	122	1	0	0	1	1	0	1	1	2400.830	15.7019	18.35	0.525	0.723	-0.05	0.660	17.99	-1.94
23	"23 - E15_0052_2_2"	122	0	0	0	1	1	0	1	1	0.1900444	-3.15275	33.47	2.66	1.60	-0.33	0.	26.05	1.37
24	"24 - E15_0052_2_3"	122	0	0	2	1	1	0	0	0	0.4508168	-1.49959	35.41	1.02	-0.77	-0.66	2.84	36.82	-0.73
25	"25 - E15_0052_2_4"	122	0	0	0	1	1	0	1	1	0.6350369	-0.84058	13.55	1.16	-0.59	0.901	0.	12.99	-0.38

<b>Approx. 95% CRITICAL VALUES:</b> (AV: absolute value)	< 34	< 6	AV <2	AV <2	< 6	< 36	AV <2
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Table 8: ARMA parameters

n	TITLE	PHI1	(t)	PHI2	(t)	PHI3	(t)	BPHI	(t)	TH1	(t)	TH2	(t)	TH3	(t)	BTH	(t)
1	"1 - E15_0028_2_0"	-	( -)	-	( -)	-	( -)	-	( -)	-0.81222	(-13.)	-	( -)	-	( -)	-0.83962	(-6.4)
2	"2 - E15_0028_2_1"	-	( -)	-	( -)	-	( -)	-	( -)	-0.82101	(-13.)	-	( -)	-	( -)	-0.79575	(-6.6)
3	"3 - E15_0028_2_2"	-	( -)	-	( -)	-	( -)	-	( -)	-0.56135	(-6.7)	-	( -)	-	( -)	-0.83439	(-6.2)
4	"4 - E15_0028_2_3"	-	( -)	-	( -)	-	( -)	-	( -)	-0.75032	(-12.)	-	( -)	-	( -)	-	( -)
5	"5 - E15_0028_2_4"	0.426649	( 3.9)	-	( -)	-	( -)	-	( -)	-0.60755	(-6.2)	-	( -)	-	( -)	-0.91090	(-23.)
6	"6 - E15_0028_2_5"	-	( -)	-	( -)	-	( -)	-	( -)	-0.64240	(-8.2)	-	( -)	-	( -)	-0.80051	(-7.1)
7	"7 - E15_0028_2_6"	-	( -)	-	( -)	-	( -)	-	( -)	-0.53510	(-6.3)	-	( -)	-	( -)	-0.83033	(-6.6)
8	"8 - E15_0028_2_7"	-	( -)	-	( -)	-	( -)	-	( -)	-0.45716	(-4.6)	-0.27306	(-2.8)	0.366436	( 3.9)	-0.89369	(-4.2)
9	"9 - E15_0028_2_8"	-	( -)	-	( -)	-	( -)	-	( -)	-0.66001	(-8.8)	-	( -)	-	( -)	-0.60309	(-5.3)
10	"10 - E15_0028_2_TTT"	0.469530	( 5.0)	0.361254	( 3.8)	-	( -)	-	( -)	-	( -)	-	( -)	-	( -)	-0.80915	(-5.4)
11	"11 - E15_0039_2_0"	-	( -)	-	( -)	-	( -)	-	( -)	-0.80275	(-12.)	-	( -)	-	( -)	-0.77611	(-7.3)
12	"12 - E15_0039_2_1"	-	( -)	-	( -)	-	( -)	-	( -)	-0.78601	(-13.)	-	( -)	-	( -)	-0.57600	(-6.0)
						-											
13	"13 - E15_0039_2_2"	-0.42454	(-4.3)	0.039061	(0.39)	0.21187	(-2.2)	-	( -)	-	( -)	-	( -)	-	( -)	-0.95530	(-34.)
14	"14 - E15_0039_2_3"	-	( -)	-	( -)	-	( -)	-	( -)	-0.32473	(-3.6)	-	( -)	-	( -)	-0.94320	(-30.)
15	"15 - E15_0039_2_4"	-	( -)	-	( -)	-	( -)	-	( -)	-0.76430	(-11.)	-	( -)	-	( -)	-0.81011	(-6.0)
16	"16 - E15_0039_2_5"	-	( -)	-	( -)	-	( -)	-0.39062	(-4.4)	-0.86857	(-19.)	-	( -)	-	( -)	-	( -)
17	"17 - E15_0039_2_6"	-	( -)	-	( -)	-	( -)	-	( -)	-0.63972	(-8.7)	-	( -)	-	( -)	-0.98500	(-60.)
18	"18 - E15_0039_2_7"	-	( -)	-	( -)	-	( -)	-	( -)	-0.56992	(-7.2)	-	( -)	-	( -)	-0.96783	(-40.)
19	"19 - E15_0039_2_8"	-0.22430	(-2.6)	-0.48483	(-5.5)	-	( -)	-	( -)	-	( -)	-	( -)	-	( -)	-0.85122	(-6.5)
20	"20 - E15_0039_2_TTT"	-	( -)	-	( -)	-	( -)	-	( -)	-0.60587	(-8.0)	-	( -)	-	( -)	-0.98500	(-60.)
21	"21 - E15_0052_2_0"	-	( -)	-	( -)	-	( -)	-	( -)	-0.61653	(-8.3)	-	( -)	-	( -)	-0.89727	(-4.8)
22	"22 - E15_0052_2_1"	-	( -)	-	( -)	-	( -)	-	( -)	-0.55758	(-6.8)	-	( -)	-	( -)	-0.63563	(-6.5)
23	"23 - E15_0052_2_2"	-	( -)	-	( -)	-	( -)	-	( -)	-0.14993	(-1.5)	-	( -)	-	( -)	-0.76705	(-6.8)
24	"24 - E15_0052_2_3"	-0.41641	(-3.4)	-0.22132	(-2.0)	-	( -)	-	( -)	-0.91524	(-12.)	-	( -)	-	( -)	-	( -)
25	"25 - E15_0052_2_4"	-	( -)	-	( -)	-	( -)	-	( -)	-0.77682	(-12.)	-	( -)	-	( -)	-0.73206	(-6.4)

