BANCO DE ESPAÑA

TREATMENT OF CONFLICTIVE FORECASTS: EFFICIENT USE OF NON-SAMPLE INFORMATION

Luis Julián Alvarez, Juan Carlos Delrieu and Javier Jareño

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TREATMENT OF CONFLICTIVE FORECASTS: EFFICIENT USE OF NON-SAMPLE INFORMATION ABSTRACT

The purpose of this paper is efficiently to incorporate into a univariate ARIMA model the information contained in alternative forecasts obtained through an expert opinion or from an econometric model. The aim is to merge the short-term properties of ARIMA models with the long-term path provided, fundamentally, by econometric models.

Any set of linear constraints on the future course of the series is envisaged and the introduction of uncertainty about these constraints is permitted. The problem is solved obtaining the "restricted forecast" by generalised least squares (GLS).

Keywords: ARIMA models, non-sample information, restricted forecast.

1. INTRODUCTION AND CONCLUSIONS

Statistical-econometric models play an important role in the obtaining of forecasts on the future course of economic events. On many occasions, however, various organisations use additional information not addressed by the models available when formulating their final forecast. This information, whose sources are numerous and varied, is somewhat haphazard or is received with a different frequency from that of the model.

Univariate time series models are very popular for forecasting due to their success in capturing the dynamic structure of data. In this context the natural question that arises is whether it is possible to incorporate into a model of this type the information considered by an expert or an econometric model, thereby obtaining more accurate forecasts. It should be understood that these forecasts are conditional upon the veracity of the information incorporated.

The aim of this paper is to resolve efficiently the problem of incorporating non-sample information into a univariate model, obtaining what we call <u>restricted forecasts</u>. At all moments a distinction is made between definite constraints and constraints with a certain degree of uncertainty, and the resulting method offers several advantages. First, it is shown that the solution differs according to the ARIMA model characterising the event and, therefore, the method allows results to be adjusted to the particularities of each series. Second, it allows the confidence intervals of the restricted forecasts to be calculated, unlike what would occur with any other empirical procedure (a linear distribution, for instance). Third, a statistic is furnished which provides for the testing of the compatibility of the information it is intended to incorporate with the past course of the series. Lastly, the relationship between the proposed estimator and the missing-values estimation is examined.

The paper is structured as follows. Section 2 offers the conceptual framework, highlighting the main differences between the literature on the combination of forecasts and the proposed procedure.

Section 3 details the analytical framework used when it is wished to incorporate constraints with a certain degree of uncertainty, likewise deriving the solution when the constraints are definite. Section 4 addresses the relationship to the literature of missing-values estimation. Sections 5 and 6 feature two applications relating to the conversion of annual totals for non-energy imports to a quarterly basis and to the Spanish economy's consumer price index. Lastly, an appendix is furnished showing various results contained in the text.

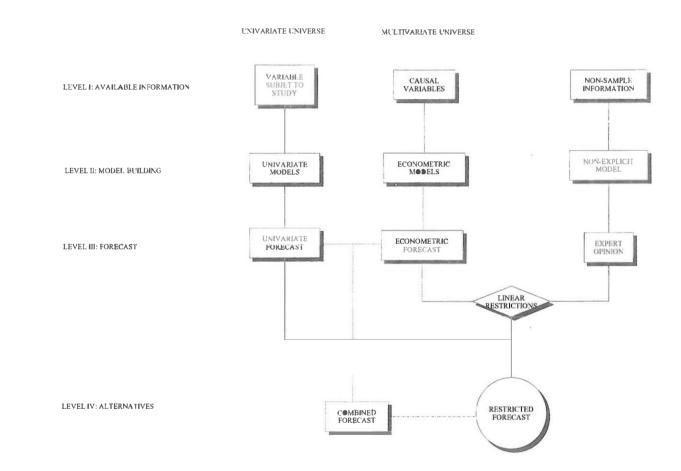
2. CONCEPTUAL FRAMEWORK

The fact that model predictions are unsatisfactory -and thus improvable- is a sign that not all relevant information has been included or that the model has been misspecified. In the latter case, econometricians should be concerned with seeking the most suitable specifications possible, since once the data generating process is obtained reliable and accurate forecasts emerge naturally.

In practice, however, it is often not possible to combine information sets efficiently. Also, when forecasting using an econometric model, the values of the explanatory variables are frequently not known and forecasts of such variables must be used. Logically, in this case, the quality of the econometric forecast deteriorates and may prove worse than the univariate one, particularly if it is sought to ascertain short-run dynamics.

In this respect, it would not seem fruitless to merge the results that can be derived from a model which detects fairly accurately the short-run dynamic structure with the properties derived from econometric models or with expert opinions for longer time-spans.

That said, the capacity to incorporate relevant information for the forecast is directly related to the statistical tools available for the study of the event in question. Fig. 1 shows the target setting for our study: the restricted forecast. The outline illustrates the different information used by the various forecasting methods, and their different



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procedures and results. Lastly, it indicates how these results can be harmonised, either with the existing methodology on the combination of forecasts or with the proposal put forward in this article.

The <u>univariate universe</u> is characterised essentially because the observation over time of an economic event detects implicitly the effect of the variables which cause it. In that way an analysis based exclusively on the variable subject to study is not excessively limited by not considering the information offered by its explanatory variables, since such information is included in the very series to be studied.

This information is processed by means of univariate models, particularly ARIMA models, on which our attention will focus. As a result of applying these models, the univariate forecast is obtained. Among its disadvantages is forecasting at times of strong changes. Its main merit is short-term forecasting due to its substantial capacity to capture the dynamics of the variable under study.

The <u>multivariate universe</u> views explicitly the information furnished by the causal variables of the event under study. Thus, compared with the univariate case, the information is captured directly; accordingly, the results obtained have greater explanatory power in addition to being more efficient.

The models employed for the use of this information are econometric ones. These make the existing relationship of the variable under study to the explanatory variables thereof explicit. In these models, when it is wished to perform forecasting, not knowing the values of the explanatory variables in the future makes it necessary to use forecasts of such values. That detracts from the quality of the econometric forecast, particularly in the short run, where univariate models show themselves to be superior.

