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Abstract

This paper analyzes the implications of a general representative agent intertemporal asset pricing model on the determination of the short-term interest rates. The model includes an extension of the Non-expected Utility Generalized Isoelastic Preferences that incorporates non-separability between private consumption and government expenditure. The model yields a generalized Fisher equation where the nominal interest rates are explained by the expected depreciation of the purchasing power of money, an endogenously determined required riskfree rate and an inflation risk premium. The econometric estimations suggest that the common rejection of the Fisher hypothesis can be, at least, partially explained by the traditional use of ad-hoc misspecified models. On the other hand, while the inflation risk premium is estimated to be small relative to the ex-ante real interest rate, its magnitude is substantially higher than the one obtained under the standard single-good expected utility models.

KEYWORDS: Fisher Equation, non-expected utility, non-separable utility, GARCH estimation.

1 Introduction

One of the most popular relations in the macroeconomic literature is the Fisher equation that relates nominal interest rates with inflation. The standard approach to estimate the Fisher equation is to assume a linear relation between the return on nominal short term bonds and some measure of the expected inflation rate allowing for a (possibly time varying) intercept which is interpreted as the required riskfree rate (see e.g. Fama (1975), Garbade and Wachtel (1978), Summers (1982) and Barsky (1987)). This ad-hoc specification is hardly compatible with the predictions of the standard intertemporal asset pricing theory. Under this theory, in the presence of inflation uncertainty, nominal bonds are risky assets which are priced in equilibrium to avoid arbitrage opportunities. Thus, the real return on nominal bonds is endogenously determined and includes an inflation risk premium over a required riskfree return. The standard approach to test the Fisher effect lacks an explicit characterization of the riskfree real rate using arbitrage conditions and ignores the existence of an inflation risk premium. As a consequence, the slope parameters from the econometric estimation of the simple Fisher equation are subject to misspecification bias and, therefore, the money illusion result of Summers (1983) and Barsky (1987) may be altered if a more complete specification of this equation is used.

The existing literature that applies standard asset pricing theory to the determination of nominal interest rates is quite small. In recent papers, Shome, Smith and Pinkerton (1988) and Evans and Wachtel (1989) use a standard version of the Consumption Based Capital Asset Pricing Model (CCAPM) to extend, by including the proper risk factors, the traditional Fisher equation relating nominal interest rates, required riskless rates and expected inflation. Those papers obtain that a significant part of the variation of the nominal rates can be attributed to both movements in the riskfree rate and movements in the risk factors. Shome et alt (1989) and Evans and Wachtel (1989) however, are unable to allocate more than a small part of the total size of the nominal interest rates to the risk premium. On the other hand, the estimated elasticity of the nominal rates to expected inflation is in both cases below unity, providing support to the money illusion hypothesis.

In the above papers, the CCAPM relations are obtained from the first order conditions of the optimization problem of a representative agent who maximizes the expected value of a time additive, constant relative risk aversion utility function (CRRA). This popular approach allows one to characterize completely the nominal interest rates by the moments of the joint distribution of aggregate consumption and the price level. Therefore, the nominal interest rates are not affected by the variability of the risky asset returns and their covariance with the inflation rate once the moments of aggregate consumption are taken into account. This somewhat counterintuitive result is a consequence of a restrictive parametrization of the utility function that imposes a perfect inverse relation between the coefficient of relative risk aversion and the elasticity of intertemporal substitution. This theoretical constraint produces an unsatisfactory characterization of the prices of nominal short term bonds, whose return is composed of a required riskfree element and an inflation risk premium that compensates investors for holding an asset whose payoff is subject to an uncertain real depreciation. While the riskfree rate component is mainly characterized by agents' willingness to substitute consumption over time, the risk premium is determined primarily by agents' attitudes toward risk.

Recently, a number of papers (Epstein-Zin (1987,1989), Weil (1989), Giovanini and Weil (1989), and Giovanini and Jorian (1989)) have introduced and exploited an isoelastic version of the Kreps-Porteus (1979) recursive preferences. This model's generalized isoelastic preferences (GIP) parametrizes independently attitudes toward risk and intertemporal substitution and includes both the growth rate of consumption and the return on the market portfolio in the determination of the marginal rate of substitution. Thus, the GIP preferences constitute a much richer theoretical framework to analyze intertemporal consumption-portfolio decisions. It provides a theoretical justification of the explanatory power for asset prices of the second order moments of the distribution of asset returns. For the case of nominal short-term bonds, the new preference structure allows one to determine the role of the variability of the market returns and its covariability with the inflation rate in the determination of the nominal interest rate. Since those terms are of much higher magnitude and variability than the moments of the joint distribution of inflation and consumption growth, the generalization is empirically attractive.

This paper investigates the formation of nominal interest rates under the intertemporal asset pricing theory derived from Generalized Isoelastic Preferences. In order to further complete the specification, I generalize the model assuming that government expenditure is not separable from private consumption and, therefore, affects the equilibrium value of asset prices. The inclusion of this variable is justifiable for at least two reasons. On one hand, government expenditure has proven to be a significant component of the marginal rate of substitution in the empirical analysis of Aschauer (1985) and Bean (1986) using standard CRRA expected utility functions. On the other hand, this variable is highly related with the inflation rate and, therefore, with the nominal interest rates.

The aim of the paper is, first, to characterize the components of the interest rates by exploiting the no-arbitrage conditions of a very general representative agent economy; second, to analyze to what extent the tractional rejection of the Fisher hypothesis is due to the use of simpler and potentially misspecified models; and finally, to study the power of the generalized asset pricing model to match the time series properties of the nominal interest rates.

The paper is organized as follows. Section 2 presents the preference model and obtains the equilibrium conditions of the representative agent. Section 3 utilizes the standard assumption of conditional lognormality of the marginal rate of substitution to obtain an explicit asset pricing relation for nominal bonds and explicit expressions for the required riskless rate and the inflation risk premium. Section 4 analyzes the empirical results concerning different evaluations of the model and the verification of the Fisher hypothesis. Section 5 provides concluding remarks.

2 The model

Consider a representative agent economy where preferences are represented by a Kreps-Porteus utility function. Assume also that the utility function is non separable in consumption and government expenditure, which appears as a substitute for private consumption. Thus, the utility function has the form

$$V_t = U\left[C_t, G_t, E_t V_{t+1}\right], \tag{1}$$

where C and G represent consumption and government expenditure respectively, and E_t is the usual t-conditional expectation operator. Assume also that U has the isoelastic form

$$U\left(H_t, E_t V_{t+1}\right) = \left[(1-\Theta)H_t^{1-\rho} + \Theta(E_t V_{t+1})^{\frac{1-\rho}{1-\gamma}}\right]^{\frac{1-\gamma}{1-\rho}}, \tag{2}$$

where

$$H_t = C_t^{1-k} G_t^k.$$

This expression constitutes a two-good generalization of the function used by Epstein and Zin (1989), Weil (1989) and Giovanini and Weil (1989). The main feature of this utility function is that the relative risk aversion coefficient (γ) and the elasticity of intertemporal substitution ($1/\rho$) are not related to one another[1]. This property is a consequence of the explicit modeling of attitudes toward early or late resolution of uncertainty through the sign of the second derivative of U with respect to its second argument $E_t(V(t+1))$. The inclusion of government expenditure as a substitute[2] for private consumption in the utility function allows me to further generalize the marginal rate of substitution and therefore the sources of risk in the asset pricing equations, provided that the parameter k is non-zero. In early studies (e.g., Aschauer (1986) and Bean (1986)), which analyzed the intertemporal allocation problem of a representative consumer in a single-asset expected utility framework, this variable proved to be empirically relevant. Therefore,

[1] Notice, in this framework, the coefficient of relative risk aversion and the elasticity of intertemporal substitution are defined over lotteries in terms of the composite good H and not over consumption lotteries.

[2] As in Bean (1986) I call C and G substitute goods. This terminology is only accurate if defined over constant levels of H and not over constant utility. An alternative approach is to assume Aschauer (1986)'s functional form where the substitutability of the goods is obtained under the usual definition over a constant level of the utility index.

it seems to be pertinent to include it in this more general environment. Notice, however, that since optimal public policy is not explicitly modeled, the variable G_t must be taken as an exogenous stochastic shock to the individual utility function at time t.

The homothetic nature of the utility function allows us to separate the choice of the optimal consumption path from the portfolio allocation problem. Thus, Define W_t as the level of wealth at period t, $R_i(t)$ $(i = 1 \cdots, N)$, as the return of asset i at period t, $w_i(t)$ as the proportion of wealth invested in asset i and $R_m(t)=\sum_{i=1}^{n} w_i(t)R_i(t)$ as the return of the portfolio of assets held in equilibrium. Then, the optimal consumption policy must satisfy the budget constraint[3]

$$W_{t+1} = R_m(t+1)(W_t - C_t). \tag{3}$$

Define the indirect utility function as

$$V(W_t, F_t) = \max_{C_t, w_i(t)} U\left[H_t, E_t V(W_{t+1}, F_{t+1})\right] = U\left[H_t, E_t V(W_{t+1}, F_{t+1})\right]. \tag{4}$$

where F_t is the individual's information set. Now, call U_H and U_V the partial derivatives of U with respect to its first and second argument respectively.

The first-order condition of the maximization problem with respect to consumption implies

$$U_H(t)\frac{dH_t}{dC_t} + U_V(t)E_t\left[V_W(t+1)\frac{dW_{t+1}}{dC_t}\right] = 0.$$

But, from (4) we have

$$V_W(t) = U_V(t)E_t\left[V_W(t+1)\frac{dW_{t+1}}{dW_t}\right].$$

[3]The budget constraint does not explicitly account for labor income. The reason is technical and arises from difficulty of solving the dynamic stochastic problem with an extra term in the budget constraint. Epstein and Zin (1987) tries to avoid the problem by assuming that the stream of labor income properly discounted can be seen as a part of the initial wealth if that source of income is non stochastic. Another possibility is to modify the market portfolio to include human capital and consider stochastic wages as returns for that asset. Neither solution is very satisfactory. The first one is unrealistic and the second one requires a very problematic estimation of the proportion of human wealth in the economy.