Table 9: Roots (of ARMA Polynomials)

n	Title	REGULAR AR INVERSE ROOTS						REGULAR MA INVERSE ROOTS					
		ROOT(1)		ROOT(2)		ROOT(3)		ROOT(1)		ROOT(2)		ROOT(3)	
		mod	per	mod	per	mod	per	mod	per	mod	per	mod	per
1	"1 - E15_0028_2_0"	-	-	-	-	-	-	0.812	-	-	-	-	-
2	"2 - E15_0028_2_1"	-	-	-	-	-	-	0.821	-	-	-	-	-
3	"3 - E15_0028_2_2"	-	-	-	-	-	-	0.561	-	-	-	-	-
4	"4 - E15_0028_2_3"	-	-	-	-	-	-	0.750	-	-	-	-	-
5	"5 - E15_0028_2_4"	0.427	2.0	-	-	-	-	0.608	-	-	-	-	-
6	"6 - E15_0028_2_5"	-	-	-	-	-	-	0.642	-	-	-	-	-
7	"7 - E15_0028_2_6"	-	-	-	-	-	-	0.535	-	-	-	-	-
8	"8 - E15_0028_2_7"	-	-	-	-	-	-	0.695	2.0	0.726	-9.6	0.726	9.6
9	"9 - E15_0028_2_8"	-	-	-	-	-	-	0.660	-	-	-	-	-
10	"10 - E15_0028_2_TTT"	0.601	-3.2	0.601	3.2	-	-	-	-	-	-	-	-
11	"11 - E15_0039_2_0"	-	-	-	-	-	-	0.803	-	-	-	-	-
12	"12 - E15_0039_2_1"	-	-	-	-	-	-	0.786	-	-	-	-	-
13	"13 - E15_0039_2_2"	0.532	-3.3	0.532	3.3	0.750	-	-	-	-	-	-	-
14	"14 - E15_0039_2_3"	-	-	-	-	-	-	0.325	-	-	-	-	-
15	"15 - E15_0039_2_4"	-	-	-	-	-	-	0.764	-	-	-	-	-
16	"16 - E15_0039_2_5"	-	-	-	-	-	-	0.869	-	-	-	-	-
17	"17 - E15_0039_2_6"	-	-	-	-	-	-	0.640	-	-	-	-	-
18	"18 - E15_0039_2_7"	-	-	-	-	-	-	0.570	-	-	-	-	-
19	"19 - E15_0039_2_8"	0.593	2.0	0.817	-	-	-	-	-	-	-	-	-
20	"20 - E15_0039_2_TTT"	-	-	-	-	-	-	0.606	-	-	-	-	-
21	"21 - E15_0052_2_0"	-	-	-	-	-	-	0.617	-	-	-	-	-
22	"22 - E15_0052_2_1"	-	-	-	-	-	-	0.558	-	-	-	-	-
23	"23 - E15_0052_2_2"	-	-	-	-	-	-	0.150	-	-	-	-	-
24	"24 - E15_0052_2_3"	0.306	2.0	0.723	-	-	-	0.915	-	-	-	-	-
25	"25 - E15_0052_2_4"	-	-	-	-	-	-	0.777	-	-	-	-	-

Table 10: Deterministic Effects

n	TITLE	TD	EE	#OUT	AO	TC	LS	REG	MO	MEAN	(t)
1	"1 - E15_0028_2_0"	1	1	0	0	0	0	0	0	0.0000	
2	"2 - E15_0028_2_1"	1	1	1	1	0	0	0	0	-0.0013	( -1.86)
3	"3 - E15_0028_2_2"	1	1	1	1	0	0	0	0	0.0000	
4	"4 - E15_0028_2_3"	0	0	2	2	0	0	0	0	0.0000	
5	"5 - E15_0028_2_4"	0	0	2	1	1	0	0	0	0.0000	
6	"6 - E15_0028_2_5"	1	1	1	1	0	0	0	0	0.0000	
7	"7 - E15_0028_2_6"	1	0	3	2	0	1	0	0	0.0000	
8	"8 - E15_0028_2_7"	1	1	6	5	1	0	0	0	0.0000	
9	"9 - E15_0028_2_8"	1	1	4	2	1	1	0	0	0.0000	
10	"10 - E15_0028_2_TTT"	1	1	3	2	1	0	0	0	0.0000	
11	"11 - E15_0039_2_0"	1	0	0	0	0	0	0	0	0.0000	
12	"12 - E15_0039_2_1"	1	0	4	1	3	0	0	0	0.0000	
13	"13 - E15_0039_2_2"	1	0	1	0	0	1	0	0	0.0319	( 6.73)
14	"14 - E15_0039_2_3"	0	0	6	4	1	1	0	0	0.0000	
15	"15 - E15_0039_2_4"	0	0	1	0	0	1	0	0	0.0000	
16	"16 - E15_0039_2_5"	1	0	1	0	1	0	0	0	0.0061	( 3.66)
17	"17 - E15_0039_2_6"	1	0	2	0	0	2	0	0	0.0000	
18	"18 - E15_0039_2_7"	1	0	0	0	0	0	0	0	0.0000	
19	"19 - E15_0039_2_8"	1	0	2	0	2	0	0	0	41.2189	( 8.66)
20	"20 - E15_0039_2_TTT"	1	0	0	0	0	0	0	0	0.0000	
21	"21 - E15_0052_2_0"	0	0	3	1	1	1	0	0	0.0000	
22	"22 - E15_0052_2_1"	1	0	1	0	0	1	0	0	0.0000	
23	"23 - E15_0052_2_2"	1	0	2	1	0	1	0	0	0.0000	
24	"24 - E15_0052_2_3"	0	0	0	0	0	0	0	0	0.0000	
25	"25 - E15_0052_2_4"	0	0	0	0	0	0	0	0	0.0000	

