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Abstract

We estimate inflation risk-neutral densities (RNDs) in the Euro area since 2009. We use Euro inflation swaps and caps/floors options, and introduce a simple and parsimonious approach to jointly estimate the RNDs across horizons. This way, we obtain the implicit RND for forward measures, like the five-on-five years inflation rate, which, although it is not directly traded in the market, it is a key rate for monetary policy. Then, we discuss several indicators derived from the information content of the historical RNDs that are useful for monetary policy and compare them in the light of the ECB's decisions and communication over the last few years. Specically, the evolution of tails risks (associated with deflation and high inflation); the balance of inflation risks; measures of risk aversion from the ECB's Survey of Professional Forecasters (SPF); and how forward inflation rates react to the ECB's non-conventional monetary policies (Longer Term Renancing Operations, LTRO, Securities Market Programme, SMP, Asset Purchase Programme, APP, and its variants and extensions).

Keywords: inflation compensation, inflation options, risk-neutral densities, inflation risk aversion, balance of inflation risks.

JEL classification: E31, E44, G13.

Resumen

En este documento de trabajo estimamos, para la inflación, las funciones de densidad neutrales al riesgo (RND) en la zona del euro diariamente desde 2009. Para ello, utilizamos *swaps* de inflación y opciones *calls/puts,* e introducimos un enfoque simple y parsimonioso para estimar conjuntamente las RND en distintos horizontes temporales. De esta manera, es posible obtener las RND implícitas para medidas *forward,* como la inflación a cinco años en cinco años, que, aunque no se negocian directamente en los mercados financieros, son indicadores de referencia para la política monetaria. Así, una vez obtenidas estas medidas, discutimos varios indicadores derivados de estos RND que pueden ser útiles para la política monetaria y estudiamos su evolución histórica a la luz de las decisiones y las comunicaciones del BCE durante los últimos años. Especialmente interesante es la evolución de los riesgos en las colas (asociadas con las probabilidades de deflación y de alta inflación, respectivamente); el balance de los riesgos de inflación; medidas de aversión al riesgo derivadas de la Encuesta a Expertos en Previsión Económica (SPF) del BCE, y la forma en que las tasas de inflación anticipada reaccionan a las políticas monetarias no convencionales del BCE (operaciones de financiación a plazo más largo, LTRO; Programa para los Mercados de Valores, SMP; Programa de Compra de Activos, APP, y sus variantes y extensiones).

Palabras clave: compensación por inflación, opciones de inflación, distribuciones neutrales al riesgo, aversión al riesgo de inflación, balance de riesgos de inflación.

Códigos JEL: E31, E44, G13.

1 Introduction

This paper use inflation-linked derivatives to recover the full spectrum of Risk-Neutral Densities (RNDs; a Q–measure), and derive indicators that are specially useful for monetary policy purposes. Inflation swaps (ISL, since April 2004) and options (caps and floors since April 2009) are regularly traded in the euro area, and are more liquid than in other similar markets (e.g., the UK or US). The maturity of these instruments ranges from 1 to more than 10 years, and their prices contain precious information from which it is possible to recover these RNDs.

A simple approach is estimating, from the most liquid strikes, the RNDs of all maturities (i.e., in the euro area, ten different RNDs for horizons spanning 1 to 10 years). From these RNDs, we can derive inflation moments as well as other important indicators for those horizons of interest. In trading rooms, practitioners usually do a similar fitting exercise. An equity model is calibrated to index calls and puts (and to yield curve and dividend inputs), using the full spectrum (i.e., different maturities) of liquid options. Then, the model is used to price exotic securities, like path-dependent or forward-starting options to name a few. This approach has two advantages; (1) it is parsimonious, since only one model is calibrated (over-fitting concerns are diminished), and (2) consistency, since all exotic securities are priced from the same model.

In this paper, we follow a similar approach to estimate the euro area inflation RNDs: We calibrate a single model (a Gaussian 1st-order autoregressive process) for euro inflation by using swap rates and all liquid options (across different strikes and maturities). In doing so, we depart from the literature, that fits a density for each maturity independently (e.g., Smith, 2012; Kitsul and Wright, 2013; Scharnagl and Stapf, 2015; Fleckenstein et al., 2017). The main advantage of our full spectrum approach is that, from the Q dynamics we calibrate, it is possible to derive (in closed-form or by numerical integration) any inflation measure. Specifically, we can focus on the forward 5-on-5 year inflation rate, which is the main metric usually tracked for medium term inflation expectations by the ECB (Draghi, 2014), as well as other monetary policy–makers. The reason for this is that when policy–makers use an inflation targeting, they have to avoid that supply shocks to short-term inflation expectations (like oil and food prices, indirect tax changes, terms of trade shocks, interest rate changes) affect the monetary policy assessment (Bernanke and Mishkin, 1997). A way to do so, is to concentrate the focus on forward measures, that cancel this undesired short-run noise.

The estimation of RNDs is especially challenging in the case of inflation compensations, since it is limited by the characteristics of available option data (sample size, limited number of strikes, and reduced liquidity). By contrast, an advantage of the over-the-counter nature of these options is that it implies that the maturity of all contracts that we use is nicely constant along the sample period. The underlying variable for all contracts is an index price level, the HIPCxT, and two types of options exist. That is, a single option, which is based on a zero coupon cap/floor, and year-on-year options, based on a portfolio of caplets/floorlets on zero coupons. Like other works and markets, we focus on the former because they are the most liquid ones. For the same reason, we only calibrate low strike caps and high strike floors, because the most liquid options are out-of-the-money (e.g., Aït-Sahalia and Lo, 1998; Jiang and Tian, 2005), implying that the put-call parity relationship is often violated. For at-the-money options, put-call parity is less of a concern.

An inflation-linked swap, or ILS, is a forward contract on the index price level. They are more liquid than caps/floors with the same maturity. Because of this, we calibrate the model in two steps: First, we get the model's first moment exclusively from ILS. Second, we calibrate the rest of the parameters from out-of-the-money caps/floors. This is a robust approach reducing the sensitivity of RNDs to the quality of the option data. This practice is similar to the calibration of equity models, where yield-curve and dividend-yield inputs are directly plugged into the model.

The model assumes that inflation compensation is Gaussian. Although the lack of asymmetry and kurtosis of this approach may look conflicting, Jarrow and Yildirim (2003), Mercurio (2005), and Fleckenstein et al. (2017) also use normal densities for inflation option-implied distributions. Away from the options literature, treating inflation compensations as Gaussian is standard in the literature (e.g., Ang et al., 2007, 2008; Christensen et al., 2010; Chernov and Mueller, 2012; Haubrich et al., 2012; Christensen et al., 2016, among many others).

This paper is part of a growing literature on inflation expectations as a main driver of monetary policy, and is specially relevant now, given the deflationary scenarios from 2008 to the present. Deflation, as well as the high inflation counterpart, is seen as a major threat to growth and price stability with uncertain consequences on the economic activity (e.g., Fisher, 1933; Hamilton, 1992; Cecchetti, 1992; Atkeson and Kehoe, 2004; Killian and Manganelli, 2007). Since the papers of Jarrow and Yildirim (2003), and Mercurio (2005) that established a framework for pricing inflation options, and option data availability since mid–2009, the literature on inflation expectations have started to pay attention to these derivatives. Smith (2012) look at RNDs estimation for UK inflation. Kitsul and Wright (2013) complete a similar study on US inflation (but also using inflation time-series data), Fleckenstein et al. (2017) focuses more on inflation risk-premium (by using a market approach borrowed form term-structure models), and Scharnagl and Stapf (2015) address euro area inflation (using similar option prices and SPF data to ours). However, all these papers are related with a single maturity and do not pay special attention to long-term forward inflation rates (e.g., the five-on-five inflation rate), that requires to jointly analize options at different maturities and impose some structure into the relationship between them. This is precisely our main focus and novelty, given their relevance for monetary policy purposes.

Secondly, we relate all these estimations to an event study. Specifically, we consider how the monetary policy channel, via changes on the ECB's non-conventional monetary policy since 2010 (e.g., Kilponen et al., 2015; Speck, 2016; Altavilla et al., 2017), has produced changes in the inflation compensation RNDs (i.e., means, variances, correlations, probabilities of deflation and high-inflation scenarios, and balances of risks).

Our main findings are that deflation risk-neutral probability is time-varying, responds to different market events (e.g., the January 2015 ECB's expanded asset purchase programme), and is highly priced by the market in some problems (e.g., if associated to stagnation). On the other side, the scenario of high inflation is also time-varying and has been becoming smaller in magnitude and less persistent in recent years. We have observed that non-conventional monetary policy produced different outcomes on the RNDs of inflation compensation, from the initial SMP to the later CSPP programmes, highlighting that the design of the programme is relevant depending on the objective policy–makers have.

The paper is organized as follows. In section 2, we explore the different type of financial assets whose cash-flows depend on inflation, and whose price can be used to estimate market inflation expectations. In section 3, we present the link between derivative prices and inflation and how to extract risk-neutral measures of inflation expectations. In section 4, we show how the measures proposed in the previous sections can be used to analyze the evolution of inflation risk and expectations in the case of the euro area and we conduct an event study for the effects of the different non-conventional monetary policy measures adopted by the ECB. In section 5, we discuss the relationship between the risk neutral and the objective measures of inflation risk, and propose a measure of risk aversion. Finally, in section 6, we sum up the main findings.

2 Financial assets linked to Inflation Expectations

Financial asset prices aggregate information on investors' expectations about the evolution of market drivers, such as economic growth, commodity prices, interest rates, or inflation. The later case is specially relevant for monetary policy purposes, since inflation expectations influence the determination of salaries and prices, that finally will drive the actual inflation evolution. For this reason, monetary policy decision–makers actively monitor inflation markets, since their high frequency is useful to provide early risk warnings of deviations from inflation policy targets.