Then, by the budget constraint we obtain the envelope result

$$V_W(t) = U_H(t)\frac{dH_t}{dC_t}.$$

Therefore, we can write the Euler equation

$$1 = E_t\left[\frac{U_V(t)}{U_H(t)}\frac{\frac{dH_{t+1}}{dC_{t+1}}}{\frac{dH_t}{dC_t}}U_H(t+1)R_m(t+1)\right].$$

Since, by standard arguments this expression must also hold for each asset return, using the isoelastic parametrization of preferences (2), and after tedious algebra (see Appendix A), we can write

$$1 = E_t[\Lambda(t+1)R_i(t+1)] \qquad i = 1\cdots N, \tag{5}$$

where

$$\begin{aligned}
\Lambda_{t+1} &= Q\left[\frac{C_{t+1}}{C_t}\right]^A\left[\frac{G_{t+1}}{G_t}\right]^B R_m(t+1)^D,\\
Q &= \Theta^{\frac{1-\gamma}{1-\rho}},\\
A &= -\rho\frac{1-\gamma}{1-\rho} - (1-\gamma)k,\\
B &= k(1-\rho),\\
D &= \frac{1-\gamma}{1-\rho} - 1.
\end{aligned}$$

It is then clear that if $k = 0$, expression (5) becomes the first order condition for utility maximization of Epstein and Zin (1989) and Weil (1989). Similarly, if $\gamma = \rho$ but $k = 0$, (5) becomes a first order condition of the standard expected CRRA utility function. Finally, if $\gamma = \rho$ and $k \neq 0$, (5) is equivalent to the Euler equation considered by Bean (1986).

Note that (5) constitutes a set of orthogonality conditions which are directly testable. The general method of moments analyzed in Hansen (1982) and Hansen and Singleton (1982) can be used to test those conditions. Furthermore, this approach provides a method to estimate parameters k, ρ and γ and test the *separability* constraint ($k = 0$) and the expected utility restriction ($\gamma = \rho$). This approach (followed by Epstein and Zin (1989)) for the $k = 0$ case deals with the estimation and testing problem without specifying a particular distribution function for

$\Lambda(t)$ and the individual returns. However, the simple verification of the orthogonality constraints lacks the attractiveness of estimating explicit asset pricing relations. An alternate estimation approach (followed by Giovanini and Weil (1988) and Giovanini and Jorian (1989)) consists of assuming joint lognormality of the marginal rate of substitution $\Lambda(t+1)$ and the asset returns conditional on information available at period t. This distributional assumption yields appealing asset pricing relations where the conditional expected value of the excess return of the risky assets over the riskfree rate is explained by the conditional covariances of the asset return with both the growth rate of consumption and the market return. This approach is explored in the next section to obtain a generalized specification of the Fisher Equation.

3 The Generalized Fisher Equation

Assume for the rest of the paper that there is one risky asset in positive net supply (the market portfolio), which pays off a real return $R_m(t+1)$ between t and $t+1$. Call $I(t+1)$ the dollar-return of a (default-free) nominal bond. This claim suffers the depreciation of the purchasing power of money[4] represented by $P(t-1)/P(t)$ where $P(t)$ is the price level at period t. Finally call $R_f(t+1)$, the real return of a perfectly safe asset (an inflation indexed bond). Since both the nominal bond and the safe asset are in zero net supply, they are priced so that the representative agent demands no claims of those assets at the equilibrium prices.

According to the Euler Equation (5), the equilibrium conditions for the market return, the nominal interest rates and the riskless return are:

$$E_t\left[\Lambda(t+1)R_m(t+1)\right] = 1, \tag{6}$$

$$Et\left[\Lambda(t+1)I(t+1)\frac{P(t)}{P(t+1)}\right] = 1, \tag{7}$$

$$Et\left[\Lambda(t+1)\right] = \frac{1}{R_f(t+1)}. \tag{8}$$

Assume that the growth rates of consumption (c) and government expenditures (g) together with the log of the market return (r_m) and the inflation rate (π) at period t are (conditional on

[4]Notice that, since money plays no role in this economy the variable P(t)/P(t+1) must be considered as an exogenous term which negatively affects the rate of return of the bonds. The alternative would be to construct a *Cash-in-Advance* version of the model which would endogenously determine the inflation rate.

the information available at period t) jointly normally distributed with mean vector $\xi(t)$ and a variance-covariance matrix $\Omega(t)$, where

$$\Omega(t) = \begin{pmatrix} \sigma_{mm}(t) & \sigma_{mp}(t) & \sigma_{mc}(t) & \sigma_{mg}(t) \\ & \sigma_{pp}(t) & \sigma_{pc}(t) & \sigma_{pg}(t) \\ & & \sigma_{cc}(t) & \sigma_{cg}(t) \\ & & & \sigma_{gg}(t) \end{pmatrix}. \tag{9}$$

Then, from the Euler equations we have that

$$\begin{aligned} E_t r_m(t+1) &= -E_t\lambda(t+1) - \frac{1}{2}\{Var_t r_m(t+1) + Var_t\lambda(t+1)\} \\ &- Cov_t(\lambda(t+1), r_m(t+1)), & (10) \\ r_f(t+1) &= -E_t\lambda(t+1) - \frac{1}{2}Var_t\lambda(t+1), & (11) \\ i(t+1) &= -E_t\lambda(t+1) - \frac{1}{2}Var_t\lambda(t+1) \qquad (rf) \\ &+ E_t\pi(t+1) - \frac{1}{2}Var_t\pi(t+1) \qquad (pp) \\ &+ Cov_t(\lambda(t+1), \pi(t+1)), \qquad (rp) & (12) \end{aligned}$$

where $\lambda(t) = \log\Lambda(t)$, $r_f(t) = \log R_f(t)$, and $i(t) = \log I(t)$.

Equations (10) and (11) are asset pricing relations for a representative agent model, provided that the marginal rate of substitution and the asset returns at every period t are jointly lognormally distributed conditional on the information available at time t. Equations (11) and (12) show that nominal interest rates are composed of three different elements: the theoretical riskfree rate (rf), the expected change in the purchasing power of money (pp) and the inflation risk premium (rp).

Thus, when preferences are GIP and the joint conditional distribution of the marginal rate of substitution ($\Lambda(t)$) and asset returns is lognormal, equation (12) has the form

$$\begin{aligned} i(t+1) &= E_t\pi(t+1) - \frac{1}{2}\sigma_{pp}(t) \qquad (pp) \\ &- \log Q - AE_t[c(t+1)] - BE_t[g(t+1)] - DE_t[r_m(t+1)] \\ &- \frac{1}{2}\left[A^2\sigma_{cc}(t) + B^2\sigma_{gg}(t) + D^2\sigma_{mm}(t)\right] \\ &+ AB\sigma_{cg}(t) + AD\sigma_{mc}(t) + BD\sigma_{mg}(t) \qquad (rf) \end{aligned}$$

$$+ \quad A\sigma_{pc}(t) + B\sigma_{pg}(t) + D\sigma_{mp}(t) \qquad (rp). \qquad (13)$$

Equation (13) shows how the preference parameters interact with the first and second moments of inflation, market return, consumption and government expenditure growth to build the equilibrium nominal interest rate. This expression constitutes a Generalized Fisher equation which departs from the standard formulations (e.g. Garbade and Wachtel (1977) and Summers (1982)) in three aspects. First, nominal interest rates respond with unit elasticity to the expected rate of depreciation of money which under inflation uncertainty is not the expected inflation rate as the simple specification suggests[5]. The term $-.5\sigma_{pp}(t)$ must be added in order to correctly specify the equation. Second, the risk-free rate is endogenously determined by the conditional first and second order moments of the marginal rate of substitution. Third, the implicit assumption of risk neutrality is dropped to include a general specification of the risk premium associated with inflation uncertainty which is defined by the covariability of the different components of the marginal rate of substitution with the inflation rate.

Equation (13) includes, as an explanatory variable, the conditional mean of the market return. Accurate estimates of this variable are usually very difficult to obtain. However, using the asset pricing relation (10) and tedious but straightforward algebra, we can write the Generalized Fisher Equation as

$$\begin{aligned} i(t+1) &= \alpha_0 + E_t\pi(t+1) - \frac{1}{2}\sigma_{pp}(t) + \alpha_1 E_t c(t+1) + \alpha_2 E_t g(t+1) \\ &+ \alpha_3\sigma_{cc}(t) + \alpha_4\sigma_{gg}(t) + \alpha_5\sigma_{cg}(t) + \alpha_6\sigma_{mm}(t) \\ &+ \alpha_7\sigma_{pc}(t) + \alpha_8\sigma_{pg}(t) + \alpha_9\sigma_{mp}(t), \qquad (14) \end{aligned}$$

where

$$a_0 = -\log Q \qquad \text{and}$$

[5]This point has been recently made by Shome et al. (1988) who obtain a specification of the Fisher equation using the CCAPM.

$$\alpha_1 = -\frac{A}{1+D} \qquad \alpha_4 = -\frac{B}{2(1+D)} \qquad \alpha_7 = A$$
$$\alpha_2 = -\frac{B}{1+D} \qquad \alpha_5 = -\frac{AB}{1+D} \qquad \alpha_8 = B$$
$$\alpha_3 = -\frac{A}{2(1+D)} \qquad \alpha_6 = \frac{D}{2} \qquad \alpha_9 = D.$$

It is then evident that the general preference structure used in this paper provides a much richer specification of the Fisher equation than the standard CCAPM approach. Thus, the Generalized Fisher Equation contains expressions for the theoretical riskfree return and the inflation risk premium that involve the conditional moments of the market return and government expenditure in addition to the conditional moments of the aggregate consumption. In particular, the GIP specification of the Fisher equation differs from the expected utility formulation in two terms. Those are the variance of the market return as a component of the riskfree rate and the covariance of the inflation rate with the market return as a part of the risk premium. The inclusion of government expenditure adds three terms to the specification of the riskfree return related to the first and second order moments of that variable and completes the inflation risk premium by adding the covariance of government expenditure with the inflation rate.