Table 11: Outliers

1	"1 - E15_0028_2_0"						
2	"2 - E15_0028_2_1"	AO01(1296, 3.20)					
3	"3 - E15_0028_2_2"	AO01(0400, 3.28)					
4	"4 - E15_0028_2_3"	AO01(0195, -3.37)	AO02(0704, 3.39)				
5	"5 - E15_0028_2_4"	AO01(0897, -5.20)	TC01(0998, 5.38)				
6	"6 - E15_0028_2_5"	AO01(1296, -3.81)					
7	"7 - E15_0028_2_6"	AO01(0495, 6.30)	AO02(0898, 4.30)	LS01(0295, -3.99)			
8	"8 - E15_0028_2_7"	AO01(0797, 10.06)	AO02(0498, 6.95)	AO03(0801, 4.47)	AO04(1204, 3.74)	AO05(0699, 4.29)	TC01(0497, 3.86)
9	"9 - E15_0028_2_8"	AO01(0102, 4.39)	AO02(0600, -3.78)	TC01(0599, -3.07)	LS01(0204, 3.08)		
10	"10 - E15_0028_2_TTT"	AO01(0797, 6.03)	AO02(0498, 4.44)	TC01(1204, 3.53)			
11	"11 - E15_0039_2_0"						
12	"12 - E15_0039_2_1"	AO01(0803, 4.54)	TC01(0103, 4.29)	TC02(0195, -3.32)	TC03(0603, -3.77)		
13	"13 - E15_0039_2_2"	LS01(0296, -5.32)					
14	"14 - E15_0039_2_3"	AO01(0899, -10.08)	AO02(0798, -7.93)	AO03(0796, -5.27)	AO04(0896, 4.42)	TC01(0198, 4.42)	LS01(0799, 3.76)
15	"15 - E15_0039_2_4"	LS01(0103, -3.42)					
16	"16 - E15_0039_2_5"	TC01(0803, -3.70)					
17	"17 - E15_0039_2_6"	LS01(1101, -4.98)	LS02(1000, 3.27)				
18	"18 - E15_0039_2_7"						
19	"19 - E15_0039_2_8"	TC01(1203, -4.83)	TC02(0603, -3.28)				
20	"20 - E15_0039_2_TTT"						
21	"21 - E15_0052_2_0"	AO01(0202, 4.50)	TC01(0301, -3.83)	LS01(0695, 6.82)			
22	"22 - E15_0052_2_1"	LS01(0698, -3.95)					
23	"23 - E15_0052_2_2"	AO01(0199, -3.90)	LS01(0101, -4.15)				
24	"24 - E15_0052_2_3"						
25	"25 - E15_0052_2_4"						

Table 12: Calendar Effects

n	TITLE	TD1 (t)	TD2 (t)	TD3 (t)	TD4 (t)	TD5 (t)	TD6 (t)	LY (t)	EE (t)
1	"1 - E15_0028_2_0"	0.010765 ( 6.2)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	-0.08040 (-2.8)
2	"2 - E15_0028_2_1"	0.007432 ( 2.5)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	-0.11237 (-2.3)
3	"3 - E15_0028_2_2"	0.008545 ( 4.3)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	-0.11969 (-3.6)
4	"4 - E15_0028_2_3"	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)
5	"5 - E15_0028_2_4"	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)
6	"6 - E15_0028_2_5"	0.012439 ( 9.7)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	-0.09112 (-4.3)
7	"7 - E15_0028_2_6"	0.011786 ( 7.0)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)
8	"8 - E15_0028_2_7"	0.010635 ( 5.3)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	-0.09219 (-3.2)
9	"9 - E15_0028_2_8"	0.009161 ( 8.2)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	-0.07315 (-4.2)
10	"10 - E15_0028_2_TTT"	0.010139 ( 7.5)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	-0.09530 (-4.9)
11	"11 - E15_0039_2_0"	0.006904 ( 5.6)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)
12	"12 - E15_0039_2_1"	0.007263 ( 4.2)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)
13	"13 - E15_0039_2_2"	0.007843 ( 4.9)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)
14	"14 - E15_0039_2_3"	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)
15	"15 - E15_0039_2_4"	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)
16	"16 - E15_0039_2_5"	0.008634 ( 3.2)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)
17	"17 - E15_0039_2_6"	0.007043 ( 4.6)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)
18	"18 - E15_0039_2_7"	0.010302 ( 6.1)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)
19	"19 - E15_0039_2_8"	10.7055 ( 9.3)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)
20	"20 - E15_0039_2_TTT"	0.009292 ( 7.9)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)
21	"21 - E15_0052_2_0"	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)
22	"22 - E15_0052_2_1"	277.975 ( 5.0)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)
23	"23 - E15_0052_2_2"	0.013425 ( 3.7)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)
24	"24 - E15_0052_2_3"	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)
25	"25 - E15_0052_2_4"	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)	- ( -)