To extract market inflation expectations, research has mainly focused on two financial markets: inflation-linked bonds and inflation swaps. The market of inflation–linked bonds (ILB) is specially active in the US, where Treasury Inflation-Protected Securities (TIPS) are issued by the US Treasury in a sufficient volume to make the market liquid enough to ensure a smooth price formation. By contrast, in Europe, inflation–linked bonds are fragmented among regular issuances by French, German, Italian Treasuries, and less frequent ones by other governments like Greek; and, since 2014, Spanish Treasuries. Under an inflation–linked bond, cash–flows match the evolution of realized inflation. Therefore, differences in yields–to–maturity of nominal bonds (where cash–flows are predetermined in advance) and inflation–linked bonds (where cash-flows are inflation dependent) give an indication on the compensation that investors are willing to pay for the protection of their investments from inflation. In purity, this cannot be considered strictly as inflation expectations, since yields–to–maturity also include a term premium to compensate for the growing uncertainty in the future market evolution in the longer horizons (i.e., it can only be considered inflation expectations under a risk neutral valuation). Furthermore, differences between nominal and inflation-linked bonds will be also consequence of different liquidity premia between both types of bonds (the traded volume of nominal bonds is much higher than the one of ILBs, and as a consequence, the liquidity is also higher; thus, the measure of inflation compensation includes a differential liquidity premium that bias the signal). Finally, given that ILB coupons are paid in a fixed day, and the seasonal nature of inflation, some seasonality correction in the price of ILBs is needed, adding an additional layer of measurement error to the signal on inflation expectations.

Inflation Linked Swaps (ILS), on the other side, are more liquid in Europe than in the US. ILS are private contracts (traded over the counter), where the buyer of protection agrees to pay a fixed amount of money in exchange of receiving another amount of money that is linked to a price index. For instance, let say that in a 1-year ILS, the fixed part of the contract agree to pay a 2% of 1 million euros in exchange of a proportion of 1 million euros equivalent to the euro area growth rate of the price index (e.g., the euro area Harmonized Index of Consumer Prices ex–Tobacco, HIPCxT) published next year. At the end of the contract horizon (i.e., 1 year), if inflation is above 2%, the buyer of inflation protection will receive the difference, but if inflation is bellow the 2% threshold, it will have to pay that difference. The main advantage of ILS is that they provide a clearer signal on inflation compensation than ILB, since there is no liquidity distortion between nominal and real assets. Other advantage of (offered) prices of over–the–counter assets is that maturity is fixed relatively to the trading day (e.g., 1 year from now instead of a given day such as in January, 1st 2017), so there is no need for seasonality corrections. Nevertheless, although credit risk distortions are mitigated by the absence of ex–ante transfers of money (in contrast with ILBs), they are not totally free of them, since there is no Central Clearing Counterparty and all contracts are bilateral (a small collateral credit risk might apply). As in the case of ILBs, ILS rates can be considered inflation expectations only under the risk–neutral measure (Q–measure), since under an objective measure (P–measure) they may include a term premium to account for the uncertainty growing with the horizon that the ILS is protecting from inflation. So, both in ILS and ILB markets, the Q–measure is also known as inflation compensation, which includes the objective inflation expectation (P–measure) and the term premium.

Finally, inflation options (i.e., Caps and Floors) provide protection if the price index moves above or below (cap and floor options, respectively) a given threshold (i.e., the strike price or rate). As in the case of ILS, they are traded over–the–counter, without a Central Clearing Counterparty that might reduce the collateral credit risk. The market for Inflation Options is more developed for the euro area inflation than for other currency areas, in line with the development of ILS that are used as the underlying asset (Smith, 2012). Market contributors provide information on cap and floor options for both zero–coupon (single option with different maturities), and year–on–year options (portfolio of zero–coupon caps (caplets) and floors (floorlets) with periodical maturities as in a coupon bond, that can be considered a portfolio of strips).[1] Inflation options with different strike rates give additional information about the uncertainty/risk surrounding the mean rate, potentially producing a full density distribution.

3 Derivatives linked to Price Indexes

We denote by I_t the price level at time t; by $\pi_{t,t+\tau}$ the annual inflation rate between time t and $t+\tau$ (see equation 1);[2] Q_τ is the τ–forward measure, which we refer as the risk–neutral measure; and $\mathcal{P}_\tau$ denotes the corresponding objective measure.

$$\frac{I_{t+\tau}}{I_t} = (1 + \pi_{t,t+\tau})^\tau \ , \ \tau \geq 0 \tag{1}$$

3.1 Inflation swaps and options

We consider two types of inflation–linked derivatives: swaps and options (caps/floors). For zero–coupon inflation swaps,[3] f_τ denotes the fixed rate necessary to build a par swap against a leg on zero–coupon appreciation on the euro area HIPCxT. In this swap contract, the payout at maturity τ will be,

$$(1 + f_\tau)^\tau \ \text{ vs. } \ \frac{I_\tau}{I_0} - 1. \tag{2}$$

In a par swap, the price is zero. Therefore, both legs in equation 2 are expected to be equivalent under the risk-neutral measure (Q_τ). Therefore, it holds that

$$(1 + f_\tau)^\tau = E_t^{Q_\tau} \left[(1 + \pi_\tau)^\tau \right], \tag{3}$$

[1] Although 26 floor prices (-1,00%; -0,75%; -0,50%; -0,25%; 0,00%; 0,25%; 0,50%; 0,75%; 1,00%; 1,25%; 1,50%; 1,75%; 2,00%; 2,25%; 2,50%; 2,75%; 3,00%; 3,25%; 3,50%; 3,75%; 4,00%; 4,25%; 4,50%; 4,75%; 5,00%; 6,00%) and 28 cap prices (-2,00%; -1,50%; -1,00%; -0,75%; -0,50%; -0,25%; 0,00%; 0,25%; 0,50%; 0,75%; 1,00%; 1,25%; 1,50%; 1,75%; 2,00%; 2,25%; 2,50%; 2,75%; 3,00%; 3,25%; 3,50%; 3,75%; 4,00%; 4,25%; 4,50%; 4,75%; 5,00%; 6,00%) can be found, most of them do not change in a daily basis, so we restrict the analysis to the 5 floor (-1%; -0.5%; 1%; 2% and 3%) and 6 cap (1%; 2%; 3%; 4%; 5%; and 6%) zero-coupon prices that change daily, and are, therefore, more reliable.

[2] When possible, we denote $\pi_{t,t+\tau}$ by π_τ.

[3] We do not consider in the paper year–on–year swaps where a set of periodical cash–flows are paid, because they have a considerable lower level of trading activity compared to zero–coupon ones, and their treatment add and additional layer of complexity.

which is an expectation (under the Q_τ–risk–neutral measure) of the annual inflation rate.

In the case of inflation options, zero-coupon caps and floors (p) are European options, similar to calls and puts respectively, on inflation.[4] The price of a cap and a floor with maturity τ and strike k ($c_t(\tau,k)$ and $p_t(\tau,k)$ respectively) will be the discounted (risk–neutral) expected payout, that is,

$$\begin{aligned} c_t(\tau,k) &= P(t,t+\tau) \times E_t^{Q_\tau}\left[\max\left\{(1+\pi_\tau)^\tau - (1+k)^\tau, 0\right\}\right], \\ p_t(\tau,k) &= P(t,t+\tau) \times E_t^{Q_\tau}\left[\max\left\{(1+k)^\tau - (1+\pi_\tau)^\tau, 0\right\}\right]. \end{aligned} \tag{4}$$

where $P(t,t+\tau)$ is the zero–coupon bond price between t and $t+\tau$. The equivalences between the observed values of swap and option prices and the expectations operators in equations 3 and 4 allow to infer the distribution under the risk neutral Q_τ measure of the inflation rate π_τ.

3.2 Risk-neutral probabilities implied in inflation options

We use a simple approach to get risk-neutral cumulative probabilities. Taking derivatives in equation 4 with respect to the strike price, we obtain

$$\begin{aligned} -P(t,t+\tau)^{-1} \times \tfrac{dc_t}{dK} &= E_t^{Q_\tau}\left[1_{\{\pi_\tau \geq k\}}\right] \quad \text{and} \\ P(t,t+\tau)^{-1} \times \tfrac{dp_t}{dK} &= E_t^{Q_\tau}\left[1_{\{\pi_\tau < k\}}\right], \end{aligned} \tag{5}$$

where $K = (1+k)^\tau$; $E_t^{Q_\tau}\left[1_{\{\pi_T \geq k\}}\right]$ is the probability under Q_τ of the annual inflation rate between t and $t+\tau$ of being equal or above the strike rate k; and $E_t^{Q_\tau}\left[1_{\{\pi_\tau < k\}}\right]$ is the probability under Q_τ of the annual inflation rate between t and $t+\tau$ of being below the strike rate k. Both events are complementary, so the sum of their probabilities is equal to one. Therefore, from equation 5, we know that

$$P(t,t+\tau)^{-1} \times \left(\frac{dp_t}{dK} - \frac{dc_t}{dK}\right) = 1. \tag{6}$$

These math derivatives are the prices of binary options and, in particular, the Q–forward cumulative probability, which is a nonparametric (i.e., model–free) result. We will use the equality in equation 6 to reassure that option prices are error free. In

[4] As in the case of swaps, year–on–year options have less activity than the zero-coupon counterparts.

particular, we can redefine and force that the two math derivatives hold the previous constraint; i.e.,

$$\begin{aligned} &\text{from } P(t,t+\tau)^{-1} \times \frac{dp_t}{dK} \quad \text{to} \quad \frac{P(t,t+\tau)^{-1} \times \frac{dp_t}{dK}}{P(t,t+\tau)^{-1} \times \left(\frac{dp_t}{dK} - \frac{dc_t}{dK}\right)} = \frac{\frac{dp_t}{dK}}{\frac{dp_t}{dK} - \frac{dc_t}{dK}} \quad \text{and} \\ &\text{from } P(t,t+\tau)^{-1} \times \frac{dc_t}{dK} \quad \text{to} \quad \frac{\frac{dc_t}{dK}}{\frac{dp_t}{dK} - \frac{dc_t}{dK}}, \end{aligned} \tag{7}$$

where the discount factor cancels (and hence, it is unnecessary to consider it). Since the number of strike prices is discrete, these derivatives are approximated numerically; i.e.,

$$\begin{aligned} E_t^{Q_\tau}\left[1_{\{\pi_\tau < k\}}\right] &\approx 1 + P(t,t+\tau)^{-1} \times \frac{c_t(k+\Delta) - c_t(k-\Delta)}{2\Delta}, \text{ or} \\ E_t^{Q_\tau}\left[1_{\{\pi_\tau < k\}}\right] &\approx P(t,t+\tau)^{-1} \times \frac{p_t(k+\Delta) - p_t(k-\Delta)}{2\Delta}. \end{aligned} \tag{8}$$

If we combine equations 7 and 8, we obtain

$$\begin{aligned} E_t^{Q}\left[1_{\{\pi_T < K\}}\right] &\approx 1 + \frac{\frac{c_t(K+\Delta) - c_t(K-\Delta)}{2\Delta}}{\frac{p_t(K+\Delta) - p_t(K-\Delta)}{2\Delta} - \frac{c_t(K+\Delta) - c_t(K-\Delta)}{2\Delta}}, \text{ or} \\ E_t^{Q}\left[1_{\{\pi_T < K\}}\right] &\approx \frac{\frac{p_t(K+\Delta) - p_t(K-\Delta)}{2\Delta}}{\frac{p_t(K+\Delta) - p_t(K-\Delta)}{2\Delta} - \frac{c_t(K+\Delta) - c_t(K-\Delta)}{2\Delta}}, \end{aligned} \tag{9}$$

which does not depend on $P(t,t+\tau)$.