The generalization of the specification of the risk premium is particularly relevant. The conditional covariances of the inflation rate with government expenditure and with the market return happen to be of higher magnitude, in absolute terms, than the inflation-consumption covariance in the last 30 years. Those facts suggests that the inflation risk premium under the standard CRRA approach is likely to be underestimated if the restrictions of that model do not hold. Notice also that the Generalized Isoelastic Preferences not only adds an empirically relevant factor in the inflation risk premium specification, it also introduces an important modification in the interpretation of the role played by the covariance of consumption with the inflation rate. Under the standard CRRA expected utility approach with positive coefficients of relative risk aversion, the inflation risk premium is positive if and only if the conditional covariance of inflation with consumption is negative as it is observed in the U.S. data of the postwar period (see Table 1). However, this implication does not hold if attitudes toward risk and intertemporal substitution are independently modeled. As an example, for the empirically relevant case of values of the risk aversion parameter (γ) and the elasticity of substitution ($1/\rho$) below one, a negative covariance between consumption growth and inflation tends to decrease the inflation risk premium. In this case, however, the contribution to the inflation risk premium of a negative covariance of the inflation rate with the market return is positive. It then follows that, for the above case, the relevant factor to explain a positive inflation risk premium is not a negative covariance of inflation with consumption as the restricted model suggests, but a negative covariance of inflation with wealth.

Estimation of (14) requires specifying the conditional mean of c, g, and π, plus a model for the conditional variance-covariance matrix, Ω. Equation (14) does not include the conditional expected value of the market return but only second order moments of this variable. However, previous studies (see, e.g., French, Schwert and Stambaugh (1987)) have shown that the explicit modeling of a time varying conditional mean of the asset returns does not substantially alter the estimated conditional second order moments. This result allows one to obtain the conditional second order moments of the market return using a simple ad-hoc specification of its conditional mean. In the empirical section of this paper I use a first-order autoregresive process.

Thus, consider the following VAR model for the market return, the inflation rate, the consumption growth and the government expenditure growth.

$$X(t+1) = E_t X(t+1) + U(t+1), \qquad t = 1 \cdots T, \tag{15}$$

with

$$X(t) = \begin{pmatrix} r_m(t) \\ \pi(t) \\ c(t) \\ g(t) \end{pmatrix} \quad \text{and} \quad U(t+1)|F_t \sim N(0,\Omega(t)). \tag{16}$$

Now, assume that the conditional variance-covariance matrix follows the GARCH(p,q) process

$$vec\Omega(t) = M_0 + \sum_{j=1}^{q} M_1(j) vecU(t-j)U(t-j)' + \sum_{j=1}^{p} M_2(j) vec\Omega(t-j).$$

A standard simplifying assumption is to restrict M_1 and M_2 to be diagonal, so that every term in the conditional variance-covariance matrix is only related to its past value and the corresponding element in the outer product of the past innovation vector. Thus we can write

$$\Omega(t) = T_0 + \sum_{j=1}^{q} T_1(j) \bullet U(t-j)U(t-j)' + \sum_{j=1}^{p} T_2(j) \bullet \Omega(t-j), \tag{17}$$

where $\bullet$ represents element by element multiplication.

Equations (14), (15), and (17) constitute a restricted specification of a multivariate GARCH in mean model (Engle, Lilien and Robbins (1987) and Bollerslev, Engle and Wooldridge (1988)).

The (log) likelihood function of the model for X has the form

$$\begin{aligned} & L\left(T_0', \{vecT_1(i)'\}, \{vecT_2(j)'\}, \quad i = 1\cdots q, j = 1\cdots p\right) = \\ = \; & f - \sum_{t=1}^{T} \log|\Omega(t)| - \frac{1}{2}U(t)'\Omega(t-1)^{-1}U(t), \end{aligned} \tag{18}$$

where f is a constant.

The asymptotic properties of the multivariate GARCH estimates are not yet well understood. However, under some regularity conditions (see Wooldridge (1986)) the maximum likelihood estimates of the parameters are consistent and asymptotically normal with a variance-covariance matrix equal to the inverse of the information matrix. Those conditions include almost sure positive definiteness of the matrix $\Omega(t)$. A straightforward way to impose this condition is to restrict the parameter matrices T_0, $T_1(i)$ and $T_2(j)$ to be positive definite for all i, j and estimate their Cholesky Factors. This procedure, however, severely constrains the dynamics of the variance and covariance terms[6] and introduces additional nonlinearities in an already complex likelihood function.

Adding an error term to (14), assuming that this is conditionally normally distributed, estimation of the Fisher equation together with specifications (15) and (17) can be performed using full information maximum likelihood techniques. However, in this paper, due to computational reasons, I estimate the Generalized Fisher Equation using a two-step procedure. In the first step I estimate the VAR- GARCH system (15)-(17) and in the second step I estimate (14) with the conditional mean and variances of the vector X generated in the first step. To correct for the es-

[6] In particular, that restriction would imply that covariances should be less persistent than variances. This restriction is hardly consistent with the data.

timated regressor problem, I obtain consistent standard errors by using the scores corresponding to the likelihood function of the whole system.

4 Empirical Results

4.1 Data

To implement the above model, I used quarterly data from 1947 to 1988. I defined the market return as the return on the value weighted portfolio of the New York Stock Exchange. Nominal interest rates are one-month Treasury bill compound returns. Both yields are obtained from the CRSP tapes. Consumption is real per capita personal consumption of nondurables and services. Inflation series are quarterly growth rates of the personal consumption deflator. Finally, the public expenditure series consists of state and local real expenditure on goods and services. The omission of federal government expenditure is due to the large component of defense expenses in this item. It is difficult to justify some degree of substitutability between military expenses and private consumption. Data on inflation, consumption and government expenditure has been seasonally adjusted.

Table 1 presents some summary statistics of the variables included in the analysis. The figures illustrate that the covariances between inflation and stock returns and the growth rates of consumption and government expenditure are negative and vary across time. The covariance of inflation with the stock returns is of much higher magnitude and more unstable over subperiods than its covariance with consumption and government expenditure. The latter is also non-negligible and larger in absolute terms than the covariance between inflation and consumption growth. Finally, unlike the variance of inflation, the covariance of inflation with both consumption and the market return reach their maximum absolute values precisely in the period with highest inflation (1970-1979). These empirical facts underscore the relevance of the empirical analysis of the risk factors which affect the nominal interest rate under the framework I presented in the previous section.

4.2 GMM Estimation Results

In order to test the generalized model without imposing strong distributional assumptions, I performed a GMM estimation of the Euler Equations corresponding to the first order conditions of the optimization problem of the representative agent.

The GMM procedure can be directly applied to equations (6) and (7). Since the riskfree rate is not observable in economies without inflation indexed bonds[7], equation (8) cannot be directly estimated. However, we can use that equation to obtain an approximation to the magnitude of the inflation risk premium in this set up. Thus, taking expectations of both sides of (8), we obtain

$$E\left[\mathrm{A}(t+1)\right]-\mu=0, \tag{19}$$

where (20)

$$\mu=E\left[\frac{1}{R_f(t)}\right]. \tag{21}$$

By comparing μ with the expected value of the inverse of the real return on nominal bonds we get a first approximation to the mean value of the inflation risk premium. Thus, define IR and RR such that $1/IR = E[P(t+1)/(I(t+1)P(t)]$ and $1/RR = \mu$. Then, the variable $RP = IR - RR$ provides such an approximation.

As it is described in Appendix B the GMM procedure provides consistent estimates of the parameters of the marginal rate of substitution (Q, A, B and D) and the riskless rate term μ. In addition, this method yields a test of the overidentifying restrictions of the model.

The GMM estimation procedure requires the use of a set of instrumental variables. This set is not explicitly determined by the theoretical model and the estimation procedure only requires that the moment equations satisfy some mild regularity conditions. Sensitivity of the results to the choice of instruments is checked by using different sets of instrumental variables which include straight lags of the endogenous variables (as in Hansen and Singleton (1982)) and some nonlinear functions of those lagged variables (as in Epstein and Zin (1987)). The estimation procedure[8] starts by obtaining consistent but non-efficient parameter estimates using the identity matrix as

[7] In the U.S. inflation indexed bonds are not heavily traded and have not been available until 1988 when the Inflation Plus CD's appeared.

[8] The GMM estimation is performed using, with slight modifications,the Gauss code developed by Hansen, Heaton and Ogaki (1988).

a weighting matrix. Efficiency is reached after an iteration over a new weighting matrix which is constructed using a consistent estimate of the variance-covariance matrix obtained from the parameter estimates of the first iteration. However, in order for estimates and the orthogonality test to be less dependent on the starting values of the algorithm, I performed additional rounds of estimation until the probability value of the overidentifying restriction tests converged.

Results of the GMM estimation of equations (6), (7) and (19) are reported in Table 2. In all the specifications, the test of the overidentifying restrictions of the model cannot be rejected at the 5% significance interval. The expected CRRA utility restriction ($D = 0$) is overwhelmingly rejected in all cases despite the variation of the estimate of D across specifications. Those results are partially consistent with the ones obtained by Epstein and Zin (1987)[9] and provide support for the Generalized Isoelastic Preference model. Less satisfactory are the results concerning the point estimates of the exponents of consumption and government expenditure growth in the Euler equations (A and B). Those parameters are very unprecisely estimated and vary widely with the choice of instrumental variables. As a consequence, the point estimates of the preference parameters are not substantially meaningful besides the fact that the coefficient of relative risk aversion seems to be significantly different from the inverse of the elasticity of intertemporal substitution. The estimated inflation risk premium also shows some variation across specifications of the moment equations. However, the estimated premium is in all cases positive and in a range from .98% to 12.16% of the average level of the nominal interest rate. This is a moderate but non-negligible magnitude.