So far we have considered ways of treating systematically the information obtained. Frequently, though, sporadic, non-recurrent (due to its nature or its source) information is generated; yet this proves most relevant when making future forecasts. Non-sample information is thus

considered as that which is not presented systematically over time and with high informativeness. Examples of this type of information are the announcement of high-impact economic policies and of economic goals, legislative changes, etc.

The nature of this information precludes analysis thereof through statistical models; it can only be treated through the subjective filter of experts, on the basis of their knowledge and experience. Their opinion will thus be obtained and this, unlike previous forecasts, will include the non-sample information.

In sum, the situation at hand can be distinguished by three types of alternative forecasts with different characteristics: a) univariate forecasting, with good short-run characteristics; b) econometric forecasting, with good long-run characteristics; and c) expert opinion, the leading advantage of which is that it includes non-sample information⁽¹⁾.

Given the difficulty of combining efficiently these information sets, the alternative solution should be to combine the properties of the different models on the future.

In this connection, there is abundant literature about the <u>combination of forecasts</u>. This expressly rejects the combination of information sets, seeking only to achieve more accurate forecasts (see, inter alia, Bates and Granger (1969), Newbold and Granger (1974), Granger and Ramanathan (1984) and the review by Clemen (1989)). The bases for improvement are: a) a forecast may take into account information that others do not have; and b) forecasts may have different initial assumptions. The end result gives the optimal forecast as a linear

⁽¹⁾ This is not an assertion that experts are infallible. Indeed, experts' forecasts occasionally contain substantial errors. This is why the monitoring of forecast errors (see Jenkins (1982)) is so important when subjective forecasts are involved.

combination of alternative forecasts, without considering explicitly the problem of which of these is the most suitable (2).

Virtually in its entirety, the literature has focused on the combination of forecasts with models of matching periodicity; recently, however, a series of papers based on the combination of forecasts of differing periodicity has emerged, aimed at obtaining forecasts both for the low-frequency period (see Corrado and Greene (1987), Corrado and Haltmaier (1987) and Howrey, Hymans and Donihue (1991) and the high-frequency period (see Fuhrer and Haltmeier (1989)).

Nonetheless, the application of the combination-of-forecasts methodology is not always possible, especially if non-systematic forecasts are involved. Consequently, it would be useful to expand upon the findings of this literature to cover new possibilities so that more extensive predictors, enabling systematic and non-systematic forecasts to be combined, may be obtained.

It is often worthwhile in itself to have forecasts that satisfy specific constraints, in that this enables targets to be evaluated and monitored. Here, the problem is how to incorporate non-sample information into a quantitative model. And the solution proposed, for the case of a univariate ARIMA model, is the <u>restricted forecast</u>. This entails a revision of the univariate forecasts in such a way that the information provided by an econometric model or by an expert is satisfied, thereby attaining efficient forecasts⁽³⁾ in the sense of minimising the forecast error. This problem has been addressed, by different approaches, in the research by Cholette (1982), Guerrero (1989), and Trabelsi and Hillmer

⁽²⁾ The specification of the weightings in the linear combination is really related directly to the standard deviation of each forecast and, therefore, the resulting forecast will be closer to that of minimum variance.

⁽³⁾ Generally, the exercises conducted to date to incorporate this type of information into the forecasts were confined to establishing a linear distribution (weighted or unweighted) of the difference between the univariate forecast and the expert's forecast, without observing efficiently the dynamics of the event.

(1989). Although the latter paper sets the most general framework, it is shown that the three solutions are equivalent under certain conditions. In addition, Pankratz (1989) extends the results to the case of a VARMA model.

The proposed methodology provides for the combination of econometric and univariate forecasts. It is thus sought to harness the short-term qualities of univariate models with the long-term qualities of multivariate models, there being great similarity in this case with the literature on the combination of forecasts.

3. ANALYTICAL FRAMEWORK

3.1 Statistical framework

Let us assume a \mathbf{Z}_{t} series that can be suitably represented by a univariate ARIMA model

$$\phi^*(L)Z_+ = \Theta(L)a_+ \tag{1}$$

where $\Theta(L) = (1 - \Theta_1 L - \ldots - \Theta_q L^q)$ and $\phi*(L) = (1 - \phi_1 L - \ldots - \phi_p L^p)$ are polynomial operators in the lag operator L. so that LZ_{t-t-1} . The two polynomials do not have common factors, and the moving average polynomial has its roots outside the unit circle, whereby the process is invertible. The autoregressive operator may have roots on the unit circle. Furthermore, we will assume that the stationary transformation of the series has a zero mean and that a_t is a white noise process comprising normal non-correlated random variables with constant variance

$$a_t \sim iid N(0, \sigma_a^2)$$
 (2)

The process can also be written in the form of a moving average as

$$Z_{t} = \frac{\Theta(L)}{\phi * (L)} a_{t} = \sum_{i=0}^{\infty} \psi_{i} a_{t-i}$$
 (3)

where Ψ_o = 1 and the rest of the coefficients can be obtained if we equal coefficients in

$$\phi^{\star}(L) \Psi(L) = \Theta(L) \tag{4}$$

On the basis of the coefficients Ψ_i and the past innovations a_{t-1} , the h steps ahead forecast error can be instantly obtained, which consider only the information contained in the past of the series $\Omega_z = \{Z_t, Z_{t-1}, \ldots\}$. Box and Jenkins (1970) show that the optimal predictor, in the sense of minimising the mean square error, is given by

$$\hat{Z}_{t}(h) = E[Z_{t+h} \mid \Omega_{z}] = \psi_{h} a_{t} + \psi_{h+1} a_{t-1} + \psi_{h+2} a_{t-2} + \dots$$
 (5)

Moreover, it is possible to decompose the series into a systematic part (the forecast) and a non-systematic part (the error), both being orthogonal

$$Z_{t+h} = \hat{Z}_t(h) + e_t(h)$$
 (6)

where $Z_t(h)$ denotes the h steps ahead optimal predictor, and e_t the h steps ahead forecast error. This h steps ahead forecast error can be expressed as a linear combination of future innovations

$$e_{t}(h) = a_{t+h} + \psi_{1}a_{t+h-1} + \dots + \psi_{h-1} a_{t+1}$$

$$= \sum_{i=0}^{h-1} \psi_{i} a_{t+h-i}$$
(7)