3.3 Risk-neutral parametric distribution of Inflation

The procedure showed in section 3.2 produces cumulative probabilities from observed options. Nevertheless, they are only useful if there are options traded for the desired maturity and a sufficient number of strike rates (k) to recover a full density distribution. In the case of the euro area inflation, although it is the most traded underlying inflation index, we have a reduced number of strike prices available (see figure 1). So, for any given day and maturity, using equation 9, we get 4 probabilities from floor prices and 5 from cap prices, although combined we just have 7 probabilities, since 2 caps and 2 floors coincide for the same strikes (see figure 2).[5]

[5]For the strikes where it is possible to obtain the cumulative probability from both caps and floors, we opt to use the one (either cap or floor) that is out of the money, since that option should be more traded, and the price, more reliable.

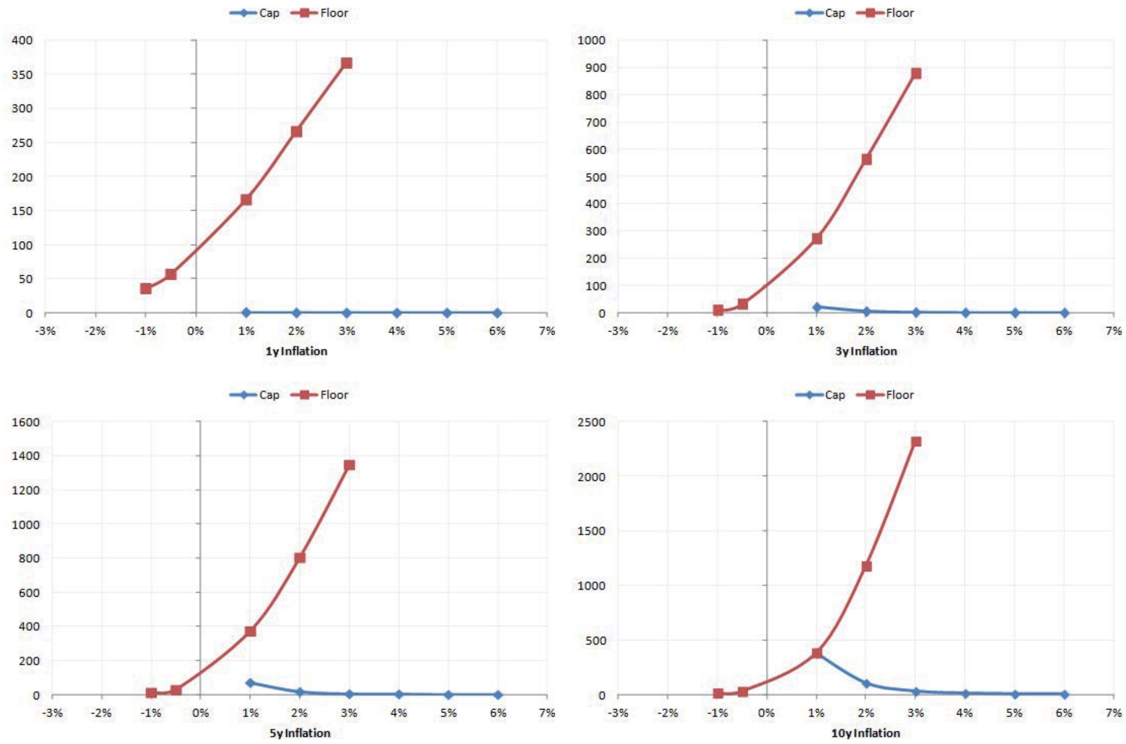

Figure 1: Cap (Blue) and Floor (Red) inflation option prices for 1 year (up-left), 3 year (up-right), 5 year (down-left) and 10 years (down-right) maturities. January, 15th 2015.

As can be seen in figure 2, for those cases where we have both cap and floor prices for the same strike prices, probabilities obtained from the options that are deeply in–the–money (floors in the case of the example showed in figure 2) tend to provide worse approximations to actual probabilities (producing values that are even above 100%). This is a consequence of the illiquid nature of in–the–money options, which increases the measurement error contained in their reported prices. Thus, and as is standard in the literature (e.g., Aït-Sahalia and Lo, 1998; Jiang and Tian, 2005), whenever we have both caps and floors, we only use the ones that are out–of–the–money.

A further step to better take advantage of these probabilities is to interpolate them using an appropriate density distribution. This is basically the calibration exercise that traders do everyday with vanilla options (e.g., in equity markets). Nevertheless, the reduced number of observed probabilities limit the possibilities for estimating a full density model. We have opted for considering that the annual inflation growth rate follows a simple Gaussian distribution, like in Jarrow and Yildirim (2003), Mercurio (2005), or Fleckenstein et al. (2017). This assumption has clear advantages:

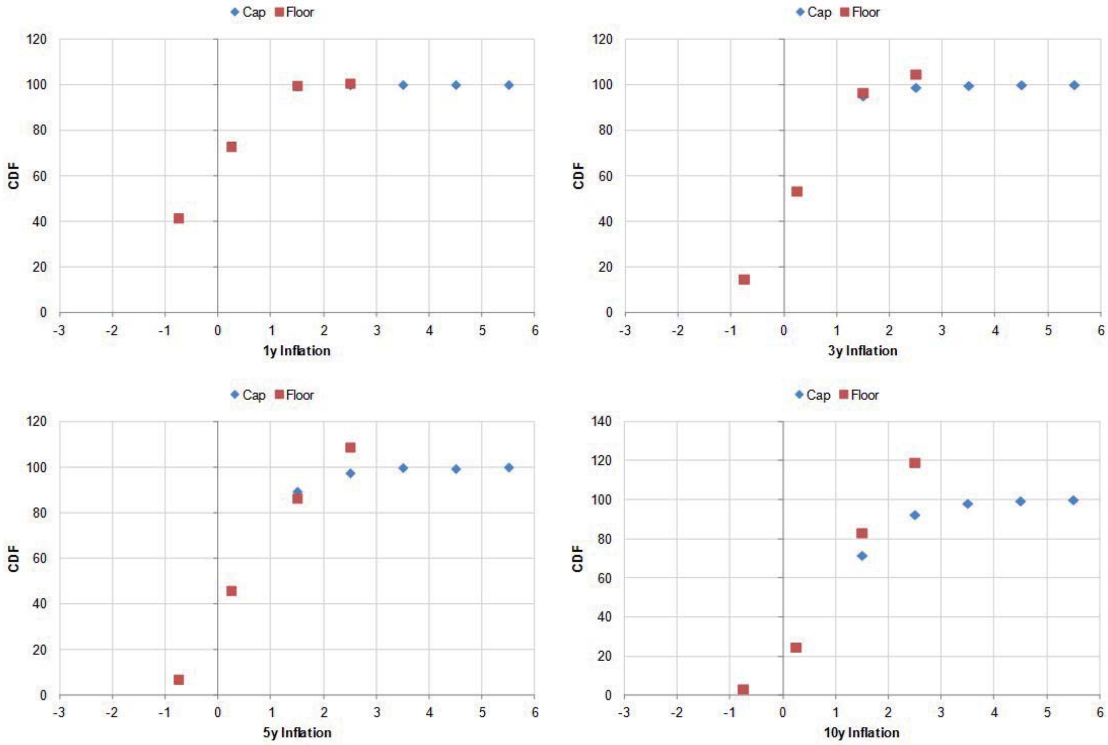

Figure 2: Cumulative probabilities derived from Cap (Blue) and Floor (Red) inflation option prices for 1 year (up-left), 3 year (up-right), 5 year (down-left) and 10 years (down-right) maturities. January, 15th 2015.

inflation values can be both positive and negative, and we just need two parameters to characterize any maturity.

In order to derive implicit forward measures of inflation, we depart from the literature (e.g., Smith, 2012; Kitsul and Wright, 2013; Scharnagl and Stapf, 2015) by fitting a single risk-neutral model to the whole maturity spectrum of inflation options available. For instance, prices in 2–year zero-coupon inflation options contain information not only about the inflation compensation for the second year, but also about the inflation compensation for the first year that, combined with 1–year inflation options, can contribute for a better estimation of RNDs for both 1–year and 2–year inflation compensations. We can go as far as to the 10 year inflation options, which contains information on inflation rates for each and every one of the next ten years. Therefore, we approximate the Q–measure by a Multivariate Gaussian Distribution,

$$\mathcal{M}(\Pi) = \Phi\left(\Pi, M, \Sigma\right), \tag{10}$$

where M, is a vector of the mean of inflation compensations for each year and Σ is the variance-covariance matrix. Main challenge to estimate (10) is the number of parameters involved. In a multivariate context, the number of parameters dramatically increase with the dimensions (i.e., periods). Thus, for p dimensions, the number of parameters to estimate would be p means, p variances, and most importantly $p(p-1)/2$ covariances. That is, in the case of maturities ranging from 1 to 10 years, we need to estimate 65 parameters (10 means, 10 variances, and 45 covariances), where the number of available data is just around 7 point estimates of the cumulative probability per each maturity.

In order to alleviate the over-parametrization problem, we have opted for imposing two restriction strategies:

1. We consider that inflation follows an AR(1) process, thus reducing all coefficients in Σ to just two parameters: the variance of the inflation in the first year (σ_1), and the correlation of inflation between two consecutive years (ρ). From those two parameters, it is possible to recover every variance (equation 11) and covariance (equation 12) elements of Σ matrix.

$$\sigma_m^2 = \sigma_{m-1}^2 + \sigma_1^2 \cdot \rho^{m-1} \tag{11}$$

$$\sigma_{m,n} = \rho^{|m-n|} \cdot \sigma_m \cdot \sigma_n \tag{12}$$

2. In the case of mean values, we take advantage of the information contained in ILS prices. In fact, by definition, ILS prices should be equal to the means of the Q–measures, as we showed in equation 3. Thus, we restrict the mean for each period to be equal to the corresponding 1 year forward rate implicit in ILS prices.