In order to evaluate the relevance of using a generalized specification of preferences in computing the inflation risk premium, I replicated the GMM analysis imposing the (statistically rejected) CRRA expected utility constraint ($D = 0$). For this comparison, I used the first four sets of instruments consisting of lagged values of the endogenous variables. Results are reported in Table 2b. The estimated average inflation risk premium is negative in two specifications. In those specifications where this estimate obtains a positive value, the estimate is about ten times smaller than the one obtained for the same set of instruments in the non-constrained GIP specification.

[9] Unlike the results reported here, EZ do reject the Generalized model for the set of instruments consisting of straight lags of the relevant variables. However that study uses monthly data and does not account for substitution between consumption and government expenditure.

Finally, in regard to the relevance of including the government expenditure term, the evidence is mixed and sensitive to the choice of the instrument set. The exponent of the Government Expenditure growth rate in the marginal rate of substitution appears as significant in three out of seven GIP specifications and in three out of four specifications under the expected utility constraint.

So the empirical results of the GMM estimation provide statistical support for the generalized model and suggest that the inflation risk premium is likely to be understated under the standard CRRA expected utility approach. In order to obtain a more complete view of the empirical implications of the model for the determination of nominal interest rates, we have to analyze the estimation results of the lognormal version of the model presented in section 3.

4.3 Estimation of the Generalized Fisher Equation

The estimation of the Generalized Fisher Equation (14) requires the conditional first and second order moments of the joint distribution of the market return, the inflation rate and the growth rates of Consumption and Government Expenditure. Those moments were estimated using a VAR-GARCH model as it is indicated in equations (15) and (17)[10].

Results are summarized in Table 3. Note that I have specified the conditional mean of the market return to be a a linear function of the lagged return. However, other specifications of the conditional mean did not significantly affect the conditional variance parameters. To reduce the dimensionality of the problem, I only allowed σ_{mm}, σ_{mp}, σ_{pp} σ_{pc} and σ_{pg} to be time varying, leaving σ_{cc}, σ_{cg}, σ_{gg}, σ_{mc}, and $\sigma_{mg}(t)$ as constants. In the first three cases the constancy assumption relies on the small magnitude and variability of those terms relative to the other components of the real riskfree rates. The constancy of the covariance of the market return with consumption and government expenditure is justified by their absence in the Generalized Fisher Equation (14). The cost of these omissions in terms of efficiency of the parameter estimates has proven, however, to be almost negligible.

The estimation procedure was as follows. Consistent estimates of the parameters were obtained by running limited information maximum likelihood estimation on different pairs of equa-

[10]The maximum likelihood estimation was performed using the Berndt-Hall-Hall-Haussman algorithm with numerical derivatives. The program was written on the basis of a Fortran code supplied by J. Wooldridge.

tions. Those parameters served as initial values in the BHHH algorithm defined on the likelihood function of the system. Without imposing positive definiteness on the parameter matrices, the algorithm had problems completely fulfilling the convergence criterion. However, the estimated model obtained by this procedure significantly outperformed a positive definite constrained specification in terms of the value of the likelihood function. Furthermore, the estimated variance-covariance matrix in the unconstrained estimation is positive definite at all the sample points. Thus, on the basis of those results I chose to use the estimates corresponding to the unconstrained specification.

The next step was to estimate equation (14). I added a noise term $\epsilon(t)$ to this equation and assumed it to follow the process

$$\epsilon(t) = \eta\epsilon(t-1) + \omega(t), \tag{22}$$

$$\text{with} \quad \omega(t) \sim N(0, h(t))$$

$$\text{and} \quad h(t) = l_0 + l_1 w(t-1)^2 + l_2 h(t-1). \tag{23}$$

This specification of the error term tries to make inference more robust to specification error and errors in variable problems, and increase the efficiency of the estimates. The estimation of equation (14) was performed by limited rather than full information maximum likelihood. However, I obtained corrected standard errors by running one iteration of the FIML routine taking the LIML estimates as initial values and taking into account possible remaining autocorrelation and heteroskedasticity (see Appendix C).

In table 4, I present the estimation results of an unrestricted version of equation (14). The dependent variable is the ex-ante real interest rate $(i_t\text{-}E_t(\pi(t+1)+.5\sigma_{pp}(t))$. The figures show that the generalized model (like the more restrictive CRRA specification of Shome, Smith, and Pinkerton (1988)) is unable to explain the high degree of autocorrelation present in nominal interest rates. On the other hand, while the terms related to first-order moments are highly significant and with the expected sign, the coefficients of the second-order moments are much less precisely estimated. Among them, however, the estimated coefficient of $\sigma_{mp}(t)$ is close to twice its corrected standard error.

Table 5 contains estimation results of the restricted model. The specification is not rejected according to the likelihood ratio test and the point estimates of the preference parameters, γ and ρ have reasonable values. The estimates indicate the relevance of including government expenditure, but the substitution parameter k is estimated to be too high (more reasonable values are within two standard deviations). More surprising is the inability to reject the expected utility constraint ($\gamma = \rho$). Comparing results from tables 4 and 5, this fact appears to be due to the failure of the required riskfree rate component of nominal interest rates to incorporate the variability of the market return and not to the irrelevance of the additional term in the risk premium ($\sigma_{mp}(t)$).

Table 6 contains estimates of the model when the term in $\sigma_{mm}(t)$ is omitted. The results show that this specification fits the data better than the one represented by equation (14) and confirms that the covariability of the market return with inflation has a leading role in explaining a positive inflation risk premium. Therefore, the results suggest that while the generalized model provides a more accurate evaluation of the inflation risk premium, it fails to substantially improve the specification of the required riskfree rate component of the nominal interest rates.

4.4 The Fisher Hypothesis

Tables 7 through 9 present some tests of the existence of the so called Fisher effect or unit elasticity of nominal interest rates with respect to expected rate of depreciation of money. The literature does not provide strong support for the empirical verification of this hypothesis in specifications which do not account for the existence of an inflation risk premium and an endogenously determined required riskless rate. (See, e.g., Summers (1983) and Barsky (1987).).

In order to better deal with low frequency properties of the data, I do not include a first order serial correlation term in the residuals[11]. However, I still use a GARCH specification in the error term and autocorrelation-heteroskedastic robust standard errors. The specification estimated now is the one represented by equation (14) where the term $E_t\pi(t+1) + 1/2\sigma_{pp}(t)$ has a free coefficient (F). Table 7 shows a point estimate for F equal to .84 and values of the t-statistic and the Likelihood ratio that do not reject the Fisher effect at a 99% confidence interval.

To study the sensitivity of the results with regard to different specifications of the marginal rate of substitution, I estimated two constrained specifications of the model. In the first specification I

[11]Note that the estimated η is relatively close to one. So the model is close to being in first differences (as in SSP).

imposed the expected utility restriction ($D = 0$), and in the second one I dropped the government expenditure term of the marginal rate of substitution ($B = 0$). The results are shown in tables 8 and 9. Consistent with the estimation of equation (14), the use of the non-expected utility preferences does not alter the inability to reject the Fisher hypothesis. On the other hand, when the test is performed for the generalized model with the single good restriction $k = 0$ imposed, the unit elasticity hypothesis is soundly rejected. This rejection is caused by both a lower level of the point estimate of the elasticity and a lower standard error with respect to the estimation of the more general model. These results indicate that while the rejection of the Fisher hypothesis is essentially insensitive to the specification of the inflation risk premium, it can be explained, at least partially, by the misspecification of the required riskfree rate component of the Fisher Equation.

4.5 How Well Does the Lognormal Model Fit the Data

The estimation of the Fisher equation has assumed a particular distribution for an error term added to specification (14). Then, the likelihood function of the model was maximized to obtain parameter estimates. Recall, however, that this procedure just exploits the moment restriction:

$$E\left[\frac{\partial L_t}{\partial \phi}\right] = 0, \tag{24}$$

where ϕ is the parameter vector and L_t is the log of the likelihood function. Notice first that if the error term does not satisfy the assumed distribution, it is not easy to establish conditions under which this estimation procedure is asymptotically more efficient than the ones that exploit other sets of moment equations, even when the market return, government expenditure and consumption follow the assumed conditional lognormal distribution so that equation (14) holds[12]. Second, and more important, since nominal interest rates are known ex-ante, equation (14) establishes an exact relation which should be only subject to measurement error problems if the model is correctly specified. Given these two concerns, the evaluation of the model should include some analysis of its ability to fit the unconditional moments of the dependent variable. In addition,

[12]Thus, even when the lognormality assumption holds, the GMM estimates could be more efficient than the ones obtained by QML if the error in the nominal interest rate equation is not normal.

it is interesting to compare the performance of the generalized model under the assumption of lognormality for different sets of preference parameter values. Thus, I picked estimates of γ and ρ from the GMM best behaved specifications (with and without imposing the expected utility restriction) and the LIML estimates. I restricted the time preference parameter Q to be equal to the standard value of .99 and allowed for three values of k (0, .3 and .5). This yields fifteen specifications. In table 10 the mean, standard deviation, and first and second order autocorrelations of the fitted ex-ante real interest rate for the different models are presented, along with the statistics obtained using the observed data on nominal interest rates. I also include the correlation between real and fitted values and the mean values of the predicted real riskless rates and inflation risk premium. The results show that all of the expected utility models overestimate the mean of the ex-ante real rate. On the other hand, they also tend to underestimate the volatility of this rate and the autocorrelation of the series. The latter problems are partially solved when the coefficient of relative risk aversion increases but at the cost of additional overestimation of the mean value.

The examples of non-expected utility models exhibit those problems to a lesser degree, suggesting that the additional degree of freedom can significantly reduce the difference between the unconditional moments of fitted and observed series. However, as the difference between γ and ρ increases, the correlation between the two series declines. As I argued before, this result is a consequence of the absence of a response of the nominal interest rates to the variance of the market return, and the fact that the weight of this variable in the Generalized Fisher Equation increases as the gap between γ and ρ increases.