In matrix form, equation (7) is expressed as

$$e = \Psi a \tag{8}$$

where a is a column vector a = $(a_{t+1} \dots a_{t+h})$ ' and Ψ is a square h x h matrix

$$\psi = \begin{bmatrix}
1 & 0 & \dots & 0 \\
\psi_1 & 1 & \dots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
\psi_{h-1} & \psi_{h-2} & \dots & 1
\end{bmatrix}$$
(9)

and it is shown that the h steps ahead forecast errors have a zero mean and a matrix of variances and covariances:

$$E[ee'] = \sigma_a^2 \psi \ \psi' \tag{10}$$

Furthermore, the decomposition into systematic and non-systematic parts can be written more concisely as

$$Z = \hat{Z} + e \tag{11}$$

where Z, Z, and e are h column vectors, whereby

$$Z = [Z_{t+1}...Z_{t+h}]', \hat{Z} = [\hat{Z}_{t}(1)...\hat{Z}_{t}(h)]' y$$

$$e = [e_{t}(1)...e_{t}(h)]'$$
(12)

Thus, using (8) and (11) gives

$$Z = \hat{Z} + \Psi a \tag{13}$$

On the basis of similar considerations, Guerrero (1989) solves the problem by proposing an optimisation programme in which it is wished to minimise the mean square error of the forecast subject to the constraints imposed by an expert being met. We propose a different approach based on ideas developed by Durbin (1953) and Theil and Goldberger (1961)⁽⁴⁾, This approach, considering the most general case possible, enables the solution to be obtained straightforwardly. The following section addresses this point.

3.2. Univariate ARIMA models and incorporation of additional information⁽⁵⁾

The constraints may be either approximate or stochastic, either because an econometric model is used to derive them, whereby it is possible to calculate the matrix of variances and covariances of the forecast errors associated with these constraints, or because information is available on the accuracy of the source. Note that the first situation is interesting in that it is common to have econometric models with annual or quarterly data and, at the same time, to have univariate models of a greater frequency (e.g. monthly or daily).

⁽⁴⁾ These authors show how the estimation of the parameters of a regression model is affected when non-sample information is incorporated.

⁽⁵⁾ It is assumed in this section that the forecasts are obtained with an ARIMA model. Generally, however, we can specify the matrix of variances and covariances of the errors h steps ahead, and the procedure would likewise be valid, for instance, for single equation econometric models.

Thus, the problem involves finding the optimal predictor which satisfies the stochastic constraints included as

$$AZ = b + u \tag{14}$$

where u is a vector of r random variables distributed normally with a mean of zero and, generally, different variances; A is an r x h matrix with r \leq h and rank r, r being the number of constraints, Z is a h x 1 vector which includes the future values of the variable and b is an r x 1 vector of constants. The general form includes as particular cases the following possibilities, and in any of these the constraints is imposed with a certain margin of variability provided by the variance of the error term:

1. <u>Isolated constraints</u>. Information is available about the value the event will take in a future moment of time

$$Z_{t+1} = b_0 + u_0$$
 (15)

2. <u>Sum or mean constraints</u>. The value of the mean or sum of a certain number of values is estimated: e.g. 12

$$Z_{t+1} + Z_{t+2} + Z_{t+3} + \dots Z_{t+12} = b_1 + u_1$$
 (16)

3. <u>Increment constraints</u>. Information is available about the increase a variable will record over an interval of time

$$Z_{t+i} - Z_{t+j} = b_2 + u_2$$
 (17)

It is thus possible for different constraints to be satisfied jointly and for each of them to have a different variance. In general, moreover, we will allow correlation to exist between the latter and the ARIMA model forecasts, and we will assume that

$$u_i \sim N(0, \sigma_i^2)$$
 (18)

$$\mathbb{E} [u_i u_i] = \sigma_{ii}$$

The existing information can thus be summarised as follows:

$$Z = \hat{Z} + e$$

$$AZ = b + u$$
(19)

where, in general,

$$\begin{pmatrix} e \\ u \end{pmatrix} \sim N \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_a^2 \psi \psi' & \sum_{eu} \\ \sum_{ue} & \sum_{u} \end{pmatrix}$$
 (20)

 $\Sigma_{\rm u}$ being the matrix of variances and covariances of the disturbances associated with the constraints, $\Sigma_{\rm su}$ the matrix of covariances between forecast errors h steps ahead and disturbances associated with the constraints and $\Sigma_{\rm ue} = \Sigma_{\rm eu}$.

The problem of finding an estimator that satisfies the stochastic constraint taking into account the properties of the error terms becomes clearer if we write expression (19) in matrix form:

$$\begin{bmatrix} \hat{Z} \\ b \end{bmatrix} = \begin{bmatrix} I \\ A \end{bmatrix} Z + \begin{bmatrix} -e \\ -u \end{bmatrix}$$
 (21)

The problem resolves itself if we consider that we are looking for the estimator using generalised least squares (GLS), along the same lines as those proposed by Durbin (1953) and Theil and Goldberger (1961). Then the optimal estimator is:

$$Z^* = \begin{bmatrix} I | A' \end{bmatrix} \begin{bmatrix} \sigma_a^2 \psi \psi & \sum_{eu} \\ \sum_{ue} & \sum_{u} \end{bmatrix}^{-1} \begin{bmatrix} I \\ A \end{bmatrix}^{-1} \begin{bmatrix} I | A' \end{bmatrix} \begin{bmatrix} \sigma_a^2 \psi \psi' & \sum_{eu} \\ \sum_{ue}^* & \sum_{u} \end{bmatrix}^{-1} \begin{pmatrix} \hat{Z} \\ b \end{bmatrix}$$
(22)