Taking both assumptions together, the number of parameters to estimate drops to just two, a number that can be easily estimated with the 70 available values for the cumulative probabilities obtained from section 3.2.

Therefore, for the fitting exercise, we will consider put options with S maturities and different strike prices K_j (all calls are in puts terms). Let $\theta = (\sigma_1, \rho)$ be the two parameters associated to the (restricted) multivariate Gaussian distribution. Then, θ^* solves

$$\theta^* = \min_{\theta} \sum_{T=1}^{S} \sum_{j=1}^{J} \Big(\mathcal{M}_T\left(K_j, \theta\right) - E_t^Q\left[1_{\{\pi_T < K_j\}}\right]\Big)^2, \tag{13}$$

$$\text{s.t.:} \ \ (1+s_m)^m = E\left[\prod_{j=1}^{m}(1+\pi_j)\right], \text{ for } m = 1, \ldots, 10$$

where s_m is the ILS spot zero-coupon rate with maturity m, and $E_t^Q\left[1_{\{\pi_T<K_j\}}\right]$ is approximated numerically as stated in section 3.2. An example of the estimated RNDs for a single day can be seen in figure 3 (cumulative distribution) and in figure 4 (density distribution).

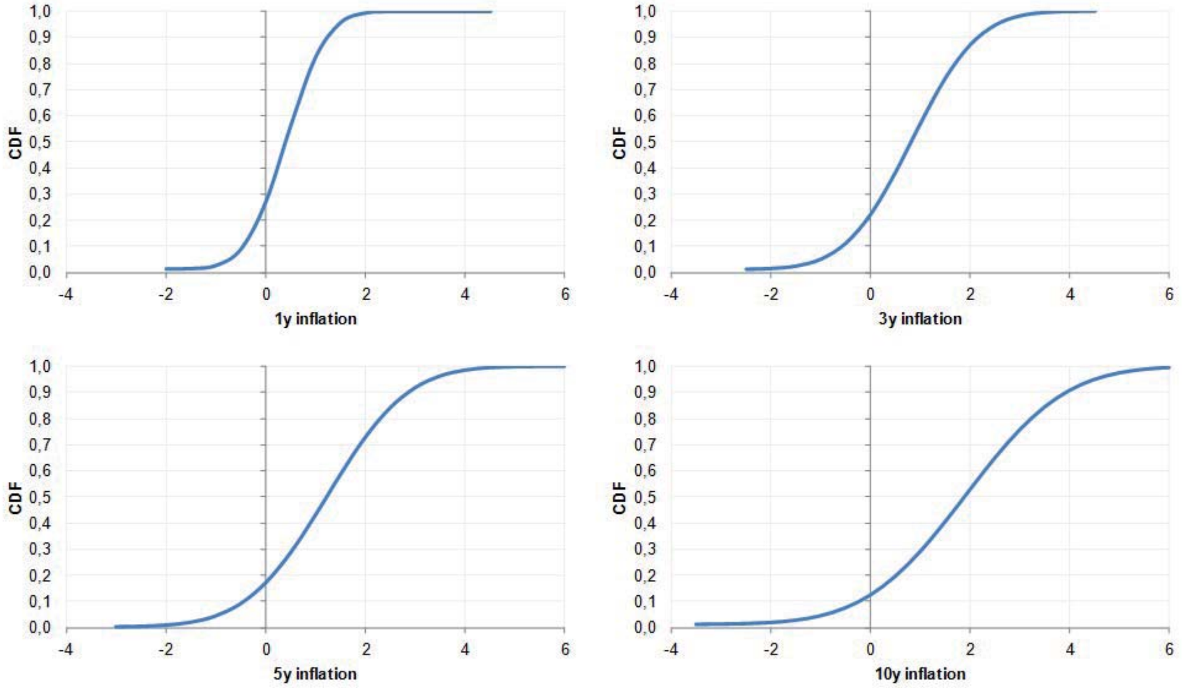

Figure 3: Cumulative distributions estimated of a Gaussian Multivariate Distribution with AR(1) covariance matrix, for 1 year (up-left), 3 year (up-right), 5 year (down-left) and 10 years (down-right) maturities. January, 15th 2015.

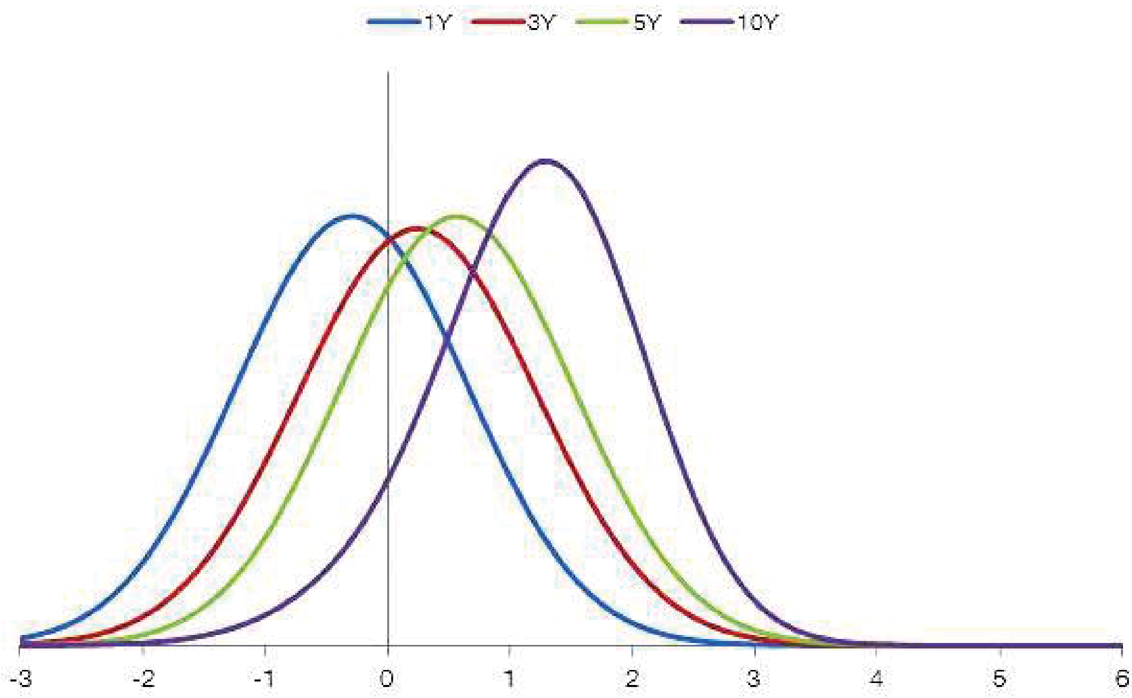

Figure 4: Density probabilities estimated of a Gaussian Multivariate Distribution with AR(1) covariance matrix) for 1 year (up-left), 3 year (up-right), 5 year (down-left) and 10 years (down-right) maturities, January, 15th 2015.

3.4 Alternative specifications

The parametric model proposed, is specially interesting when faced with a market with limited liquidity and with a scarcity of available strike prices to derive the CDFs. An advantage of the parametric approach over a non-parametric kernel specification (Smith, 2012) is that we can make extrapolations outside the limits of the strike prices. This is specially relevant in moments where inflation expectations are close to the boundaries set by those strikes (i.e., below -0.5% or above 5.5%), leaving sizable probabilities outside the estimation possibilities of a nonparametric model. As we have previously argued, the Gaussian assumption is not new in the literature, and has been previously proposed in relationship with options (e.g., Jarrow and Yildirim, 2003; Mercurio, 2005; Fleckenstein et al., 2017) as well as with ILSs and ILBs (e.g., Ang et al., 2007, 2008; Christensen et al., 2010; Chernov and Mueller, 2012; Haubrich et al., 2012; Christensen et al., 2016, among many others).

As an exercise to show the overfitting problems of more complex distributions we have also estimated Gaussian mixtures to capture possible skewness or kurtosis in the estimated densities. However, as shown in figure 5, the obtained RNDs are difficult to interpret and produce fitting errors that are not far from the ones of a single Gaussian distribution. In other cases, the estimation outputs favor the single distribution: the weight of each distribution in the mixture tends to be equal to either one or zero. Thus, we consider that the more parsimonious alternative of a single Gaussian distribution is a better approach.