The relative importance of each component of the ex-ante real interest rate is very sensitive to the specification of preferences. It is evident that the non-expected utility models estimate a considerably lower average level for the riskless return. In addition, the inclusion of the government expenditure has a significant effect on the magnitude of the riskless rate. However, the direction of this effect depends heavily on the combination of the preference parameters considered.

The inflation risk premium term averages a relatively small magnitude which is not highly sensitive to the inclusion of the government expenditure term, but which varies widely with small departures from the expected utility specification. This fact is better illustrated in graphs 1 and 2.

In graph 1 the inflation risk premium corresponding to the parameters ($\gamma = .87$, $\rho = 1.27$)[13], obtained as point estimates in one of the GMM specifications, is plotted using the lognormal specification of the risk premium. For comparison, I also included the series corresponding to the lognormal constrained estimates ($\gamma = 1.27, \rho = 1.30$) and the expected utility specification ($\gamma = \rho = 1.27$). The series illustrates how a small discrepancy between attitudes about risk and intertemporal substitution has a large effect on the estimated premium. Thus, a difference of .03 in those coefficients doubles the average risk premium and a difference of .4 multiplys this average by almost 15. In graph 2 the risk premium associated with an expected utility model that depicts moderate risk aversion ($\gamma = \rho = 1.27$) is compared to a high risk aversion one (γ=ρ = 4.7) and to the non-expected utility specification ($\gamma = 1.56$, $\rho = .34$) obtained in another GMM estimation. This graph perfectly illustrates how the level and volatility of the risk premium associated with the non-expected utility model is not attainable for the expected utility specification except for unreasonably high risk aversion coefficients.

[13] I assumed $k = 0$ for these calculations, given the small modification to the value of the inflation risk premium the inclusion of this variable provides.

5 Conclusions

This paper has used advances in asset pricing theory derived from non-expected utility generalized isoelastic preferences to explain the behavior of short term nominal interest rates. The model allows for an independent parametrization of attitudes toward risk and intertemporal substitution, and it assumes that government expenditure is a substitute for consumption. A direct estimation of the Euler equations derived from this preference structure has illustrated the relevance of the generalization by unambiguously rejecting the standard expected utility CRRA restrictions. Using distributional assumptions on the components of the marginal rate of substitution, I obtain a rich characterization of nominal bond pricing equations which is composed of a term measuring the expected evolution of purchasing power, an endogenously determined riskless rate and an inflation risk premium. Those terms involve first and second order moments of the market return, the inflation rate and the consumption and government expenditure growth rates. This model has been used to test the Fisher Hypothesis and to evaluate the relative magnitudes of the different components of the interest rates.

The inclusion of government expenditure in the preference structure has reversed the traditional rejection of the hypothesis of unit elasticity of nominal rates with respect to expected inflation. This result indicates that the traditional rejection of the Fisher hypothesis can be partially explained by the use of misspecified models that do not consider a sufficiently general specification of the required risk free rate.

On the other hand, the relaxation of the hypothesis of expected utility has not substantially improved the specification of the real riskless return component of the Fisher equation. Thus, contrary to the predictions of the non- expected utility model, the variance of the market return does not significantly affect the required riskless rate. However, the estimation of the Generalized Fisher Equation has suggested that while the inflation risk premium has a relatively small magnitude, this term is considerably larger than the one obtained under the standard expected utility models. The results show that the inflation risk premium is responsible at most for 10% of the ex-ante real return of nominal bonds, but the use of the covariance of consumption with inflation as the single risk factor tends to severely understate it. More precise measures of the inflation risk premium are hardly obtainable given the difficulty in obtaining accurate estimates of the preference parameters and given the sensitivity of the results to small variations in those parameters.

Finally, some serious caveats apply to the ability of the log- linear intertemporal asset pricing model to match the time series properties of the nominal interest rates even in this general non–expected utility set up. In general, the specifications that correlate better with movements in the observed interest rates tend to overestimate the means and underestimate volatility and autocorrelation.

In summary, the generalized model has provided a more accurate evaluation ofthe components of the nominal interest rate and a better framework in which to analyze the Fisher hypothesis. However, this model is far from succeeding in satisfactory closing the observed gap between the predictions of the equilibrium representative agent models and the time series behavior of the nominal interest rate.

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APPENDIX A

Derivation of the Marginal Rate of Substitution for the Generalized Isoelastic Preferences with Non-separability between Consumption and Government Expenditure

Consider the utility function (2) and define $\Lambda(t+1)$ as the marginal rate of substitution between consumption at periods t and $t+1$. Then

$$\begin{aligned}\Lambda(t+1) &= \frac{U_H(t+1)U_V(t)}{U_H(t)}\frac{\frac{dH_{t+1}}{dC_{t+1}}}{\frac{dH_t}{dC_t}} \\ &= \Theta\left[\frac{H_{t+1}}{H_t}\right]^{-\rho}\left[\frac{E_tV_{t+1}}{V_{t+1}}\right]^{\frac{1-\rho}{1-\gamma}-1}\frac{\frac{dH_{t+1}}{dC_{t+1}}}{\frac{dH_t}{dC_t}}.\end{aligned}$$

Conjecture a solution for the indirect utility function

$$V(W_t,F_t) = S(F_t,G_t)W_t^{(1-k)(1-\gamma)}. \tag{25}$$

Then, substituting (25) in the function (2) and obtaining the first order conditions of the maximization problem yields:

$$\begin{aligned} C_t &= z(F_t,G_t), \\ \text{where} &\quad z \text{ is such that} \\ (1-\Theta)z^{(1-k)(1-\rho)-1}G_t^{k(1-\rho)} &= \Theta\vartheta_t(1-z)^{-\rho}, \qquad (26) \\ \text{and} & \\ \vartheta_t &= \left\{E_t\left[s(F_{t+1},G_{t+1})\right]R_m(t+1)^{(1-k)(1-\rho)}\right\}^{\frac{1-\gamma}{1-\gamma}}. \qquad (27)\end{aligned}$$

Then, solving for ϑ_t in (26), and using (25) and (27), we find that

$$S(F_t,G_t) = (1-\Theta)^{\frac{1-\gamma}{1-\rho}}z(F_t,G_t)^{-\rho\frac{1-\gamma}{1-\rho}-k(1-\gamma)}G_t^{k(1-\gamma)}. \tag{28}$$

Now, using the budget constraint (3), the definition of ϑ_t in (27), and (26), we have that

$$E_tV(t+1) = \vartheta_t^{\frac{1-\gamma}{1-\rho}}\left[1-z(F_t,G_t)\right]^{(1-k)(1-\gamma)}W_t^{(1-k)(1-\gamma)} \tag{29}$$

and then,

$$\frac{E_t V_{t+1}}{V_t} = \frac{\vartheta_t^{\frac{1-\gamma}{1-\rho}}}{S(F_{t+1}, G_{t+1})} R_m(t+1)^{(1-k)(1-\gamma)}. \tag{30}$$

Replacing ϑ_t by its expression in (27) and using the budget constraint we obtain

$$\frac{E_t(V_{t+1})}{V_{t+1}} = \Theta^{-1-\frac{1-\gamma}{1-\rho}} \left[\frac{C_{t+1}}{C_t}\right]^{\rho\left(1-\frac{1-\gamma}{1-\rho}\right)+k(\gamma-\rho)} R_m(t+1)^{\frac{1-\gamma}{1-\rho}}.$$

But then, from (25)

$$\Lambda(t+1) = \Theta^{\frac{1-\gamma}{1-\rho}} \left[\frac{C_{t+1}}{C_t}\right]^{-\rho\frac{1-\gamma}{1-\rho}-k(1-\gamma)} \left[\frac{G_{t+1}}{G_t}\right]^{k(1-\rho)} R_m(t+1)^{\frac{1-\gamma}{1-\rho}-1},$$

which is the result stated in the text.

APPENDIX B

Model's Estimation and Tests under the Generalized Metbod of Moments

Consider a vector of m variables $Z(t)$ which belong to the agent's information set at period t (F_t). Then, by the law of iterated expectations we can write the set of orthogonality conditions

$$E\left[Z(t)\left(\Lambda(t+1)R_m(t+1)-1\right)\right] = 0, \tag{31}$$

$$E\left[Z(t)\left(\Lambda(t+1)I(t+1)\frac{P(t)}{P(t+1)}-1\right)\right] = 0, \tag{32}$$

$$E\left[\Lambda(t+1)\right] - \mu = 0. \tag{33}$$

Now, define

$$Y(t) = \left(\frac{C(t+1)}{C(t)}, \frac{G(t+1)}{G(t)}, R_m(t+1), I(t+1)\frac{P(t+1)}{P(t)}, Z(t)\right)'$$

and

$$\phi_0 = (Q, A, B, D, \mu)'.$$

Then, the moment equations (31), (32), and (33) can be compactly written as

$$E\left[f\left(Y(t), \phi_0\right)\right] = 0 \tag{34}$$

for a function $f : \Gamma \times \Phi \rightarrow \Re^{2m+1}$, where Γ is a subset of $\Re^{m+4}$ and Φ is a compact parameter space whose interior includes ϕ_0.

Define $S_T(\phi_0)$ as $\frac{1}{T}\sum_{t=1}^{T} f(Y(t), \phi_0)$ and consider a $(2m+1 \times 2m+1)$ positive definite matrix V_T. Then, a GMM estimator is

$$\hat{\phi} = \arg\min_{\phi \in \Phi} \left[S_T(\phi)' V_T S_T(\phi)\right].$$

Under some regularity conditions (see Hansen (1982) and Bates and White (1985)), the above estimate is consistent and asymptotically normal. Conditional on a set of instruments, the most efficient estimate is obtained by choosing V_T to be a consistent estimate $(\hat{\Delta})$ for

$$\Delta = \left\{Var\left[\frac{1}{T^{1/2}}\sum_{t=1}^{T} f(Y(t)\phi_0)\right]\right\}^{-1}.$$

Then, defining

$$s(\phi_0) \quad = \quad E\left[\frac{dS(\phi)}{d\phi}\right]_{\phi=\phi_0}, \tag{35}$$

it can be shown that

$$T^{1/2}(\hat{\phi} - \phi_0) \rightarrow N(O, \Sigma) \tag{36}$$

$$\text{where} \qquad \Sigma = \left[s(\phi_0)' \Delta s(\phi_0)\right]^{-1}.$$

Finally, the $(2m + 1 - 5)$ overidentifying restrictions can be tested by means of the statistic

$$Q = T S_T(\hat{\phi}) \Delta S_T(\hat{\phi}), \tag{37}$$

which under the null hypothesis has an asymptotic $\chi^2(2m + 1 - 5)$ distribution.