In practice, however, the particular case where $\Sigma_{\rm eu}$ =0 is particularly relevant since, on occasions, it may prove not to be overly simple to specify these matrices of covariances between errors. Moreover, in many cases the information sources may be sufficiently independent as to consider that this assumption is not especially restrictive. It would thus seem worthwhile considering this particular case in greater detail. It is shown in the appendix that, when there is no correlation between the disturbances of the constraints and the disturbances of the ARIMA model, the optimal predictor Z^* will be given by

$$Z^* = \hat{Z} + (\sigma_a^2 \psi \psi') A' [A (\sigma_a^2 \psi \psi') A' + \Sigma_u]^{-1} (b - A\hat{Z}) =$$

$$= \hat{Z} + P^* (b - A\hat{Z})$$
(23)

where

$$P^* = (\sigma^2 \psi \psi') A' [A(\sigma^2 \psi \psi') A' + \Sigma_n]^{-1}$$
 (24)

This expression proves more interpretable than the previous one and indicates that the optimal predictor is a linear combination of the free

ARIMA predictor and the <u>new</u> information that the constraints contain (6). The term Σ_u reflects the precision associated with the different constraints. Thus, for a given divergence between the ARIMA forecast and the vector of constraints, revisions will be all the greater the lesser the variance associated with this constraint. At the other extreme, if a constraint is substantially inaccurate, the optimal predictor will not differ, virtually, from the ARIMA forecast.

That said, information is often generated which is sporadic and not periodic. This is due either to its nature or source, but is of great significance when making forecasts. The singularity of this type of non-sample information may, occasionally, lead us to consider it as valid⁽⁷⁾. In these cases, it is easy to derive from expression (23) what the forecast subject to the constraint would be, merely by cancelling the term $\boldsymbol{\Sigma}_{u}.$ Namely,

$$Z^{**} = \hat{Z} + P^{**}(b - A\hat{Z})$$
 (25)

where Z^{**} is the optimal predictor that satisfies our optimisation problem, Z is the ARIMA model forecast without any constraint and P^{**} is a h x r weighting matrix which is given by:

$$P^{**} = (\psi \ \psi') A' [A(\psi \ \psi') A']^{-1}$$
 (26)

Equation (25), which is that derived in Guerrero (1989), provides a readily interpretable solution where the optimal restricted predictor is obtained as a linear combination of the ARIMA forecast and

⁽⁶⁾ Note that if A = I, the (Bayesian) standard formula of combination of information weighted by relative precision is obtained.

 $^{^{(7)}}$ It might be worthwhile using the assumption that non-sample information is valid for evaluating objectives. See the application relating to the CPI.

the difference between the vector of constraints and the univariate optimal predictor of the constraint (A \hat{z}). As before, the term (b-A \hat{z}) reflects the new information introduced into the forecast, with a relative significance measured by matrix P^{**} .

In any event, the expression arrived at discloses that the optimal restricted estimator will differ according to the dynamic structure characterising the data and, therefore, the ARIMA model that generates the process under study. Evidently, the restricted predictors \mathbf{Z}^* and \mathbf{Z}^{**} satisfy the constraints stochastically or exactly, respectively.

Furthermore, since the optimal predictor can be obtained as a generalised least squares (GLS) estimator, the expression of the matrix of variances of the estimator's errors of expression (22) turns out as

$$\operatorname{Var}(\mathbf{Z}^* - \mathbf{Z}) = \left[\begin{bmatrix} \mathbf{I} & \mathbf{A}' \end{bmatrix} \begin{bmatrix} \sigma_a^2 & \psi \psi' & \Sigma_{eu} \\ \Sigma_{ue} & \Sigma_u \end{bmatrix}^{-1} \begin{pmatrix} \mathbf{I} \\ \mathbf{A} \end{pmatrix} \right]^{-1}$$
(27)

and it can be seen in the appendix that, where $\Sigma_{\text{eu}}\text{=0,}$ the preceding expression becomes:

$$Var(Z^* - Z) = \sigma_a^2 \psi \psi'(I-PA)' + P \sum_u P'$$
 (28)

If, moreover, we consider that the constraints have no associated uncertainty, that gives

$$Var (Z^{**}-Z) = \sigma_{\perp}^2 \psi \psi' (I-PA)'$$
 (29)

Since the matrix of differences between the matrices of free and restricted forecast error variances is positive semi-definite, it is instantly given that the variance of forecast error of any linear combination of restricted predictions is less than that of this same linear combination of ARIMA forecasts. This result is as intuitively expected, since the introduction of supposedly correct information on the future course of the event lessens our degree of uncertainty in relation to that prevailing before having such information.

At the same time, when the constraints are stochastic and, therefore, compliance therewith is uncertain, the matrix of variances is greater than when the constraints are satisfied with equality. Specifically, if the stochastic constraints have a high variance, they are not very informative and lessen our uncertainty to a lesser degree.

2.3. A compatibility test

An implicit assumption used in deriving the optimal restricted estimator was that the constraint is compatible with past course of the event. Accordingly, in this section we set out a compatibility test that enables us to detect which constraints are incompatible with the past course of the series. This test is crucial in that if it is rejected, it is implicitly assumed that there will be a structural change. If this were so, the results would have to be viewed with all due caution since they are obtained under the assumption of stability.

In the appendix it is shown that, under the null hypothesis of satisfaction of the constraints, the statistic obtained -in line with those proposed by Box and Tiao (1976) and Lütkepohl (1988)- is, if the covariance between the forecast errors and the disturbances associated with the constraints is null,

$$Q = (b-A\hat{Z})'[\sigma_{a}^{2} A \psi \psi' A' + \sum_{n}]^{-1}(b-A\hat{Z})$$
 (30)

which is distributed as an χ^2 with r degrees of freedom, r being the number of constraints. In practice, however, σ_a^2 , Ψ and , Σ_u are unknown, whereupon they will have to be replaced by their efficient estimators to obtain a feasible statistic.

4. THE RELATIONSHIP BETWEEN THE PROPOSED RESTRICTED PREDICTOR AND THE ESTIMATION OF MISSING VALUES

One problem which frequently arises in practice is that only incomplete series are available. This is because i) data are missing in some periods (isolated or in groups); ii) the frequency of the observation changes; or iii) one or more of the observations is clearly wrong. Although the statistical literature has addressed this matter (see Brubacher and Tunnicliffe Wilson (1976), Peña and Maravall (1991) and the references quoted thereunder), the aim of this section is to show that the estimator proposed for making forecasts with constraints may be used to conduct optimal interpolation. The attraction of this is that the problem can be tackled from an alternative approach.