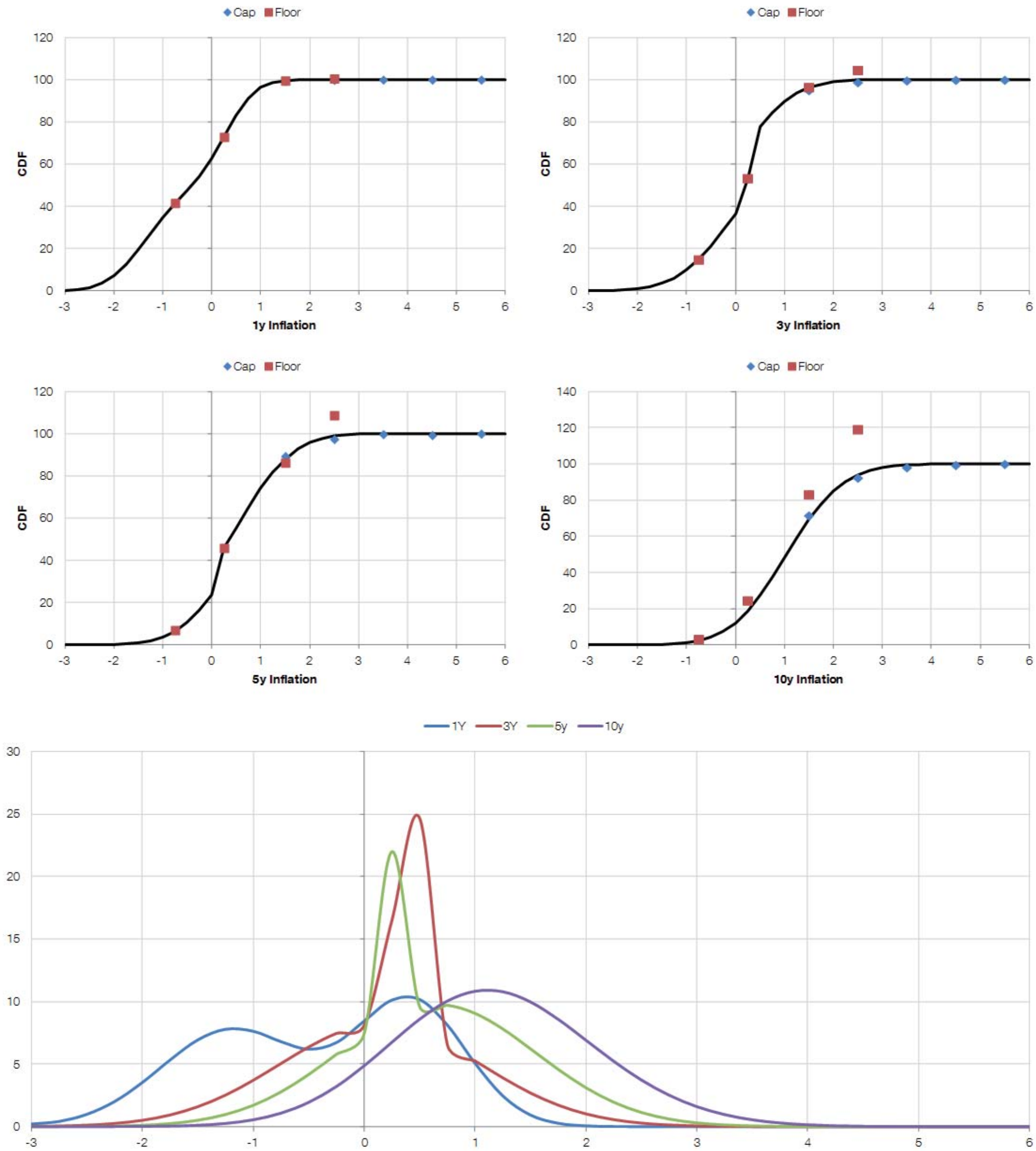

Figure 5: Cumulative RNDs estimated of a mixture of two Gaussian Multivariate Distribution with AR(1) covariance matrix) for 1 year (up-left), 3 year (up-right), 5 year (down-left) and 10 years (down-right) maturities, January, 15th 2015. The bottom chart represent the corresponding densities.

The other important advantage of the proposed assumptions is that we are able, not only to estimate the values of the CDFs for each individual year, but also to obtain the correlations between them, and get any other aggregate distribution (e.g., five–on–five forward rates). Alternative specifications could have been, for instance a unit root process. However, as Campbell et al. (2009) argue, a unit root process for expected inflation has unreasonable implications, being the most important one that any shock to realized inflation (i.e., changes in oil prices) will have permanent impact on future expected inflation. Thus, a more realistic approach is to consider, as in Stock and Watson (2007) or Primiceri (2006), autoregressive models. Other alternative, would have been using some moving average model, but then the problem would have been that regardless of what happens to short-term inflation, this will never affect long term inflation expectations. By considering an AR(1) process for annual inflation we consider that we are getting nice properties from both worlds, with the possibility of persistence of short term shocks if ρ gets close to one, and perfect anchoring of inflation expectations if ρ is equal to zero.

The third strategy we have used is that we have forced mean values to correspond with those of the ILS rates. If markets where perfectly liquid, this would not be

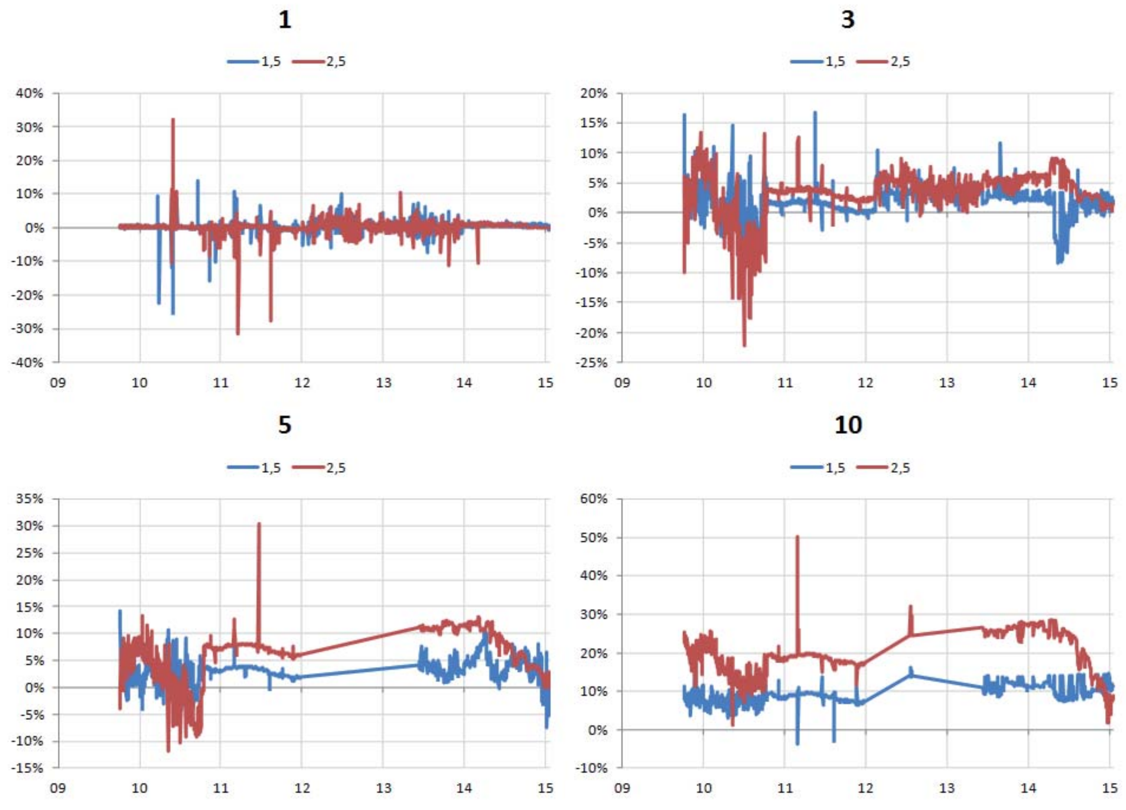

Figure 6: Deviations from the put call parity for rates of 1.5% (blue) and 2.5% (red) for 1 year (up-left), 3 year (up-right), 5 year (down-left) and 10 years (down-right) maturities.

necessary, since put-call parity will ensure that option prices contain the same information than ILS. However, as we have mentioned earlier, options are not specially liquid, and violations of this parity are common (see, for instance, figure 2). In a more general way, figure 6 presents deviations of the put-call parity for different maturities across the sample.

The consequence of trying to estimate the first moments of the RNDs with such noisy data is that we will obtain values that are away from those that we could consider reasonable, as shown (for a single day) figure 7.

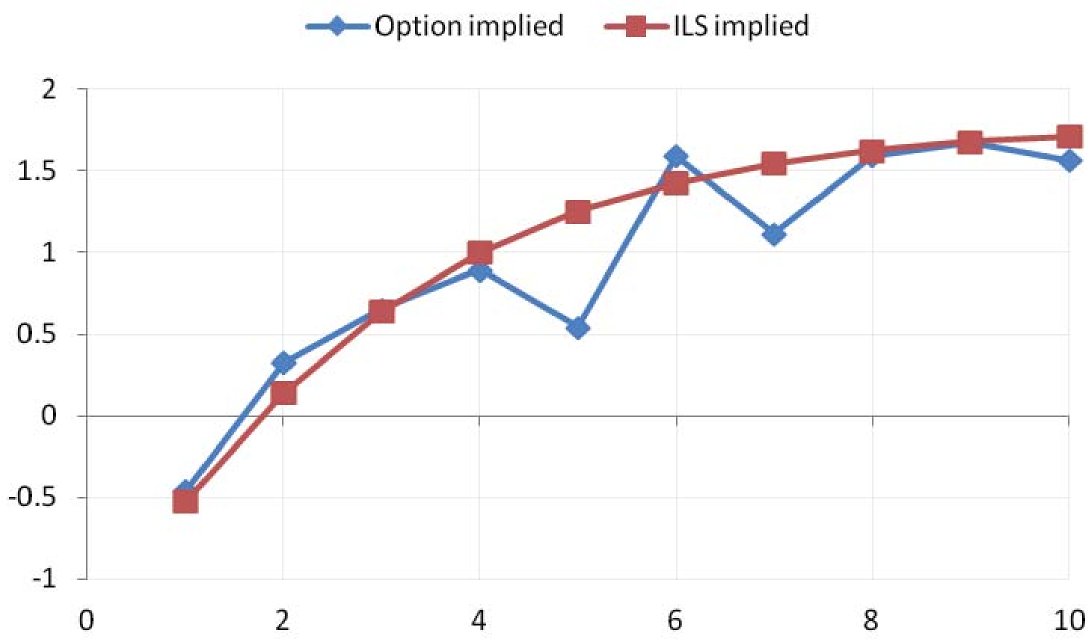

Figure 7: Means of estimated RNDs for January, 15th 2015, using ILS (red) and estimations from option prices (blue).

Taking all the above considerations, we have opted for a Gaussian AR(1) multivariate model with the first moments consistent with ILS rates, because this produces more plausible outcomes than other alternative parameterizations.

4 Empirical results on the Eurozone expected inflation

One of the main advantages of using a multivariate approach is that we do not only have estimates of inflation RNDs for the quoted maturities, but we can also get density estimates for forward rates as in the five–on–five (5y5y) forward rate that is regularly tracked by policy makers. As we will show in this section, the depth of the potential analysis increase considerably when we go from a single point estimate of inflation compensation (i.e., the mean), as we would get from just looking at swap rates, to a whole range of measures once we have the full density distribution. In this section, we present daily estimates of forward inflation rate densities at different horizons (e.g., 5y5y, 2y2y, 1y1y and 1y4y), from October the 5th, 2009 to December the 31st, 2016.

4.1 Moments

The first direct measures we get from the computation of the RNDs are, obviously, the distribution moments. By construction, mean inflation compensations coincide with the values we would have got from just looking at ILS. But in addition to them, we are now able to recover the implicit volatilities of those inflation compensations. An increase in implied volatility would mean a growth in the uncertainty regarding the inflation compensation. Thus, even if the mean value is not changed, an increase in the volatility would be signaling that there are rising concerns on inflation, which could be a problem for monetary policy decision–makers. In the same vein, an increase in inflation correlation implies that short term shocks to inflation might have an effect on long term inflation compensations, an indication of a reduction of the perceived efficiency of monetary policy.