Table 13: SEATS, General

n	Title	Preadj	Model Changed	Approx. to NA	NewModel						SD(a)	Spectrum Factor	Check on ACF	Check on CCF	TC	CMT			
					m	p	d	q	Bp	Bd						Bq	S	U	Trans
1	"1 - E15_0028_2_0"	Y	N	N	0	0	1	1	0	1	1	.5981E-01	0	0	0	N	Y	N	N
2	"2 - E15_0028_2_1"	Y	N	N	1	0	1	1	0	1	1	.1107	0	0	0	N	Y	Y	N
3	"3 - E15_0028_2_2"	Y	N	N	0	0	1	1	0	1	1	.7730E-01	0	0	0	N	Y	Y	N
4	"4 - E15_0028_2_3"	Y	N	N	0	0	1	1	0	0	0	.2486	0	0	0	N	N	Y	N
5	"5 - E15_0028_2_4"	Y	N	N	0	1	1	1	0	1	1	.2490	0	0	0	N	N	Y	N
6	"6 - E15_0028_2_5"	Y	N	N	0	0	1	1	0	1	1	.4867E-01	0	0	0	N	Y	Y	N
7	"7 - E15_0028_2_6"	Y	N	N	0	0	1	1	0	1	1	.6610E-01	0	0	0	Y	Y	Y	N
8	"8 - E15_0028_2_7"	Y	Y	Y	0	0	1	2	0	1	1	.6796E-01	0	0	0	N	Y	Y	N
9	"9 - E15_0028_2_8"	Y	N	N	0	0	1	1	0	1	1	.4336E-01	0	0	0	Y	Y	Y	N
10	"10 - E15_0028_2_TTT"	Y	N	N	0	2	1	0	0	1	1	.5007E-01	0	0	0	N	Y	Y	N
11	"11 - E15_0039_2_0"	Y	N	N	0	0	1	1	0	1	1	.4469E-01	0	0	0	N	Y	N	N
12	"12 - E15_0039_2_1"	Y	N	N	0	0	1	1	0	1	1	.6394E-01	0	0	0	N	Y	Y	N
13	"13 - E15_0039_2_2"	Y	N	N	1	3	0	0	0	1	1	.5768E-01	0	0	0	Y	Y	N	N
14	"14 - E15_0039_2_3"	Y	N	N	0	0	1	1	0	1	1	.6938E-01	0	0	0	Y	N	Y	N
15	"15 - E15_0039_2_4"	Y	N	N	0	0	1	1	0	1	1	.1422	0	0	0	Y	N	N	N
16	"16 - E15_0039_2_5"	Y	N	N	1	0	1	1	1	0	0	.8680E-01	0	0	0	N	Y	Y	N
17	"17 - E15_0039_2_6"	Y	N	N	0	0	1	1	0	1	1	.5496E-01	0	0	0	Y	Y	N	N
18	"18 - E15_0039_2_7"	Y	N	N	0	0	1	1	0	1	1	.6438E-01	0	0	0	N	Y	N	N
19	"19 - E15_0039_2_8"	Y	N	N	1	2	0	0	0	1	1	45.73	0	0	0	N	Y	Y	N
20	"20 - E15_0039_2_TTT"	Y	N	N	0	0	1	1	0	1	1	.4323E-01	0	0	0	N	Y	N	N
21	"21 - E15_0052_2_0"	Y	N	N	0	0	1	1	0	1	1	.2047	0	0	0	Y	N	Y	N
22	"22 - E15_0052_2_1"	Y	N	N	0	0	1	1	0	1	1	2378.	0	0	0	Y	Y	N	N
23	"23 - E15_0052_2_2"	Y	N	N	0	0	1	1	0	1	1	.1874	0	0	0	Y	Y	Y	N
24	"24 - E15_0052_2_3"	Y	N	N	0	2	1	1	0	0	0	.4500	0	0	0	N	N	N	N
25	"25 - E15_0052_2_4"	Y	N	N	0	0	1	1	0	1	1	.6350	0	0	0	N	N	N	N