The fact this is so is extremely clear. Generally, the minimum mean square error estimator of the missing observations is the expectation conditional on the observations at hand. If we denote the observed series $Z_{(m)}$ as the series with k missing values in the periods t+1, t+m₁, t+m₂,.., t+m_{k-1} where m_1,\ldots,m_{k-1} are positive integers, the optimal estimator of the missing k values is given by

$$\mathbb{E}\left[Z_{m}|Z_{(m)}\right] \tag{31}$$

where Z_m encompasses the values of the series in t+1, t+m₁,..., t+m_{k-1}. To verify that the proposed estimator coincides with the latter estimator, it need merely be observed that it is always possible to take position at the moment immediately prior to the first missing observation and make the necessary forecasts to reach the end of the series. The question then arises as to which constraints are necessary so that both estimators may

coincide. The reply involves imposing that the constraints should coincide with the known values as from the first missing value. Since the information set is the same in both cases, the minimum variance estimator is identical to that proposed.

To see how the proposed estimator and that habitually used in the literature coincide, we will use an AR(1) process as an example in which the penultimate observation is not known⁽⁸⁾. In this case, the optimal estimator of the missing observation is given by

$$\hat{Z}_{m} = \frac{\phi}{1 + \phi^{2}} (Z_{m-1} + Z_{m+1})$$
 (32)

On the basis of the restricted predictor expression (25), particularising for an AR(1) process with a two-period forecasting time-span, the matrix

of variances and covariances will be
$$\sigma_a^2 ~\psi \psi' = \sigma_a^2 ~\begin{bmatrix} 1 & \phi \\ \phi & 1 + \phi^2 \end{bmatrix}$$
 .

In this case, using the same notation as in previous sections, $b {=} z_{_{\text{mal}}}$ and A=[0 1].

Further,
$$\hat{Z}(1) = \phi Z_{m-1} y \hat{Z}(2) = \phi^2 Z_{m-1}$$
 and

⁽⁸⁾ The demonstration of this result for a general ARIMA model can be seen in Alvarez, Delrieu and Jareño (1992).

As a result, particularising in (25),

$$Z^{**} = \begin{bmatrix} \phi & Z_{m-1} \\ \phi^2 & Z_{m-1} \end{bmatrix} + \begin{bmatrix} \phi \\ 1 + \phi^2 \end{bmatrix} \frac{Z_{m+1} + \phi^2 Z_{m-1}}{1 + \phi^2} =$$

$$= \begin{bmatrix} \frac{\phi}{1 + \phi^2} & (Z_{m+1} + Z_{m-1}) \\ Z_{m+1} & \end{bmatrix}$$
(33)

whereby the same estimator as in (32) is obtained.

5. CONVERTING ANNUAL NON-ENERGY IMPORT FIGURES TO A QUARTERLY BASIS: AN APPLICATION

Annual-to-quarterly-basis exercises using annual National Accounts figures for the main variables of the Spanish economy are aimed at estimating the quarterly profile of these variables up to the present and at obtaining forecasts on their quarterly course for the coming years. In this connection, a frequently pursued work outline involves:

- a) Seeking an indicator that reflects sufficiently well the performance of the variable to be converted to a quarterly basis.
- b) Enlarging the time series of the indicator with forecasts (generally based on univariate ARIMA models).
- c) Using some signal extraction procedure on the indicator.
- d) Enlarging with forecasts the macroeconomic variable it is wished to convert to a quarterly basis. In many cases, this forecast is similar to that furnished by the ARIMA model for the variable in question.

e) Applying some interpolation and distribution procedure.

Evidently, the univariate models play an important role in this outline. This type of model has an adaptive forecast function, as a result of which it normally presents realistic forecasts. However, a high proportion of the Spanish economy's real-sector economic series was, following a period of strong growth as from mid-1985, affected by the adoption of various restrictive economic policy measures and, in particular, by the curbs on credit to the private sector in the summer of 1989. The outcome was a change in the course of the growth rates of these variables, a breaking point⁽⁹⁾ emerging which originated, most immediately, systematic upward bias in the forecasts of the quantitative models.

Particularising in the univariate models (although these retained their suitability for capturing short-term dynamics and, especially, seasonality), forecasts which contrasted notably with the information derived from other variables or with expert opinions were generally arrived at. In short, the resulting situation was marked by the presence of not merely alternative but clearly opposing forecasts.

Against such a background, we attempt in this section to attain clarity concerning the different results which would have ensued from converting one of the most relevant macroeconomic variables of the Spanish economy -non-energy imports at current prices- to a quarterly basis had we conjugated the properties of ARIMA models with the properties of econometric models or with expert opinions for lengthier time-frames.

The exercise presented refers to the 1990-1991 period, assuming that the quantitative information available is only to June 1990. This application is of interest since:

⁽⁹⁾ See Alvarez, Delrieu and Espasa (1992) for a study of non-energy imports. Further, Sebastián (1991) finds a change in elasticity in the demand for imports with respect to GDP.

- 1.- First, at that date univariate models had little information on change in the system. That meant that the resulting forecasts were systematically biased upwards (see the series of negative residuals as from the second half of 1989 in Charts 1 and 1 bis).
- 2.- Moreover, there were alternative quantitative models (Sebastián [1991]) which appeared to capture more suitably the slowdown in Spanish purchases of foreign goods, giving rise to certain discrepancies with the univariate models. These divergences were, moreover, ratified by foreign-sector analysts.

In any event, it is sought with this application, using the aforementioned restricted-forecast procedures, to highlight the divergences that each available alternative causes at the different stages of the process of the conversion of non-energy imports at current prices to a quarterly basis, drawing on National Accounts data.

First, the expert considered it was advisable to prolong the annual National Accounts series, assuming imports would grow by 13% in 1990 and 15% in 1991.

Further, it was decided to use the non-energy imports series published by Dirección General de Aduanas (the Spanish Customs Authorities) as an indicator of the National Accounts series since the accounting criteria both statistics define are virtually identical.