As can be seen in Figure 8, for medium and long term inflation compensations (2y2y and 5y5y), there has been a decline since mid–2014 of the risk–neutral expected inflation, as well as the implicit volatility. Nevertheless, although there is a point of inflexion around the announcement of the *Public Sector Purchase Program* (PSPP) by the European Central Bank, at the beginning of 2015, it seems that this effect was only able to stop the fall in inflation compensation, not to revert the tendency. In fact, at the end of 2015 and beginning of 2016, there was a new intensification of the downward tendency, that might have triggered the new ECB measures. Finally, only in the second half of 2016 there has been a sustained recovery of inflation compensations.

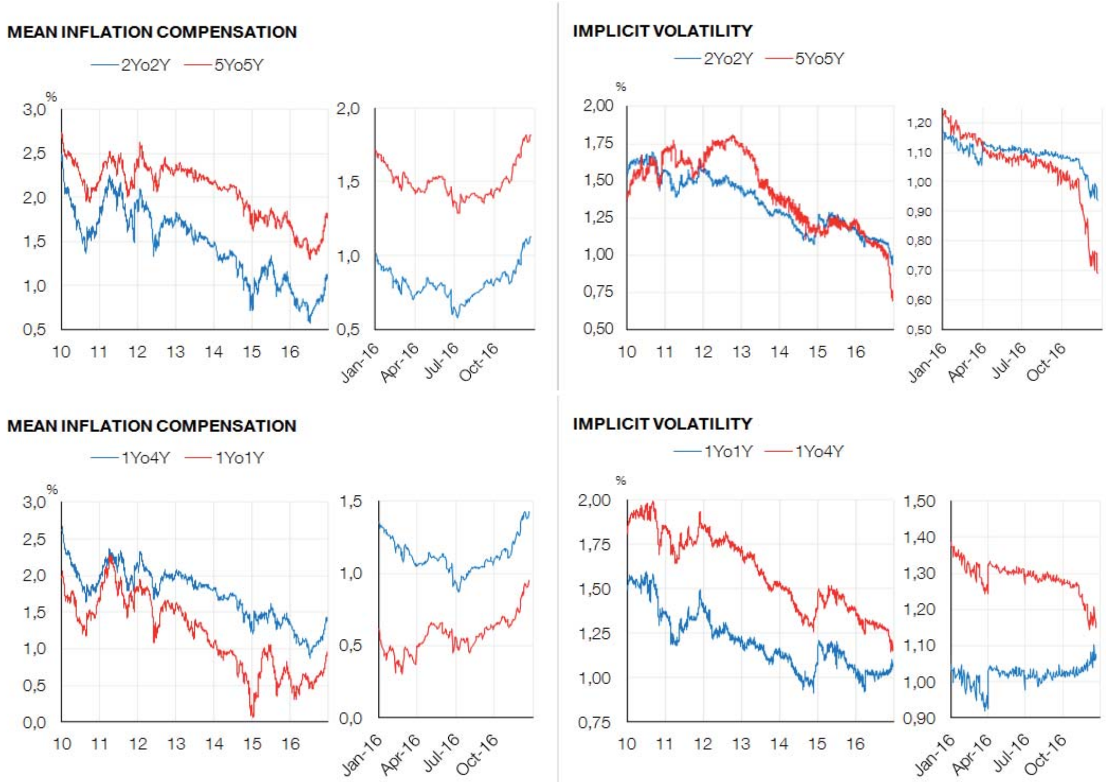

Figure 8: Daily evolution of the estimated inflation compensation (mean, left) and variance (right) of the 2y2y (blue, top); 5y5y (red, top); 1y1y (red, bottom); and 1y4y (blue, bottom) forward inflation rates from January 1st, 2010 to December 1st, 2016.

In the case of implied volatility, also displayed in Figure 8, there are two important features that can be highlighted from their evolution. Firstly, there has been a decline in volatility since 2012 in the medium term and 2013 in the long term, reflecting a decrease in inflation uncertainty. Secondly, we would expect, in normal times, that long term uncertainty (i.e., 5y5y) should be higher than medium term one (i.e., 2y2y), and that is the case for most of the sample period (2011–2014). However, both in 2010 and in 2015 we observe an inversion in both volatilities, with longer maturities showing less volatility than shorter ones. This could be a consequence of shocks to short term inflation, that investors are not expecting to be transmitted to longer horizons. In fact, both 2010 and 2015 coincide with periods of high and low oil prices, respectively, that are not expected to be permanent, thus producing lower volatilities for the longer horizons.

In the case of correlations, Figure 9 shows that the correlation between consecutive years is relatively high (between 0.6 and 0.9) but declines with the increase in the gap between the years considered. For the longer time gap (between year 1 and year 10, green line), we observe that, coincidentally with the sovereign crisis, there has been episodes with correlations close to 0.3. There is also a clear decline in the correlation after the monetary policy interventions (OMT at mid 2012, PSPP at the beginning of 2015 and March 2016), reinforcing the idea that this measures have had, at least, some success in breaking the link between short and long term inflation expectations.

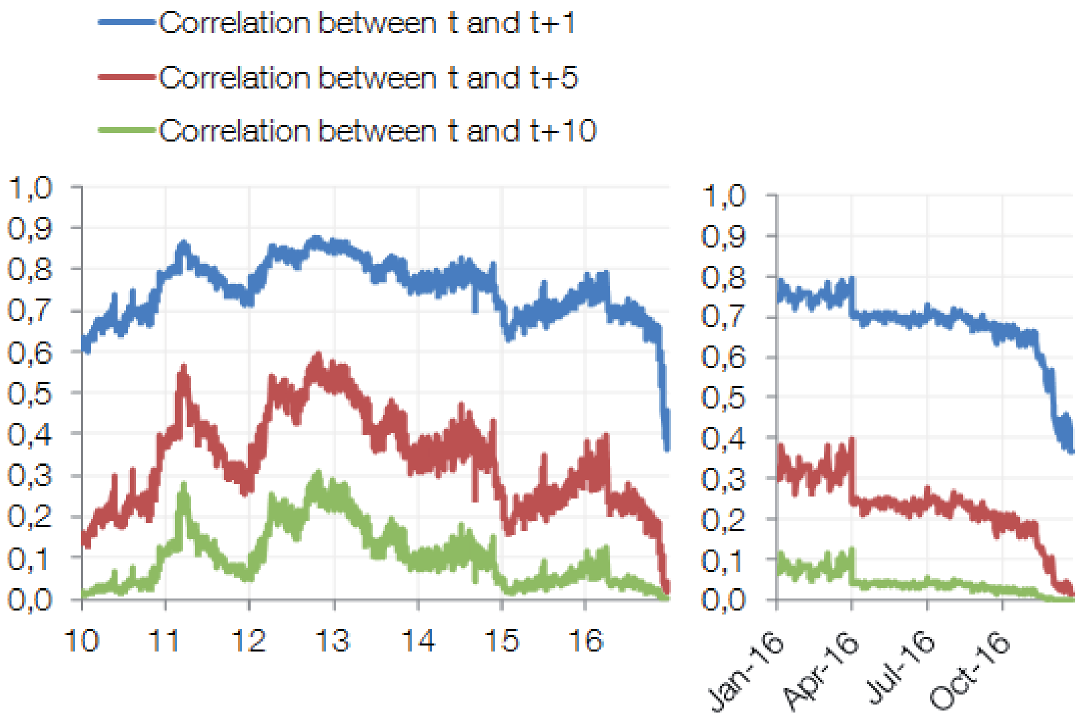

Figure 9: Daily evolution of the estimated correlation between year t and $t+1$ (blue); t and $t+4$ (red); and t and $t+9$ (green) inflation from January 1st, 2010 to December 1st, 2016.

4.2 Probabilities

Obviously, once we have RND estimations, it is possible to obtain probabilities on the intervals considered to be of special relevance. For instance, an increase in the (risk neutral) probability that long term inflation rate will be bellow 0%: $(-\infty, 0\%]$, would signal a risk of deflation; while a rise in the probability of inflation rates above 4%: $[4\%, \infty)$, would reflect inflation well above the monetary policy target (i.e., close to, but bellow, 2% for the ECB). In Figure 10, we present the risk-neutral probabilities of inflation being above 4% (bottom) or bellow 0% (top). In the first case, there has been a clear decline (especially since 2012) in the probabilities of too high inflation, and they have remained at historical low levels for the whole 2015 and the beginning of 2016. By contrast, probabilities of negative inflation, which were lower than those of higher inflation before 2014, showed an upward trend in 2013 in the medium term (i.e., 2y2y), with a spike in the end of 2014 that almost reach the 30%, before a strong decline after the announcement of the PSPP, which quickly halved the risk of deflation, although in the summer of 2015 it has experimented a new rise that could be linked to several factors (e.g., strong decline in oil and other material prices; concerns about Chinese economy; uncertainty about the continuation the the Greek program). In the case of probabilities for longer horizons (i.e., 5y5y), the probabilities of remaining in negative values have seldom been above 10%, and have been in decline from mid–2012 to mid–2014 coinciding with the decline in implied volatility. However, since the second half of 2014, this probability has started an upward tendency that has made this probability to almost double in less than two years. However, in the latter part of the sample, this probability has been reduced back to almost 0%.