Table 14: SEATS, Parameters I (Standard Errors)

n	Title	SD(innov)					SE Est. (Conc.)		SE Rev. (Conc.)		SE : Rates of Growth				
		TC	S	Trans.	U	SA	TC	SA	TC	SA	SE T11 (One Period)		SE T1Mq (Annual Period)		
											TC	SA	X	TC	SA
1	"1 - E15_0028_2_0"	.5203E-02	.6317E-02	0.000	.4975E-01	.5530E-01	.2218E-01	.1942E-01	.1606E-01	.1340E-01	0.73	2.80	6.49	3.49	6.43
2	"2 - E15_0028_2_1"	.8986E-02	.1498E-01	0.000	.9019E-01	.1000	.3997E-01	.3967E-01	.2934E-01	.2716E-01	1.26	5.72	11.93	6.17	11.76
3	"3 - E15_0028_2_2"	.1563E-01	.6113E-02	0.000	.5535E-01	.7139E-01	.3719E-01	.2433E-01	.2615E-01	.1714E-01	2.03	3.22	10.83	9.23	10.77
4	"4 - E15_0028_2_3"	.3103E-01	0.000	0.000	.2175	.2486	.1053	0.000	.7190E-01	0.000	4.24	0.00	28.47	18.86	28.47
5	"5 - E15_0028_2_4"	.3283E-01	.1399E-01	.8702E-01	.1373	.2387	.8890E-01	.6474E-01	.6457E-01	.4511E-01	4.43	10.73	29.19	19.52	29.11
6	"6 - E15_0028_2_5"	.7881E-02	.5376E-02	0.000	.3593E-01	.4414E-01	.2209E-01	.1658E-01	.1582E-01	.1158E-01	1.06	2.27	6.23	4.89	6.17
7	"7 - E15_0028_2_6"	.1413E-01	.5492E-02	0.000	.4643E-01	.6092E-01	.3222E-01	.2105E-01	.2262E-01	.1484E-01	1.82	2.75	9.53	8.28	9.48
8	"8 - E15_0028_2_7"	.1383E-01	.4209E-03	.2886E-01	.4147E-01	.6764E-01	.3810E-01	.5328E-02	.2442E-01	.3255E-02	2.04	0.66	9.65	8.05	9.65
9	"9 - E15_0028_2_8"	.5940E-02	.9807E-02	0.000	.2862E-01	.3513E-01	.1862E-01	.1863E-01	.1414E-01	.1282E-01	0.81	2.50	5.45	4.09	5.25
10	"10 - E15_0028_2_TTT"	.1243E-01	.5918E-02	.1423E-01	.2314E-01	.4559E-01	.2288E-01	.1702E-01	.1698E-01	.1192E-01	1.55	2.21	7.76	7.01	7.71
11	"11 - E15_0039_2_0"	.3953E-02	.6547E-02	0.000	.3562E-01	.3993E-01	.1662E-01	.1647E-01	.1223E-01	.1126E-01	0.55	2.36	4.88	2.71	4.80
12	"12 - E15_0039_2_1"	.5477E-02	.1753E-01	0.000	.4413E-01	.5052E-01	.2324E-01	.2842E-01	.1796E-01	.1884E-01	0.77	3.99	7.09	4.11	6.70
13	"13 - E15_0039_2_2"	.1696E-01	.1775E-02	.2130E-01	.3366E-01	.5649E-01	.2718E-01	.1110E-01	.1588E-01	.7775E-02	2.10	1.42	6.61	5.27	6.60
14	"14 - E15_0039_2_3"	.2281E-01	.2131E-02	0.000	.4465E-01	.6757E-01	.3505E-01	.1400E-01	.2288E-01	.9894E-02	2.60	1.65	12.57	11.96	12.56
15	"15 - E15_0039_2_4"	.1528E-01	.1718E-01	0.000	.1132	.1295	.5670E-01	.4869E-01	.4108E-01	.3358E-01	2.11	6.94	16.07	10.07	15.90
16	"16 - E15_0039_2_5"	.5736E-02	.5917E-01	0.000	.3711E-01	.4355E-01	.2516E-01	.3670E-01	.2036E-01	.1775E-01	0.82	4.71	9.05	4.34	6.59
17	"17 - E15_0039_2_6"	.9832E-02	.4542E-03	0.000	.4472E-01	.5458E-01	.2574E-01	.5278E-02	.1736E-01	.3445E-02	1.30	0.74	7.06	5.66	7.06
18	"18 - E15_0039_2_7"	.1364E-01	.9775E-03	0.000	.4972E-01	.6343E-01	.3122E-01	.9426E-02	.2096E-01	.6570E-02	1.76	1.28	8.93	7.66	8.93
19	"19 - E15_0039_2_8"	13.21	14.63	0.000	20.51	32.10	19.73	19.14	13.31	12.16	15.72	26.58	56.35	46.45	51.20
20	"20 - E15_0039_2_TTT"	.8460E-02	.3298E-03	0.000	.3445E-01	.4293E-01	.2065E-01	.4136E-02	.1385E-01	.2700E-02	1.10	0.57	5.76	4.80	5.76
21	"21 - E15_0052_2_0"	.3737E-01	.1102E-01	0.000	.1569	.1950	.9609E-01	.5252E-01	.6658E-01	.3696E-01	4.93	7.20	26.96	22.01	26.90
22	"22 - E15_0052_2_1"	432.0	415.6	0.000	1515.	1974.	1121.	998.6	837.2	703.6	572.12	1276.63	3345.42	2800.71	3262.29
23	"23 - E15_0052_2_2"	.7073E-01	.2484E-01	0.000	.9518E-01	.1670	.1028	.7753E-01	.7173E-01	.5521E-01	7.66	7.67	40.24	38.93	39.95
24	"24 - E15_0052_2_3"	.1689	0.000	.9011E-01	.2501	.4500	.2054	0.000	.1232	0.000	18.41	0.00	62.51	57.24	62.51
25	"25 - E15_0052_2_4"	.6200E-01	.1093	0.000	.4856	.5536	.2440	.2476	.1809	.1686	8.64	35.20	70.97	42.66	69.41