APPENDIX C

Corrected Standard Errors and Conditional Moment Tests

Define ψ as the vector of conditional mean and variance parameters in the system formed by equations (15) and (17). Call $\phi = (Q, A, B, D, l_0, l_1, l_2)$ the nuisance parameter vector in (14) with the residual specification (22). Define $\hat{\phi}$ as the (quasi) limited information maximum likelihood estimate of ϕ conditional to the VAR-GARCH estimate $\hat{\psi}$. Call $L_t(\phi, \psi|X)$ the gaussian likelihood function of the system formed by the equations corresponding to i, r_m, π, c, and g according to the specifications (14) through (17).

A consistent estimate of the asymptotic variance-covariance matrix of ϕ is then given by

$$H(\hat{\phi}) = \left[\frac{1}{T}\sum_t \frac{\partial^2 L_t}{\partial\phi\partial\phi'}\right]^{-1} \hat{\Delta} \left[\frac{1}{T}\sum_t \frac{\partial^2 L_t}{\partial\phi\partial\phi'}\right]^{-1}, \tag{38}$$

where $\hat{\Delta}$ is a consistent estimate of

$$\Delta = Var\left(\frac{\partial L_t}{\partial \phi}\right). \tag{39}$$

According to Newey and West (1987) , an estimate of Δ which is robust to remaining heteroskedasticity and autocorrelation is

$$\begin{aligned} \hat{\Delta} &= M_0 + \sum_{i=1}^{q} w(i,q)(M_i + M_i'), \\ \text{for} \quad M_i &= \sum_{t=i+1}^{T} \left(\frac{\partial L_t}{\partial \phi}\right)\left(\frac{\partial L_{t-i}}{\partial \phi}\right)' \\ \text{and} \quad w(i,q) &= 1 - \frac{\quad}{q+1}, \quad i = 0, \cdots q. \end{aligned} \tag{40}$$

In order to test the adequacy of the specification of the nominal interest rate equation, I use conditional moment tests (Newey (1985)). Suppose u is the residual from a ML estimation, X_t is the vector of explanatory variables and ν_0 the $f \times 1$ true parameter vector. Define the set of statistics

$$dm_j(X_t, \nu) = \left[\frac{u(t)}{\sigma_u(t-1)}\right]\left[\frac{u(t-j)}{\sigma_u(t-j-1)}\right] \qquad j = 1 \cdots J \tag{41}$$

and

$$dh_s(X_t,\nu) = \left[\frac{u(t)^2}{\sigma_u(t-1)^2} - 1\right]\left[\frac{u(t-l)^2}{\sigma_u(t-l-1)^2} - 1\right] \qquad l = 1\cdots S. \tag{42}$$

By definition, it must be true that

$$E\left(dm_i(X_t,\nu_0)\right) = E\left(dh_s(X_t,\nu_0)\right) = 0.$$

Thus, it can be shown that if the model is correctly specified we can construct a Lagrange multiplier test based on the discrepancy from zero of tbe sample averages of expressions (41) and (42). To test for residual autocorrelation, I regress a vector of T ones on the vectors $\{dm_j(X_t,\hat{\nu}), j = 1,\cdots,J\}$ and the f scores $\left\{\frac{\partial L_t(X_t,\hat{\nu})}{\partial \nu_i},\quad i = 1,\cdots,f\right\}$. Newey has proven that, under the null hypothesis of a correctly specified model, $T \times R^2$ in this regression has a χ^2 distribution with J degrees of freedom. Similarly, we construct a test of the conditional variance specifications using the moment equation (42) and obtain a statistic with a χ^2_S distribution.

Sample Means

	47-88	47-59	60-69	70-79	80-88
i	.136	-.226	.311	-.198	.823
π	1.044	.635	.641	1.727	1.312
c	.438	.297	.625	.477	.388
g	.675	.943	.985	.328	.336
r_m	1.722	3.147	1.349	-.226	2.282

Standard Deviations

	47-88	47-59	60-69	70-79	80-88
i	.716	.774	.253	.459	.654
π	.780	.748	.375	.652	.645
c	.595	.738	.511	.552	.448
g	1.01	1.301	.905	.715	.684
r_m	8.05	5.748	7.115	10.171	8.990

Covariances with Inflation

	47-88	47-59	60-69	70-79	80-88
π	.608	.560	.141	.426	.416
c	-.113	-.077	-.008	-.243	-.163
g	-.313	-.381	-.026	.026	-.275
r_m	-1.40 6	-.871	-.702	-2.027	-.337

Table 1: **SAMPLE STATISTICS**. i is the ex-post real compound return on 1-month treasury bills. c is the consumption growth rate. g is the government expenditure growth rate. π is the Inflation rate. r_m is the real rate of return on the value weighted portfolio (CRSP). All variables are quarterly rates in %.

Moment Equations:

$$E\{Z(t)[\Lambda(t+1)R_m(t+1)-1]\} = 0; \quad E\left\{Z(t)\left[\Lambda(t+1)\frac{P(t)}{P(t+1)} - \frac{1}{I(t+1)}\right]\right\} = 0; \quad E\{\Lambda(t+1)-\mu\} = 0$$

$$\Lambda(t+1) = Q\left[\frac{C(t+1)}{C(t)}\right]^A \left[\frac{G(t+1)}{G(t)}\right]^B R_m(t+1)^D$$

A) GENERAL MODEL

INST SET	Q	A	B	D	μ	IR.P(%)	P.V.
(1)	.9954	.6529	-.0084	-1.526	1.00006	12.16	.09
	(.0020)	(.320)	(.221)	(.042)	(.00059)		
(2)	.9943	1.292	.1333	-1.492	.99876	1.14	.29
	(.976)	(.632)	(.630)	(.079)	(.00034)		
(3)	1.0088	-1.350	-.851	- .317	.99935	6.17	.11
	(.0006)	(.551)	(.411)	(.112)	(.00060)		
(4)	1.03037	-.898	-.357	-3.155	.99887	.52	.14
	(.0150)	(3.57)	(2.79)	(.776)	(.0008)		
(5)	.9954	1.190	.772	-1.861	.99963	8.55	.08
	(.0038)	(.60)	(.49)	(.092)	(.00038		
(6)	1.00435	1.1946	-.994	-1.609	.99945	12.16	.06
	(.0030)	(.453)	(.334)	(.087)	(.00071)		
(7)	1.00613	-.985	.914	-1.963	.99987	.98	.14
	(.0073)	(.781)	(.644)	(.119)	(.00077)		

B) EXPECTED UTILITY MODEL(D=0)

INST SET	Q	A	B	μ	IRP(%)
(1)	1.0292	-2.776	-1.947	.99876	1.13
	(.006)	(1.017)	(.717)	(.00049)	
(2)	1.0293	-4.025	-2.447	.99854	-.69
	(.006)	(.976)	(.631)	(.00034)	
(3)	1.0312	-4.131	-3.201	.99870	.66
	(.018)	(2.57)	(2.02)	(.00064)	
(4)	1.0025	-.1945	-.763	.9943	-1.60
	(.007)	(1.252)	(.423)	(.00076)	

INSTRUMENTS:

(1) : $1, c(-1), c(-2), g(-1), g(-2), R_f(-1), R_m(-1)$

(2) : $1, c(-1), c(-2), g(-1), R_f(-1), R_m(-1)$

(3) : $1, c(-1), g(-1), g(-2), R_f(-1), R_m(-1)$

(4) : $1, c(-1), c(-2), g(-1), g(-2)$

(5) : $1, c(-1), g(-1), R_f(-1), [(c \cdot g/R_m)(-1)]$

(6) : $1, c(-1), c(-2), g(-1), g(-2), R_f(-1), [(c \cdot g/R_m)(-1)]$

(7) : $1, c(-1), g(-1), [(c \cdot g/R_m)(-1)]$

Table 2: **GMM ESTIMATION RESULTS**. Lower case letters represent growth rates. Standard errors are in parentheses. IRP(%) represents the inflation risk premium as a percentage of the average level of the nominal interest rates. Numbers in the P.V. column are probability values of the overidentifying restriction tests.