Nonetheless, since interest focused on the quarterly profile of the macroeconomic variables rather than Quarterly National Accounts, it was decided to use the trend of non-energy imports as an indicator (using the signal-extraction method developed by Burman (1980). The available series was thus prolonged as from June 1990 by means of monthly forecasts⁽¹⁰⁾ furnished by a univariate ARIMA model, giving rise to average growth of 13% for 1990 and 18.1% for 1991.

⁽¹⁰⁾ Note the importance of having good forecasts since both signal-extraction procedures and those used for quarterly-adjustment purposes are affected by revision errors which should ideally be minimised.

TOTAL NON-ENERGY IMPORTS

Residuals

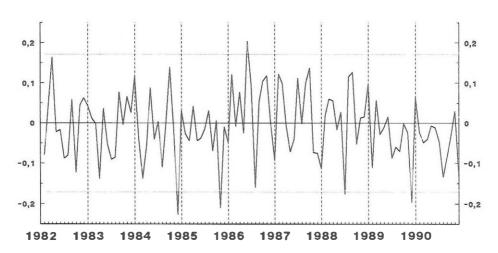
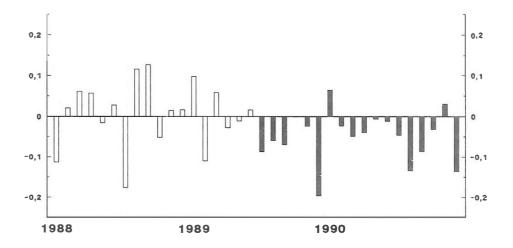


Chart 1 bis

TOTAL NON-ENERGY IMPORTS

Residuais



Using this information, non-energy imports at current prices were converted to a quarterly basis. However, had we been able to apply the proposed restricted-forecast procedure, other alternatives, which were previously not considered -either because they were sporadic (the expert opinion) or of a different periodicity (Sebastián's [1991] annual econometric models)-, would have been available. This meant it was not possible to have a quarterly indicator with annual growths for the whole of the year more reasonable than those provided by the ARIMA model. Table 1 addresses these possibilities, which are discussed below.

The ARI line thus shows the average growths obtained using the univariate model without imposing any type of constraint. The growths shown by BON express the expert forecast, which included non-sample information not considered by the models available at that time. The two following lines show the resulting average growths when a dynamic simulation as from 1989 is made using Sebastián's (1991) econometric model: it is assumed in the first case that demand-income elasticity remains constant, MSS, and, in the second, it is accepted that this elasticity changed. Lastly, with the OBS line it is sought to analyse what the monthly path of non-energy imports would have been had we restricted the model's forecasts so that the average growths in 1990 and 1991 matched those actually observed.

In the different instances restricted forecasts were obtained so that the year-on-year growths of Table 1 are satisfied, no constraints being imposed for 1992. The results provided by each of these alternatives are shown in Charts 2 to 6, from which the following conclusions may be drawn:

- 1.- Whatever the accumulated average growth forecast, the shortterm dynamic profile remains constant, there being a change in the level and slope of the path of the series (see Chart 2).
- 2.- In each case there is a monthly indicator which satisfies the constraints imposed in terms of annual growth (see Chart 3).

Table 2

NON-ENERGY IMPORTS AT CURRENT PRICES

Forecast date: June 1990

			FORECAST		
Source of the forecast	Nomenclature	1990	1991		
ARIMA model	ARI	13.0%	18.1%		
Expert	BON	13.0%	15.0%		
Econometric model dynamic similation1:					
Sebastián (1991), without changes in $\epsilon_{_{_{\Upsilon}}}$	MSS	3.8%	3.7%		
Sebastián (1991), with changes in $\epsilon_{_{_{\boldsymbol{Y}}}}$	MSC	7.9%	6.2%		
Observed average growth	OBS	5.9%	8.0%		

⁽¹⁾ Given that these models are expressed in real terms, the nominal figure has been implicitly obtained drawing on the assumption made in this paper on the non-energy imports deflactor.

Chart 2

NON-ENERGY IMPORTS

Original series and forecasts Level

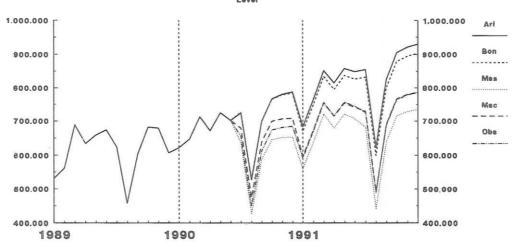


Chart 3

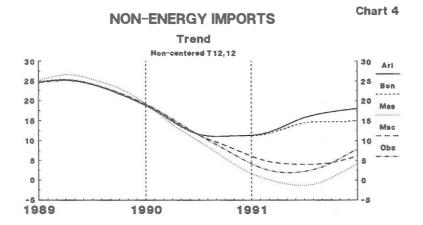
NON-ENERGY IMPORTS

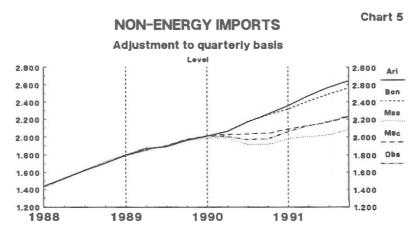
Original series and forecasts Non-centered T12,12

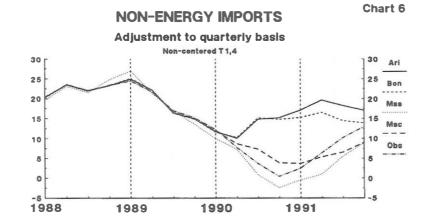
30 Ari
25 Bon
20 Mss
16 Msc
10 Obs
5
0 0
1989 1990 1991

- 3.-On extracting a stochastic trend as a non-observable component of a time series, it is advisable to prolong the original series with forecasts to avoid tail-end problems. This is because the optimal estimator is obtained using a centred mixed filter in which both past and future observations intervene, whereby unknown observations have to be replaced by forecasts. Consequently, if the forecasts are systematically biased upwards, we will be estimating a trend which will not only have been badly calculated at the end of the period in question, but also one whose growth rate may be reflecting a radically different course. In this sense, it can be appreciated in Chart 4 that the situation indicated by the ARI or BON lines is stable growth between 15% and 18%, whereas the situation arising from the rest of the lines appears to suggest that the growth at end-1991 might, at least, show a slight slowdown to growths between 4% and 9% depending on the constraint imposed.
- 4.- Accordingly, if we take the information in Table 1 to prolong the National Accounts series and use the Denton (1971) method for the time disaggregation, using the related trend as an indicator, the result is a conversion to a quarterly basis of the annual magnitude that presents a different level (see Chart 5). Furthermore, the profile shown by each alternative and traced using quarter-on-quarter growths (Chart 6) is quite different, thereby influencing the results of the annual-to-quarterly-basis exercise performed. Specifically, note how the BON or OBS lines, for example, attain a similar growth in 1991, though the course followed to attain it is radically different.