Table 15: Parameters II (Additional Features)

n	Title	Convergence (in %)				Signif. Season. (95%)			DAA	
		1Y		5Y		Hist.	Prel.	Fore.	TC	SA
		TC	SA	TC	SA					
1	"1 - E15_0028_2_0"	90.9	16.0	97.7	58.3	9	9	9	0.40	0.07
2	"2 - E15_0028_2_1"	90.4	20.4	98.0	68.1	11	9	9	1.28	0.27
3	"3 - E15_0028_2_2"	83.0	16.4	91.8	59.5	11	9	8	0.65	0.09
4	"4 - E15_0028_2_3"	96.8	0.0	100.0	0.0	0	0	0	2.84	0.00
5	"5 - E15_0028_2_4"	89.8	8.9	93.0	37.3	6	4	4	2.37	0.40
6	"6 - E15_0028_2_5"	87.0	19.8	94.7	67.1	9	8	6	0.35	0.07
7	"7 - E15_0028_2_6"	81.4	16.8	91.2	60.4	10	8	8	0.43	0.08
8	"8 - E15_0028_2_7"	96.9	1.7	97.3	8.4	12	11	11	0.77	0.10
9	"9 - E15_0028_2_8"	88.3	39.0	98.5	91.9	12	10	10	0.30	0.12
10	"10 - E15_0028_2_TTT"	72.8	19.0	88.4	65.3	11	10	10	0.18	0.04
11	"11 - E15_0039_2_0"	91.7	22.4	98.0	71.8	10	8	8	0.36	0.04
12	"12 - E15_0039_2_1"	93.2	42.2	99.2	93.6	7	7	5	0.62	0.16
13	"13 - E15_0039_2_2"	78.5	4.5	82.1	20.8	11	10	10	0.20	0.12
14	"14 - E15_0039_2_3"	77.0	5.7	81.8	25.5	8	6	6	0.29	0.08
15	"15 - E15_0039_2_4"	92.2	18.9	97.1	65.1	4	2	2	1.12	0.30
16	"16 - E15_0039_2_5"	96.2	99.4	100.0	100.0	4	3	0	0.83	0.40
17	"17 - E15_0039_2_6"	95.8	2.0	96.2	9.9	12	11	11	0.42	0.07
18	"18 - E15_0039_2_7"	91.1	3.4	92.3	16.0	8	8	8	0.43	0.11
19	"19 - E15_0039_2_8"	63.0	45.3	80.6	71.3	11	11	9	0.00	0.00
20	"20 - E15_0039_2_TTT"	95.2	2.0	95.6	9.9	10	10	10	0.18	0.05
21	"21 - E15_0052_2_0"	87.8	10.2	92.1	41.8	6	4	4	0.89	0.44
22	"22 - E15_0052_2_1"	82.6	35.4	97.2	89.5	7	5	4	0.00	0.00
23	"23 - E15_0052_2_2"	57.5	23.0	85.3	73.3	0	0	0	0.55	0.23
24	"24 - E15_0052_2_3"	97.7	0.0	100.0	0.0	0	0	0	3.24	0.00
25	"25 - E15_0052_2_4"	92.3	26.7	98.1	79.0	7	6	4	6.84	4.37

### **Identification of Problematic Series**

In large scale applications, detailed inspection of the many thousand matrix rows is, when feasible, rather cumbersome and prone to errors. However, the matrices in Excel format can be read by the macro **Problematic** –also available at the Bank of Spain web site-, and the ones with unacceptable statistics can be detected. One can enter “critical values” for these statistics or use the values set by default. While the critical values in the table Summary Models above were always 1%, irrespective of the test, the ones used by the macro Problematic may well reflect the analyst choice. For example, if the main purpose is to prewhiten the series, then one should be strict concerning the Q statistics; if seasonal adjustment is the objective, then one should be particularly careful with abnormally large values of the QS statistics.

At present, the macro checks 17 items. They are listed and briefly described in Table 16 below. Table 17 presents the series that have been detected as problematic in the 500 seirs set.

Table 16: Checks for Problematic Series

Variable name	Problem Description	Series judged problematic when	Assoc. pval. (approx.)	#detected in set of 500/Expected number in 500 series	Comment
1. Crash	Program crash	-1	---	0/-	Highly unlikely
2. Eif	Error in input file	E	---	0/-	E: Error (ex: file containing formula)
3. Q	Residual autocorrelation	> 42.8	1 in 200	1 / 2.5	Approx. $\chi^2(22)$ .
4. N	Residual Normality	> 14	1 in 1000	0/0.5	Approx. $\chi^2(2)$ .
5. Sk	Skewness of residuals	av > 3.1	1 in 500	0/1	Approx. t.
6. Kur	Kurtosis of residuals	av > 3.9	1 in 5000	0/0	Approx. t.
7. QS	Seasonality in residuals	> 6.4	1 in 25	7/20	Approx. $\chi^2(2)$ .
8. Q2	Nonlinearity in residuals	> 51.2	1 in 1000	3/0.5	Approx. $\chi^2(24)$ . Appropriate for GARCH or bilinear type of nonlinearities
9. Runs	Random signs in residuals	av > 3.1	1 in 500	1/1	Approx. t
10. Out	% of observations that are outliers	> 5	5 in 100	10/25	No more than 5% of the observations should be outliers
11. Model_Changed_SEATS	SEATS modifies the model passed by TRAMO	Y	---	38/-	The change of model usually does not yield an error
12. Approx_NA	SEATS changes the model to achieve an admissible decomposition	Y	---	9/-	SEATS has modified the TRAMO model because it did not accept an admissible decomposition
13. Bias	Bias induced in the level by the log transformation of the	> 1	---	25/-	In % of the level