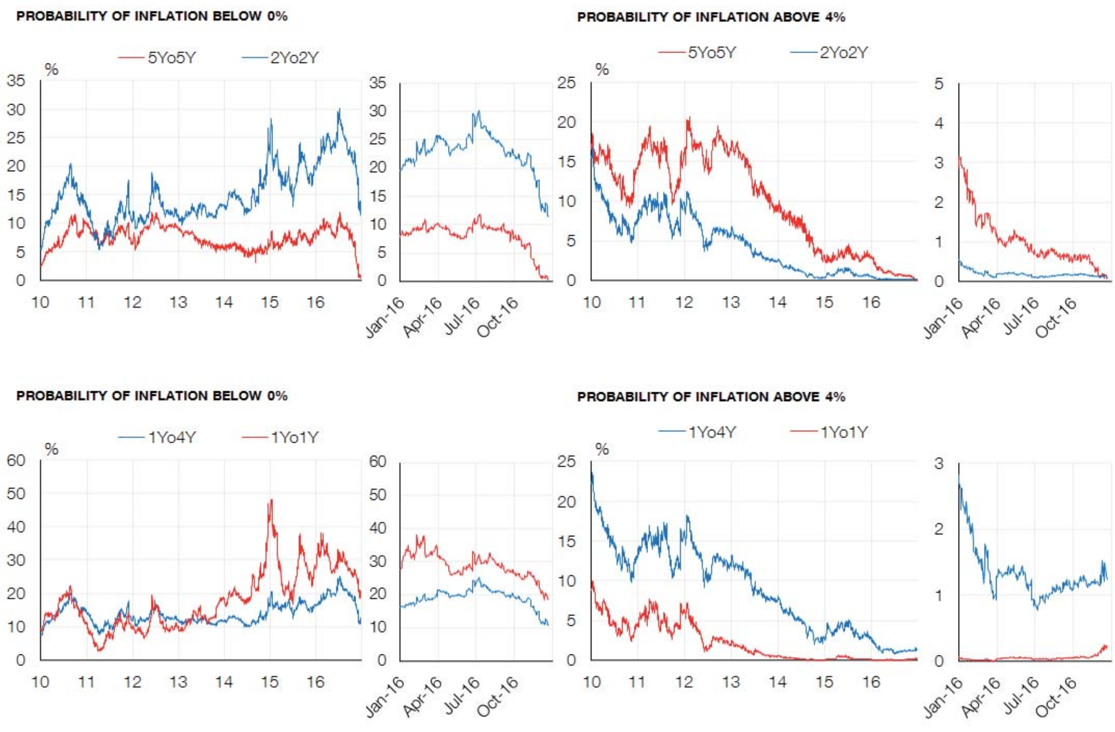

Figure 10: Daily evolution of the estimated risk neutral probabilities $P[\pi < 0\%]$ (top); $P[\pi > 4\%]$ (bottom) for the 2y2y (blue, left); 5y5y (red, left); 1y1y (red, right); and 1y4y (blue, right) forward inflation rates from January 1st, 2010 to December 1st, 2016.

An alternative way to present the same results is showed in Figure 11. These figures allow to see the clear drift to lower (risk–neutral) inflation rates observed in the euro area along the whole horizon of inflation option prices available (since the end of 2009). This is also the case independently of the forward interval considered.

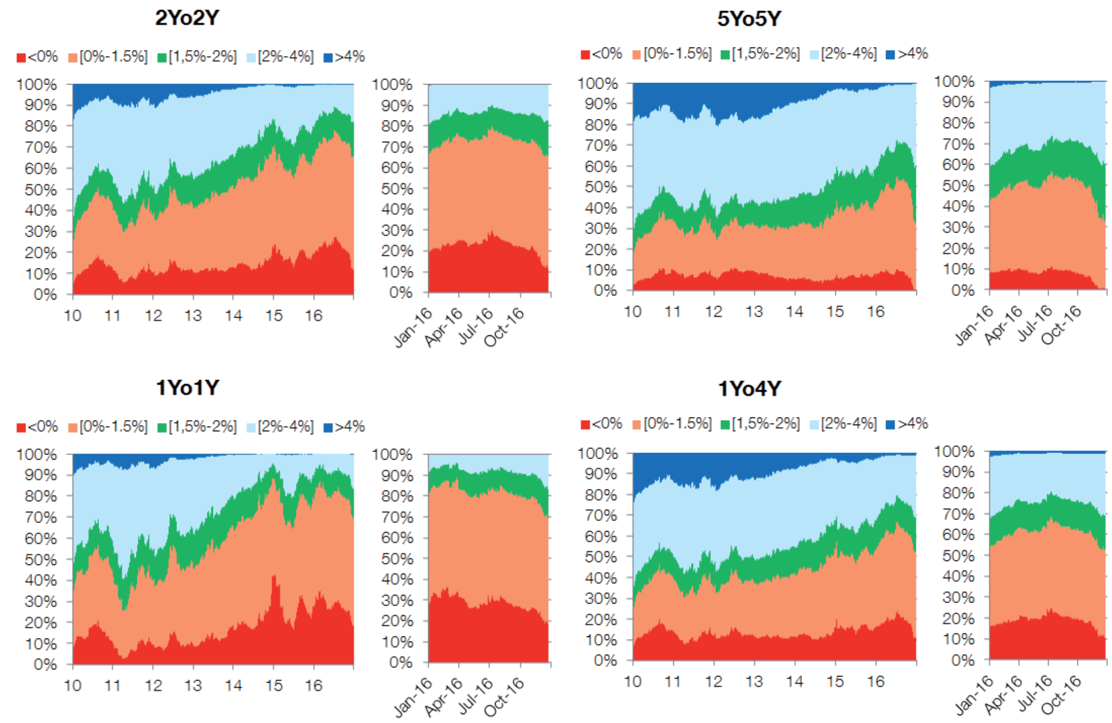

Figure 11: Daily evolution of the estimated risk neutral probabilities $P[\pi < 0\%]$ (dark red); $P[0 \leq \pi < 1.5\%]$ (light red); $P[1.5\% \leq \pi < 2\%]$ (blue); $P[2\% \leq \pi < 4\%]$ (light green); $P[\pi \geq 4\%]$ (dark green) for the 2y2y (top, left); 5y5y (top, right); 1y1y (bottom, left); and 1y4y (bottom, right) forward inflation rates from January 1st, 2010 to December 1st, 2016.

It would also be interesting to check the evolution of the risk neutral inflation rate probability of lying inside the monetary policy target. However, to do so, we would need a clear interval for the inflation target, and the ECB has not an explicit definition of price stability (i.e., close to, but bellow, 2% for the ECB), producing different alternative options for such an interval: $(x\%, 2\%)$. For instance, there is no clear evidence if the ECB considers 1.7% to be inside its mandate, or even a 1.5%. In Figure 12, we show the evolution of the probability of inflation being inside the policy target using two alternative definitions: a) $(1.5\%, 2\%)$; b) $(1.8\%, 2\%)$. Although the level changes considerably depending on the measure used, both intervals have presented a similar evolution. In the case of the longer horizons (i.e., 5y5y), the probability of being inside the target is higher since 2015 than it was during the euro area sovereign crisis (2010-2012). However, for a medium term horizon (i.e., 2y2y) this probability has actually declined.

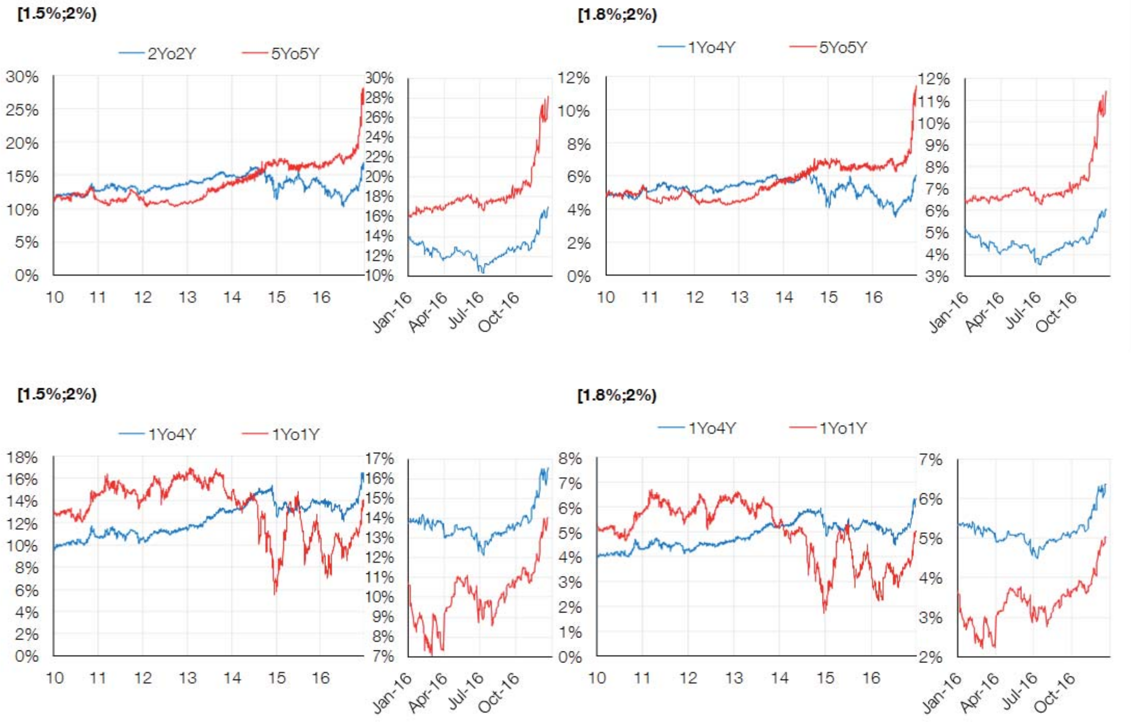

Figure 12: Daily evolution of the estimated risk neutral probabilities $P[1.5\% \leq \pi < 2\%]$ (left); and $P[1.8\% \leq \pi < 2\%]$ (right); for the 2y2y (red, top); 5y5y (blue, top); 1y1y (red, bottom); and 1y4y (blue, bottom) forward inflation rates from January 1st, 2010 to December 1st, 2016.

Another way to take advantage of these density distributions is looking to shifts in the distribution between two dates. For instance, the ECB reacted to the decline in inflation expectations by announcing the *Public Sector Purchase Programme* (PSPP, i.e., a QE-like purchase of government bonds) on January 22nd, 2015, with the purpose of easing the monetary policy. In Figure 13, we show the inflation risk–neutral probability density distributions on January 15th, 2015 (red), one week before the announcement, and also before the start of rumors about ECB possible decisions; and on January 23rd, 2015 (blue), one day after the announcement of the programme. As can be seen, there was a shift to the right in all maturities considered (2y2y top left; 5y5y top right; 1y1y bottom left; 1y4y bottom right), and also a contraction in the implied volatility, reducing the probability of low inflation without a similar increase in the probability of high inflation.