With the BON line an average growth of 13% is reached after a stage of slightly slowing growth in 1991. With the OBS line, meanwhile, the result is similar, but with a path that reflects accelerated growth in 1991 following a phase of deceleration culminating in the third quarter of 1990.







6. RESTRICTED FORECASTS AND THE CONSUMER PRICE INDEX

The consumer price index (CPI) is considered by economic agents to be the fundamental variable in the analysis of inflation⁽¹¹⁾. Economic agents thus establish their actions and attitudes indexing the variables of interest to them through the CPI. Accordingly, the economic authorities set price-growth targets based on this index, pursuing policies designed to lead to the values sought.

The effect of this behaviour by the economic authorities is that the value of interest in this exercise is not an alternative forecast as a statistical model or expert might provide; rather, our attention should focus on the target set by the economic authority and, therefore, on the possibility of meeting it. From this standpoint, application of the restricted-forecast method would enable us to obtain a future monthly path of the CPI that were consistent with both the past history of this variable and with the economic authorities' target value. This would provide for a monthly test of the coherence of this target in respect of the forecast provided by a univariate model⁽¹²⁾ and, therefore, a measure of its credibility. In this sense, the presence of systematic deviations from the reference path will be indicative of the impossibility of compliance therewith; as a result, the target would, in this case, have to be revised.

The situation in January 1992 is of added interest to us if regard is had to the effect of the rise in the intermediate rate of value added tax (VAT), along with increases in other taxes, e.g. on tobacco and hydrocarbons. Generally, the modelling of phenomena such as those referred to above is done by introducing into the statistical model deterministic variables that capture the increase in the average rate of indirect taxation within the CPI. This form of procedure is followed

⁽¹¹⁾ However, the price index for non-energy goods and services (IPSEBENE) may be a better indicator of core inflation than the general index (see Espasa et al. (1987).

⁽¹²⁾ Note that a fresh datum entails the revision of the univariate forecasts and, by extension, the modification of the target path.

principally under three assumptions: first, that the tax shift is total; second, a <u>ceteris paribus</u> assumption with respect to the demand for goods that implies the non-substitutability thereof, whereby relative prices do not alter for this reason; and third, it is assumed that agents do not anticipate the tax rise. In this application, the tax rates are taken from the findings of Pérez (1991) for the different components. As a result, it is assumed that the total shift relating to the change in the tax rate does not occur until after the first quarter of 1992.

For the specific application of the restricted forecast to the CPI, a univariate model of the general index has been considered since the economic authority's target is set in terms of this aggregate. Nonetheless, as it is only wished to analyse the compatibility of the government target with the path shown by the CPI, irrespective of the differing course of its components, it suffices to have a model for the aggregate. Growth of 5.5% in average annual terms has been considered for 1992, with December 1991 being taken as the latest observation.

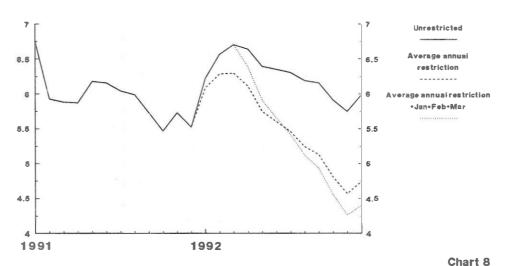
The assumptions considered in this application are as follows:

- a) ARIMA forecast.
- b) Forecast under the assumption of average growth in 1992 of 5.5%.
- c) Forecast under the assumption that average growth in 1992 will be 5.5%, but that the adjustment process will begin as from April, at which time it is assumed that the tax change will have shifted fully to prices.

Charts 7 and 8 show respectively the courses of the year-on-year rate and the T^{12}_{12} under the different assumptions. Chart 7 depicts how the year-on-year rate should run to meet target, highlighting the difference between the ARIMA forecast, which reaches 6% in December, and the restricted forecasts, which respectively entail year-on-year growth of 4.7% and 4.4% in the last month. Also salient is the different path under assumptions b) and c); here it can be seen that the

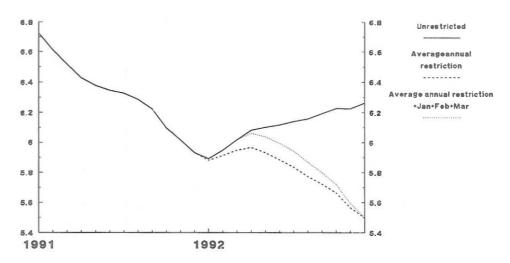
CONSUMER PRICE INDEX

Year-on-year rate



CONSUMER PRICE INDEX

Non-centred T12,12



hypothesis whereunder the process of adjustment towards the target does not begin until the second quarter of the year entails a more marked slowdown in the remaining nine months. Chart 8 shows that the paths of assumptions b) and c) are aimed progressively at the target (at the constraint set at 5.5% for December) after a surge further to the changes in indirect taxation, while the ARIMA forecast reflects average growth of 6.26%.

Table 2 gives the results obtained with information to December 1991 $^{(13)}$.

Once the monthly paths have been obtained for each case and for the target set, attention should be focused on the possibilities of meeting such target. Observation of the values of the compatibility statistic (1.5 for assumption b) and 0.99 for assumption c)) leads to the conclusion that, statistically, the target set is attainable, despite the increment entailed by the indirect-tax change.