	SA series and trend-cycle				
14. Spectr. Factor	Decomposition of the series spectrum	E	---	0/-	E: $SS(\text{error}) < 10^{-3}$ Highly unlikely
15. ACF	Disagreement with ACF of theoretical component estimator	E	1 in 100	5/5	Significant difference between Variances of stationary transform. of theoretical and empirical estimator
16. CCF	Disagreement with CCF of theoretical estimators	E	1 in 100	3/5	As above, but for lag-0 crosscorrelation theoretical estimator and empirical estimate
17. Not_treated	Not treated by TRAMO or SEATS	-1	---	0	Series is too short/contains too many zeros/too long sequences of constant values/...

Table 17: List of Detected Problematic Series

Num	Name	Q-Val	Qs	Q2	Runs	Out	MCS	ANA	ACF	CCF	BIAS
6	6 - E15_0028_2_5			62,25							
8	8 - E15_0028_2_7						E	E			
25	25 - E15_0052_2_4										4,37
27	27 - E15_0052_2_6						E				
29	29 - E15_0052_2_8						E				
31	31 - E15_0075_2_0					9					
32	32 - E15_0075_2_1						E	E			1,24
34	34 - E15_0075_2_3										1,31
37	37 - E15_0075_2_6						E				
43	43 - E15_0400_2_2	56,79	9,51				E				
49	49 - E15_0400_2_8									E	
53	53 - E15_0404_2_2								E		
54	54 - E15_0404_2_3					8					
56	56 - E15_0404_2_5		8,63								
57	57 - E15_0404_2_6						E	E			
61	61 - E15_0720_2_0										1,13
62	62 - E15_0720_2_1						E				
63	63 - E15_0720_2_2						E				
64	64 - E15_0720_2_3					24					
74	74 - E15_0732_2_3					8					
76	76 - E15_0732_2_5		6,45								
82	82 - E15_1031_2_1						E				
91	91 - E15_1051_2_0						E				
93	93 - E15_1051_2_2						E				
97	97 - E15_1051_2_6						E				
99	99 - E15_1051_2_8						E				
100	100 - E15_1051_2_TTT						E				
102	102 - E15_1053_2_1										1,16
103	103 - E15_1053_2_2		9,38								
112	112 - E15_1057_2_1					9					
114	114 - E15_1057_2_3										3,8
125	125 - E15_1058_2_4								E		
127	127 - E15_1058_2_6						E	E			
134	134 - E15_1110_2_3						E				
157	157 - E15_1130_2_6						E				
164	164 - E15_1310_2_3		6,71								
169	169 - E15_1310_2_8									E	
187	187 - E15_1811_2_6						E	E			
195	195 - E15_1815_2_4		8,78								
220	220 - E15_5200_2_TTT						E				
231	231 - E15_5400_2_0						E				
235	235 - E15_5400_2_4										4,12
236	236 - E15_5400_2_5								E		
244	244 - E15_5500_2_3					16					1,58
247	247 - E15_5500_2_6						E				
249	249 - E15_5500_2_8						E				
252	2 - E15_0028_1_1										1,09
262	12 - E15_0039_1_1										1,27
264	14 - E15_0039_1_3					7					
265	15 - E15_0039_1_4					7					
272	22 - E15_0052_1_1						E				

Num	Name	Q-Val	Qs	Q2	Runs	Out	MCS	ANA	ACF	CCF	BIAS
276	26 - E15_0052_1_5						E				
278	28 - E15_0052_1_7			64,66							
283	33 - E15_0075_1_2						E				
285	35 - E15_0075_1_4			58,6							
288	38 - E15_0075_1_7										1,64
289	39 - E15_0075_1_8					11					
302	52 - E15_0404_1_1										1,77
304	54 - E15_0404_1_3										2,14
307	57 - E15_0404_1_6				-3,13						1,23
308	58 - E15_0404_1_7										1,07
311	61 - E15_0720_1_0						E				
317	67 - E15_0720_1_6						E	E			
321	71 - E15_0732_1_0					8					
324	74 - E15_0732_1_3										4,26
335	85 - E15_1031_1_4										1,56
336	86 - E15_1031_1_5						E				
338	88 - E15_1031_1_7										1,27
345	95 - E15_1051_1_4										2,42
347	97 - E15_1051_1_6						E	E	E		
350	100 - E15_1051_1_TTT						E	E			
352	102 - E15_1053_1_1										1,53
356	106 - E15_1053_1_5								E		
364	114 - E15_1057_1_3										1,9
379	129 - E15_1058_1_8						E				
384	134 - E15_1110_1_3						E				
394	144 - E15_1120_1_3										1,3
398	148 - E15_1120_1_7						E	E			
404	154 - E15_1130_1_3										1,44
405	155 - E15_1130_1_4			6,77							
423	173 - E15_1415_1_2						E				
440	190 - E15_1811_1_OIL						E				
442	192 - E15_1815_1_1										1,34
452	202 - E15_5190_1_1						E				
465	215 - E15_5200_1_4						E				1,89
470	220 - E15_5200_1_TTT									E	
276	26 - E15_0052_1_5						E				
278	28 - E15_0052_1_7			64,66							
283	33 - E15_0075_1_2						E				
285	35 - E15_0075_1_4			58,6							
288	38 - E15_0075_1_7										1,64
289	39 - E15_0075_1_8					11					
302	52 - E15_0404_1_1										1,77
304	54 - E15_0404_1_3										2,14
307	57 - E15_0404_1_6				-3,13						1,23
308	58 - E15_0404_1_7										1,07
311	61 - E15_0720_1_0						E				
Limit Values:		42,8	6,4	51,18	3,1	5%					1