EQUATIONS:

EQ.1 : $r_m(t) = B_{m0} + B_{m1} r_m(t-1) + U_m(t)$

EQ.2 : $\pi(t) = B_{po} + B_{p1}\pi(t-1) + B_{p2}\pi(t-2) + U_p(t)$

EQ.3 : $c(t) = B_{co} + B_{c1}\pi(t-1) + B_{c2}c(t-1) + B_{c3}g(t-1) + U_c(t)$

EQ.4 : $g(t) = B_{go} + B_{g1}\pi(t-1) + B_{g2}c(t-1) + B_{g3}g(t-1) + U_g(t)$

where $U(t) = (U_m(t) \quad U_p(t) \quad U_c(t) \quad U_g(t))'$, $\quad VarU(t) = W(t)$

and $vecW(t) = T_0 + T_1 vec(U(t-1)U(t-1))' + T_{21} vecW(t-1) + T_{22} vecW(t-2)$

PARAMETER	ESTIMATE	S. ERROR	PARAMETER	ESTIMATE	S. ERROR
B_{m0}	0.208E-01	0.528E-02	$T_0(5,1)$	0.124E-05	0.701E-06
B_{m1}	0.154E-01	0.754E-01	$T_0(6,1)$	-0.562E-05	0.331E-05
B_{p0}	0.168E-02	0.608E-03	$T_0(7,1)$	-0.105E-04	0.170E-05
B_{p1}	0.622	0.812E-01	$T_0(8,1)$	0.317E-04	0.403E-05
B_{p2}	0.210	0.866E-01	$T_0(9,1)$	-0.454E-05	0.514E-05
B_{c0}	0.525E-02	0.101E-02	$T_0(10,1)$	0.872E-04	0.914E-05
B_{c1}	-0.154	0.511E-01	$T_1(1,1)$	0.489E-01	0.621E-02
B_{c2}	0.238	0.658E-01	$T_1(2,2)$	0.152	0.486E-01
B_{c3}	-0.474E-01	0.446E-01	$T_1(5,5)$	0.163	0.573E-01
B_{g0}	0.106E-01	0.197E-02	$T_1(6,6)$	0.150	0.594E-01
B_{g1}	-0.436	0.118	$T_1(7,7)$	0.781E-01	0.337E-01
B_{g2}	-0.183E-01	0.126	$T_{21}(1,1)$	1.695	0.288E-01
B_{g3}	0.960E-01	0.855E-01	$T_{21}(2,2)$	0.568	0.153
$T_0(1,1)$	0.153E-02	0.182E-03	$T_{21}(5,5)$	0.778	0.590E-01
$T_0(2,2)$	-0.262E-04	0.164E-04	$T_{21}(6,6)$	-0.845E-01	0.551
$T_0(3,1)$	0.106E-03	0.380E-04	$T_{21}(7,7)$	-1.061	0.488E-01
$T_0(4,1)$	0.900E-04	0.533E-04	$T_{22}(1,1)$	-0.949	0.288E-01

CONDITIONAL MOMENT TESTS: χ^2 (5) (See appendix B)

	EQ.1	EQ. 2	EQ. 3	EQ. 4
Autocorrelation:	1.8	6.4	4.1	5.5
Heteroskedasticity:	9.2	8.3	4.4	6.9

Table 3: **VAR–GARCH MODEL.** Quasi-Maximum Likelihood estimation results. The conditional moment tests for absence of autocorrelation and heteroskedasticity are described in Appendix B. The critical value at 95% confidence interval is 11.1.

EQUATION:

$$
\begin{aligned}
i(t) &- E_t\pi(t+1) + .5\sigma_{pp}(t) = b_0 + b_1 E_t c(t+1) + b_2 E_t g(t+1) + b_3\sigma_{mm}(t) \\
&+ b_4\sigma_{mp}(t) + b_5\sigma_{pc}(t) + b_6\sigma_{pg}(t) + \epsilon(t) \\
\epsilon(t) &= \eta\epsilon(t-1) + \omega(t) \\
\sigma^2_\omega(t) &= l_0 + l_1\omega(t-1)^2 + l_2\sigma^2_\omega(t-1)
\end{aligned}
$$

	ESTIMATE	SE (1)	SE(2)
b_0	-0.601E-02	0.171E-02	0.284E-02
b_1	0.525	0.866E-01	0.315
b_2	0.738	0.643E-01	0.261
b_3	-0.523E-01	0.111	0.135
b_4	-4.039	1.904	2.111
b_5	-13.05	24.115	37.445
b_6	8.17	11.176	14.456
η	0.897	0.300E-01	0.378E-01
l_0	0.381E-06	0.355E-06	0.379E-06
l_1	0.237	0.822E-01	0.925E-01
l_2	0.699	0.125	0.139

AV. VALUE OF LIKELIHOOD FUNCTION = 778.655

CONDITIONAL MOMENT TESTS $\chi^2(5)$:

-Autocorrelation= 2.8
-Heteroskedasticity=9.1

Table 4: **LOGNORMAL SPECIFICATION. UNRESTRICTED MODEL.** SE(1) is the ordinary standard error obtained from the score of the likelihood function of the nominal interest rate equation. SE(2) corrects for the estimated regressor problem and remaining heteroskedasticity and autocorrelation as it is described in Appendix B.

EQUATION:

$$
\begin{aligned}
i(t) \quad - \quad & E_t\pi(t+1) + .5\sigma_{pp}(t) = b_o + b_1 E_t c(t+1) + b_2 E_t g(t+1) \\
+ \quad & b_3\sigma_{mm}(t) + b_4\sigma_{mp}(t) + b_5\sigma_{pc}(t) + b_6\sigma_{pg}(t) + \epsilon(t) \\
& \epsilon(t) = \eta\epsilon(t-1) + \omega(t) \\
& \sigma_\omega(t) = l_0 + l_1\omega(t-1)^2 + l_2\sigma_\omega(t-1) \\
& b_0 = \alpha_0 - \frac{1}{2(1+D)}\left(A^2\sigma_{cc} + 2AB\sigma_{cg} + B^2\sigma_{gg}\right); b_1 = -\frac{A}{1+D}; \quad b_2 = -\frac{B}{1+D} \\
& b_3 = \frac{D}{2}; \quad b_4 = D; \quad b_5 = A; \quad b_6 = B
\end{aligned}
$$

	ESTIMATE	SE(1)	SE(2)
α_0	-0.522E-02	0.167E-02	0.262E-02
A	-0.467	0.150	0.224
B	-0.635	0.172	0.277
D	-0.981E-01	0.233	0.377
η	0.892	0.314E-01	0.503E-01
l_0	0.351E-06	0.309E-06	0.428E-06
l_1	0.220	0.727E-01	0.805E-01
l_2	0.724	0.109	0.119

AV. VALUE OF LIKELIHOOD FUNCTION= 777.00313. (P.V.=.07)

CONDITIONAL MOMENT TESTS $\chi^2(5)$:

-Autocorrelation= 3.5
-Heteroskedasticity=10.1

Table 5: **LOGNORMAL SPECIFICATION. RESTRICTED MODEL.**

	ESTIMATE	SE(1	SE(2)
α_0	-0.641E-02	0.159E-02	0.275E-02
A	2.062	1.308	2.135
B	2.993	1.985	3.076
D	-4.992	2.311	2.359
η	0.894	0.279E-01	0.136E-01
l_0	0.511E-06	0.426E-06	0.033E-05
l_1	0.264	0.129	0.272E-01
l_2	0.645	0.119	0.138

AV. VALUE OF LIKELIHOOD FUNCTION= 778.37 (P.V.=.25)

CONDITIONAL MOMENT TESTS $\chi^2(5)$:

-Autocorrelation= 3.8
-Heteroskedasticity=10.8

Table 6: **LOGNORMAL SPECIFICATION. RESTRICTED MODEL.** σ_{mm} is omitted ($b_3 = 0$).

EQUATION:

$$
\begin{aligned}
i(t) &= F\left[E_t\pi(t+1) - .5\sigma_{pp}(t)\right] + b_o + b_1 E_t c(t+1) + b_2 E_t g(t+1) + b_3\sigma_{mm}(t) \\
&+ b_4\sigma_{mp}(t) + b_5\sigma_{pc}(t) + b_6\sigma_{pg}(t) + \omega(t) \\
\sigma_\omega(t) &= l_0 + l_1\omega(t-1)^2 + l_2\sigma_\omega(t-1) \\
b_0 &= a_0 - \frac{1}{2(1+D)}\left(A^2\sigma_{cc} + 2AB\sigma_{cg} + B^2\sigma_{gg}\right); b_1 = -\frac{A}{1+D}; \quad b_2 = \frac{B}{1+D} \\
b_3 &= \frac{D}{2}; \quad b_4 = D; \quad b_5 = A; \quad b_6 = B
\end{aligned}
$$

	ESTIMATE	SE(1)	SE(2)
a_0	-0.311E-02	0.360E-02	0.679E-02
A	-0.731	0.186	0.327
B	-0.224	0.277	0.538
D	0.120	0.127	0.194
F	0.841	0.166	0.323
l_0	0.157E-05	0.651E-06	0.960E-06
l_1	0.787	0.255	0.301
l_2	0.203	0.894E-01	0.983E-01

LIKELIHOOD RATIO TEST for Ho (F=1) = 1.20 (P.V.=.25)

Table 7: **TEST OF THE FISHER HYPOTHESIS. GENERAL RESTRICTED MODEL.**

	ESTIMATE	SE(1)	SE(2)
a_0	-0.299E-02	0.372E-02	0.700E-02
A	-0.629	0.146	0.283
B	-0.237	0.256	0.488
F	0.873	0.166	0.316
l_0	0.167E-05	0.611E-06	0.882E-06
l_1	0.760	0.252	0.292
l_2	0.215	0.835E-01	0.872E-01

LIKELIHOOD RATIO TEST FOR Ho (F=1) =.70 (P.V.=.40)

Table 8: **TEST OF THE FISHER HYPOTHESIS. EXPECTED UTILITY MODEL.** (D=0)

	ESTIMATE	SE(1)	SE(2)
a_0	-0.557E-03	0.108E-02	0.195E-02
A	-0.713	0.178	0.291
D	0.140	0.118	0.182
F	0.719	0.468E-01	0.109
l_0	0.137E-05	0.638E-06	0.927E-06
l_1	0.799	0.244	0.304
l_2	0.214	0.906E-01	0.1043

LIKELIHOOD RATIO TEST FOR Ho (F=1) = 10.46 (P.V.=0.0)

Table 9: **TEST OF THE FISHER HYPOTHESIS. MODEL SEPARABLE IN GOV. EXPENDITURE** (B=k=0)