Table 3 COMPARISON OF RESULTS							
CONSTRAINT	T112	T ₁₂	Ď				
None	6.0	6.2	~				
Average annual growth	4.7	5.5	1.5				
Average annual +Jan+Feb+growth Mar	4,4	5.5	0.99				

 T^{13} The columns denote the type of restriction, the value of T^{1}_{12} and T^{12}_{12} in December 1992 and the value of the compatibility statistic.

APPENDIX

1. NON-CORRELATED STOCHASTIC CONSTRAINTS BETWEEN THE DISTURBANCES OF THE CONSTRAINTS AND THE ARIMA MODEL ERRORS

The optimal estimator in the presence of stochastic constraints is

$$Z^* = \begin{bmatrix} I | A' \end{bmatrix} \begin{bmatrix} \sigma_a^2 \psi \psi' & \Sigma_{eu} \\ \Sigma_{ue} & \Sigma_{u} \end{bmatrix}^{-1} \begin{bmatrix} I \\ A \end{bmatrix}^{-1} \begin{bmatrix} \sigma_a^2 \psi \psi' & \Sigma_{eu} \\ \Sigma_{ue} & \Sigma_{u} \end{bmatrix}^{-1} \begin{pmatrix} \hat{Z} \\ b \end{pmatrix}$$
(A.1)

Assuming there is no correlation between the disturbances of the constraints and the ARIMA model errors, $\Sigma_{\rm eu}$ =0 $\Sigma_{\rm ue}$ =0'.

To obtain an alternative expression, the following identities should be taken into account:

1.
$$(A+BDB')^{-1}=A^{-1}-A^{-1}BEB'A^{-1}+A^{-1}BE(E+D)^{-1}EBA'^{-1}$$
 (A.2)

2.
$$(A+B)^{-1}=A^{-1}(A^{-1}+B^{-1})^{-1}B^{-1}$$
 (A.3)

3.
$$B(B'A^{-1}B+D^{-1})^{-1}B'=BEB'+BE(E+D)^{-1}EB$$
 (A.4) where $E=(B'A^{-1}B)^{-1}$

$$\mathbf{Z}^{*} = [(\sigma_{a}^{2} \psi \psi')^{-1} + \mathbf{A}' \sum_{u}^{-1} \mathbf{A}]^{-1} [(\sigma_{a}^{2} \psi \psi')^{-1} \hat{\mathbf{Z}} + \mathbf{A}' \sum_{u}^{-1} \mathbf{b}] (\mathbf{A}.5)$$

Considering (A.2)

$$\begin{split} Z^* &= \hat{Z} - (\sigma_a^2 \ \psi \psi') \ A' [A(\sigma_a^2 \ \psi \psi') A']^{-1} \ A \hat{Z} \ + \\ &+ (\sigma_a^2 \ \psi \psi') A' [A(\sigma_a^2 \ \psi \psi') A']^{-1} [(A(\sigma_a^2 \ \psi \psi') A')^{-1} + \sum_u^{-1}] \\ &\qquad \qquad (A.6) \\ \cdot \ [A(\sigma_a^2 \ \psi \psi') A']^{-1} A \hat{Z} + (\sigma_a^2 \ \psi \psi') A' \ \sum_u^{-1} \ b - (\sigma_a^2 \ \psi \psi') A' \ \sum_u^{-1} \ b + \\ &+ (\sigma_a^2 \ \psi \psi') A' [A(\sigma_a^2 \ \psi \psi') A']^{-1} [(A(\sigma_a^2 \psi \psi') A^{-1} + \sum_u^{-1}]^{-1} \ \sum_u^{-1} \ b \end{split}$$

Using (A.3) and (A.4):

$$Z^* = \hat{Z} + (\sigma_a^2 \psi \psi') A' [A(\sigma_a^2 \psi \psi') A' + \sum_u]^{-1} (b - A\hat{Z})$$
 (A.7)

which is the expression sought

1.1. Variance of the restricted predictor

Re-ordering expression (23) gives

$$Z^* = P^*b + (1 - P^*A) \hat{Z}$$
 (A.8)

and using (11) then gives

$$Z^* = P^*b + Z - P^*AZ + e - P^*Ae$$
 (A.9)

Accordingly, when the constraint is verified,

$$Z^* - Z = -P^*u + (I - P^*A)e$$
 (A.10)

Calculating the variance in (A.10)

Var
$$(Z^*-Z) = (I-P^*A) \sigma_n^2 (\Psi \Psi^{\dagger}) (I-P^*A)' + P^* \Sigma_n P^{**}$$
 (A.11)

and operating gives

$$Var(Z^* - Z) = \sigma_a^2 (\Psi \Psi^*) (I - P^* A)^* + P^* \Sigma_u P^{**}$$
(A.12)

which is the expression sought.

1.2. The compatibility test

Let a ~ N (0,
$$\sigma_{\tt m}^2$$
 I)

We define the $r \times 1$ information vector v as

$$v = b - A \hat{Z} \tag{A.13}$$

Under the null hypothesis H_0 : b+u=AZ

$$v = -u + A \Psi a \tag{A.14}$$

 ${f v}$ is a r x 1 vector that follows a normal distribution, since it is a linear combination of normal variables. Moreover, its first two moments are given by

$$E[v] = 0$$

$$E[vv'] = \sum_{u} + \sigma_{a}^{2} A \Psi \Psi' A' \qquad (A.15)$$

Thus,

$$v \sim N (0, \sigma_a^2 A \Psi \Psi^{\dagger} A^{\dagger} + \Sigma_u)$$
 (A.16)

On the basis of the properties of the normal distribution,

$$Q = v'(\sigma_a^2 A \psi \psi' A' + \sum_u)^{-1} v \sim \chi_r^2$$
 (A.17)

Thus arriving at the desired expression:

Q =
$$(b - A \hat{Z})'(\sigma_a^2 A \psi \psi' A' + \Sigma_u)^{-1} (b - A\hat{Z}) \sim \chi_r^2$$
 (A.18)

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⁽¹⁾ Los Documentos de Trabajo anteriores a 1990 figuran en el catálogo de publicaciones del Banco de España.