Three of the checks (#1, 2, and 17) have to do with errors in the input or in the program. It is seen that no program crash has occurred, that there was no error in the input file, and that the full TRAMO and SEATS treatment was completed for all series. Checks #3 to #9 are the TRAMO residual diagnostics in the "Model Fit" matrix. One should be aware that the critical values should be set so as to keep the type I error under control. For example, if 10000 series are treated, a 5% size would imply that one could expect to reject 500 models that are in fact correct. Given that problematic series are meant to be analysed individually, the cost of having to analyse 500 spurious rejections for each diagnostic seems too high. By default, the critical values in Problematic are aimed at sets of 500 to 1000 series that need to be seasonally adjusted. (Seasonal adjustment is in fact the most frequent large scale routine application of the programs.) Therefore, the most tightly controlled features are the possible presence of residual seasonality, residual autocorrelation, and skewness in the residuals, followed by randomness in the residual's sign and Normality. Kurtosis and nonlinearity (of the conditional heteroscedasticity type) are of secondary importance because they have little effect on the optimality of point estimators; they would affect estimation of standard errors, which may present some variation in time. In the application, only 1 series was found problematic because of autocorrelation in the residuals; none because of non-normality, excess kurtosis, or excess skewness; 7 because of seasonal autocorrelation in the residuals; 3 because of nonlinearity; and 1 because of non-randomness in the residual signs. These seven diagnostics identify 12 problematic series among the set of 500 series.

Check #10 sets a maximum limit for the acceptable number of outliers per series. It is found that 12 series exceed the 5% limit.

Checks #11 to #16 are related to the SEATS output. Check #14 signals errors (namely, insufficient precision) of the spectral factorization of the ARIMA model, from which the unobserved component models are derived. This error is rarely encountered, and it is not present in any of the series in the set.

Check #13 detects, for series that have been modelled in logs, cases in which the underestimation bias in the level of the SA series induced by the log transformed is beyond what is considered acceptable. In the application, 5 series exceeds this limit.

Check #14 identifies series for which the variances of the stationary transformations of the theoretical and empirical estimators cannot be accepted as equal. The comparison is performed for all unobserved components, and disagreement between the two variances would indicate model misspecification and possible cases of component under/over-estimation. These diagnostics are still in the experimental phase and need to be perfected according to the results of work done at the USBC by \*\*\*\*\*. At present only the problematic series for which no other diagnostic (other than MCS) fails are printed.

Check #15 performs a similar comparison for the lag-0 crosscorrelation between all pairs of unobserved components. The remarks made for check #14 also apply, and hence the results of these checks will not be taken into consideration.

Finally, check #11 signals series for which SEATS changes the ARIMA model identified by TRAMO for the linearized (or preadjusted) series; check #12 indicates when this change has been made to replace

a model that did not accept an admissible decomposition. In the set analyzed, for 38 series SEATS changed the TRAMO model, and in 9 of these cases the reason was to replace a model with no admissible decomposition.

Then, a check is made on the accuracy of the spectral factorization that provides the component's models. No series in the application yields any problem in this respect. Next, the bias effect on the level of the SA series induced by the log transformation (in multiplicative decompositions) is considered. In the application, 26 series (about 5% of them) exceed the 1% limit set by default. Finally, a check is made to detect significant differences between the ACF and CCF of the theoretical estimators and the empirical estimates of the trend-cycle and SA series. In the example, 30 series (or 6%) of the total show differences at the 5% significance level. It should be mentioned that these ACF checks are based on a 5% significance level, which should be probably reduced (the associated type I error implies 25 false rejections). A similar check for the lag-0 crosscorrelation between each pair of component estimators is performed: when the empirical estimate differs significantly from the theoretical value, the series is judged problematic. In the application, only 3 series fail the diagnostic. (Work at the USBC indicates that these tests require more refinement.)

A total of 97 series present one or more problems. However the MCS and ANA controls should not be properly interpreted as failures but rather as an indication that the change of models should be checked because it may induce discrepancies between the series forecasts obtained with TRAMO and with SEATS (the aggregate forecast of TRAMO will be maintained).

Additional checks are being added to these diagnostics. The stability and non-parametric residual seasonality check have already been mentioned. Other examples are a test for idempotency (seasonal adjustment of the adjusted series should basically reproduce the adjusted series), a check for seasonal overdifferencing, a check for highly stationary and unstable seasonality, and –most importantly- spectral domain tools (similar to the ones in X12ARIMA) that yield information on the gain and phase effect of preliminary estimators and on residual seasonality or calendar effects.

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