OBSERVED INTEREST RATES

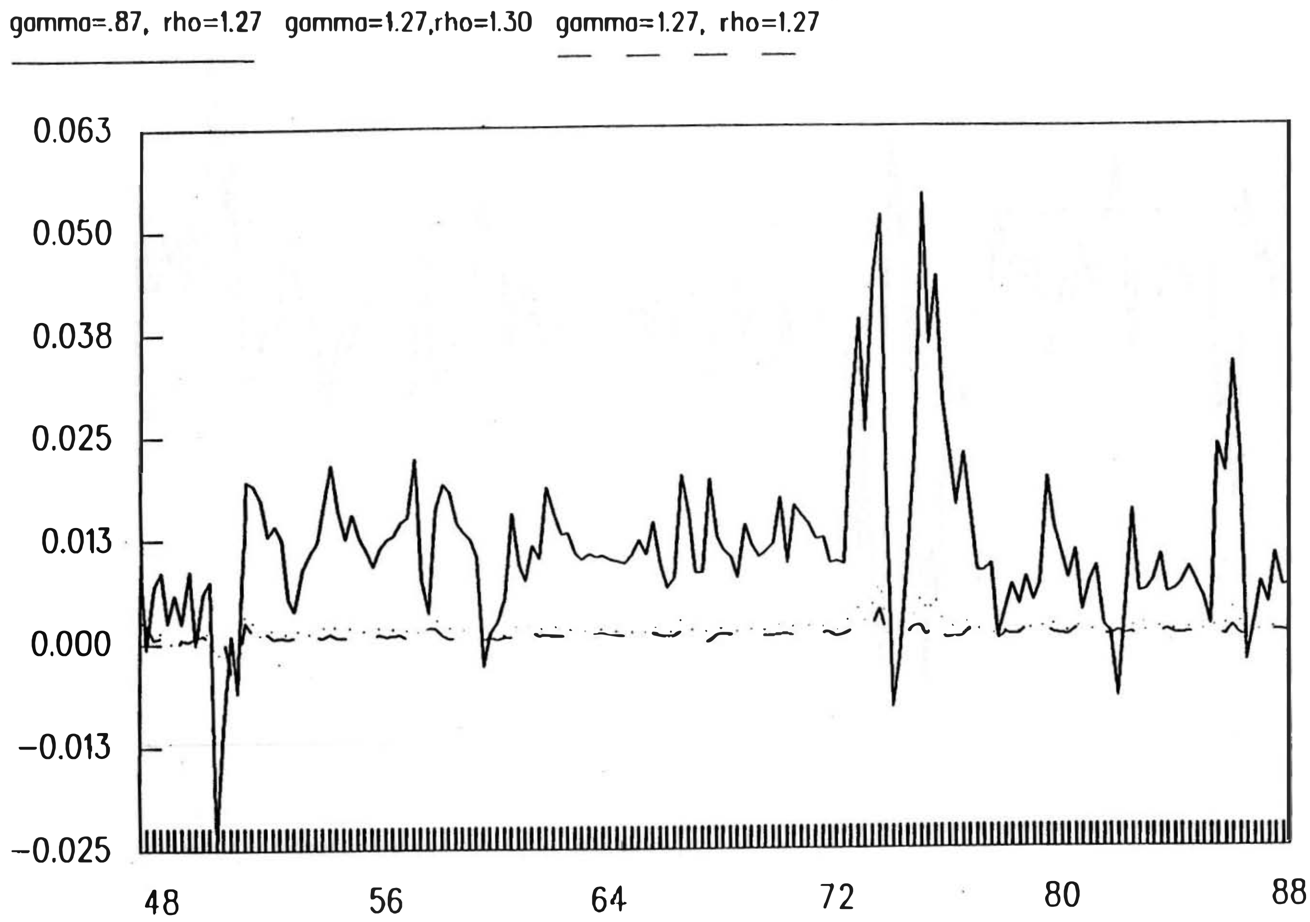
	Nominal Interest Rate	Ex-ante Real Interest Rate
MEAN=	1.1996	.1667
SDEV=	.7787	.6015
C1=	.9623	.8064
C2=	.9231	.6548

FITTED EX-ANTE REAL INTEREST RATES

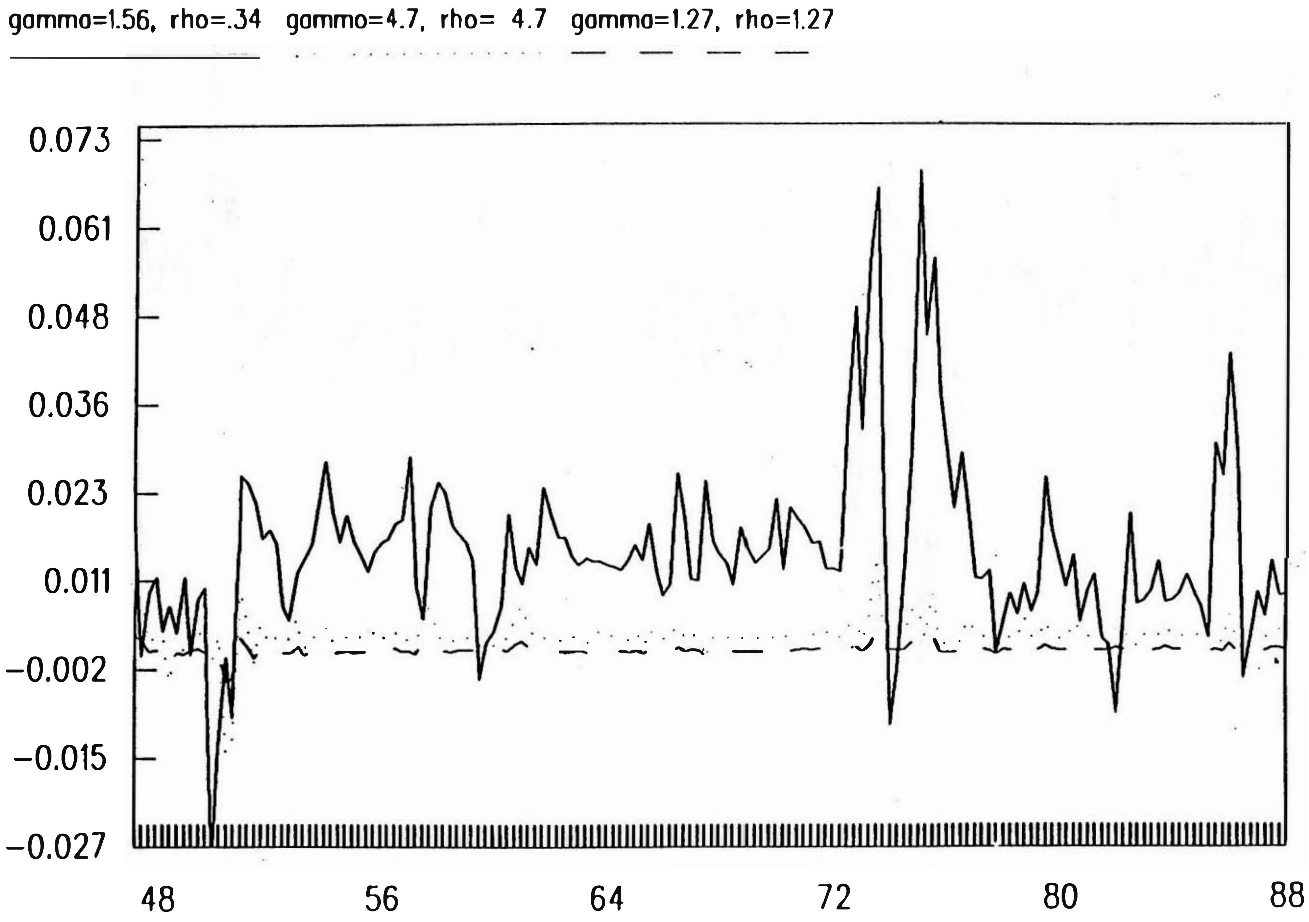
γ	ρ	k	*mean*	*sdev*	c1	c2	*corr*	*rf*	*irp*
1.56	0.34	0.0	-0.078	0.287	0.871	0.622	0.10115	-0.092	0.0142
1.56	0.34	0.3	0.165	0.350	0.847	0.640	0.16005	0.151	0.0143
1.56	0.34	0.5	0.344	0.412	0.829	0.647	0.18425	0.332	0.0141
0.87	1.27	0.0	0.462	0.375	0.683	0.489	0.21859	0.455	0.0113
0.87	1.27	0.3	0.313	0.337	0.665	0.465	0.19654	0.303	0.0112
0.87	1.27	0.5	0.204	0.317	0.656	0.455	0.17213	0.191	0.0110
1.27	1.30	0.0	0.970	0.267	0.427	0.274	0.28910	0.962	0.0014
1.27	1.30	0.3	0.998	0.271	0.495	0.341	0.28869	0.993	0.0014
1.27	1.30	0.5	1.011	0.277	0.544	0.390	0.28587	1.011	0.0014
4.7	4.70	0.0	2.461	0.948	0.408	0.257	0.29229	2.411	0.0025
4.7	4.70	0.3	2.730	1.023	0.617	0.470	0.27815	2.721	0.0026
4.7	4.70	0.5	2.921	1.150	0.716	0.589	0.25213	2.932	0.0027
1.27	1.27	0.0	0.992	0.256	0.408	0.257	0.29229	0.991	0.0007
1.27	1.27	0.3	1.012	0.258	0.468	0.314	0.29217	1.022	0.0007
1.27	1.27	0.5	1.021	0.261	0.511	0.357	0.29017	1.012	0.0007

Table 10: **UNCONDITIONAL MOMENTS OF OBSERVED AND SIMULATED INTEREST RATES.** Ex-ante real interest rate is defined as $i(t)$-$E_t(\pi(t+1))$ $+.5\sigma_{pp}(t)$. $c1$ and $c2$ are the first and second order autocorrelation coefficients. *Corr* is the correlation coefficient between fitted and real series. rf is the average estimated riskfree rate. irp is the average inflation risk premium. The preference parameter Q is assumed to be .99. All rates are quarterly data and are measured in %.

GRAPH 1: INFLATION RISK PREMIUM



GRAPH 2: INFLATION RISK PREMIUM



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(1) Los Documentos de Trabajo anteriores a 1988 figuran en el catálogo de publicaciones del Banco de España.

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