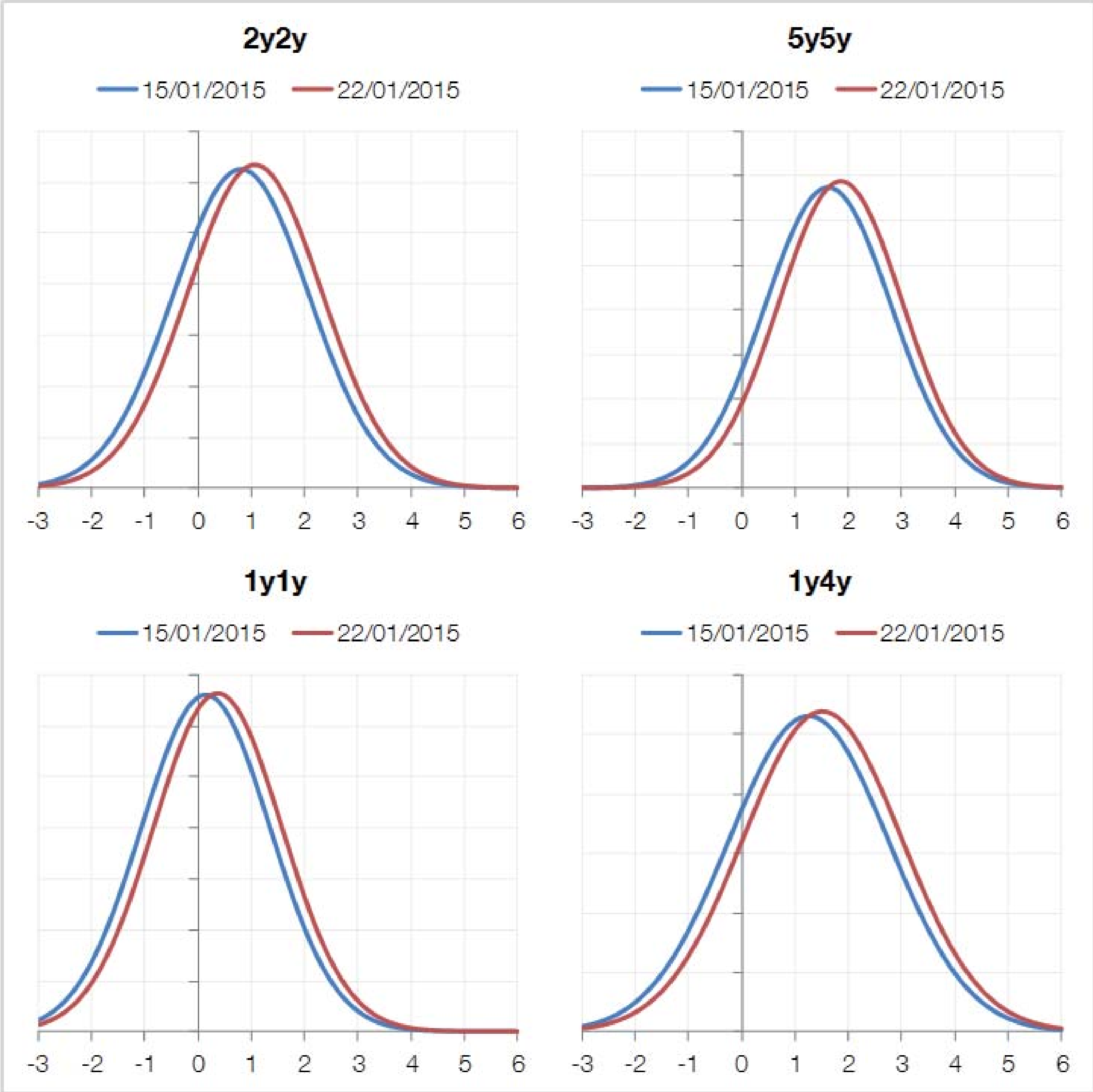

Figure 13: Risk-neutral Density forecast for the 2y2y (top left), 5y5y (top right), 1y1y (bottom left), and 1y4y (bottom right) forward inflation rates the 15th January, 2015 (blue) before the announcement of the ECB's PSPP and the 22nd January 2015 (red), just the day after the announcement of that program.

4.3 Balance of Risks

Killian and Manganelli (2007) proposed a measure (i.e., the balance of risk) based on inflation density distributions in order to assessing the tone that monetary policy should have. However, the measure of Killian and Manganelli (2007) requires knowing this density distribution, making difficult its practical use. Precisely, the inflation RNDs obtained from inflation derivatives allows to obtain such indicators straightforwardly.

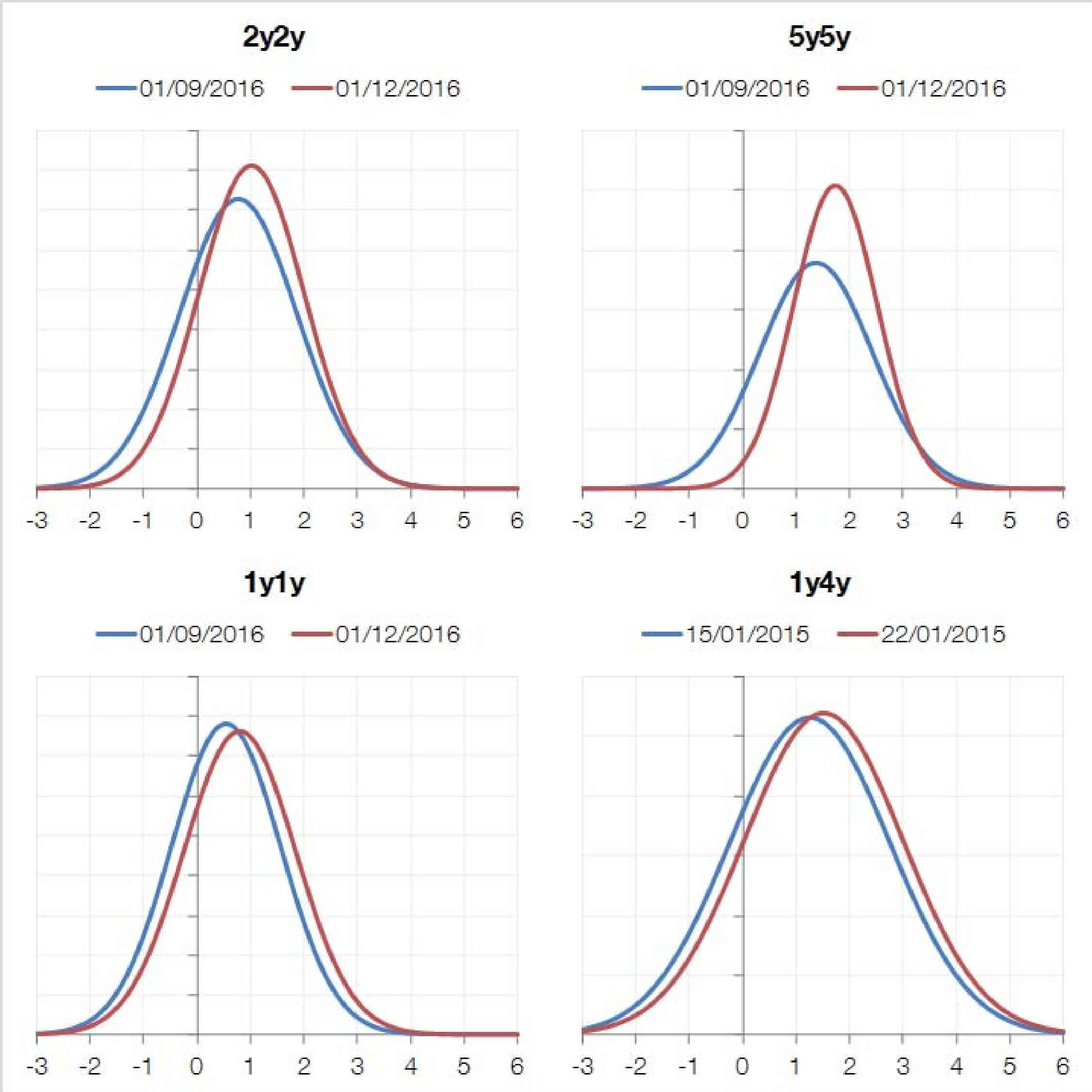

Figure 14: Risk-neutral Density forecast for the 2y2y (top left), 5y5y (top right), 1y1y (bottom left), and 1y4y (bottom right) forward inflation rates the 1st September, 2016 (blue) and the 1st December 2016 (red).

The balance of risk indicator proposed by Killian and Manganelli (2007) is based on a *loss function*, defined by the degree of concern by monetary policy-makers about having an inflation rate out of the range established as price stability. The loss function is as shown in equation 14. Parameters $\underline{\pi}$ and $\overline{\pi}$ represent the lower and upper values for the interval that is compatible with the definition of price stability. Parameters α and β represent the policy maker's risk aversion to low and high inflation respectively (i.e., the higher the value, the higher the risk aversion: if $\alpha < \beta$, it implies that policy makers are more concerned with high inflation than with low inflation). Finally, a is the weight given to the low inflation loss. Therefore, all 5 parameters in the loss function ($\underline{\pi}$, $\overline{\pi}$, α, β and a) are dependent on the preferences of the monetary policy makers.

$$
L(\pi) = \begin{cases} a\left(\underline{\pi} - \pi\right)^{\alpha} & if \quad \pi < \underline{\pi} \\ 0 & if \quad \underline{\pi} \leq \pi \leq \overline{\pi} \\ (1-a)\left(\pi - \overline{\pi}\right)^{\beta} & if \quad \pi > \underline{\pi} \end{cases} \tag{14}
$$

Given the loss function defined in equation 14, and once we have a RND function, as the ones we have obtained, it is possible to compute the expected loss, as shown in equation 15. In expression 15, the first term can be considered as an excessive inflation risk ($EIR_{\beta}(\overline{\pi}, T)$); while the second term would be the deflation (or low inflation) risk ($DR_{\alpha}(\underline{\pi}, T)$).

$$
E\left[L\right] = (1-a)\int_{\overline{\pi}}^{\infty}\left(\pi_{t,t+T} - \overline{\pi}\right)^{\beta} dQ\left(\pi_{t,t+T}\right) + a\int_{-\infty}^{\underline{\pi}}\left(\underline{\pi}_{t,t+T} - \pi_{T}\right)^{\alpha} dQ\left(\pi_{t,t+T}\right) \tag{15}
$$

The expected loss would be at the minimum if $\frac{\partial E[L]}{\partial \pi_{t,t+T}} = 0$. That is, if

$$
(1-a)\cdot\beta\cdot EIR_{\beta-1}(\overline{\pi}, T) - a\cdot\alpha\cdot DR_{\alpha-1}(\underline{\pi}, T) = 0 \tag{16}
$$

From expression 16, Killian and Manganelli (2007) define the balance of risk as the deviations from that situation where the expected loss is minimum (equation 17). The sign of this indicator inform about which risk has a higher relevance in those deviations. Thus, a positive (negative) sign implies that there is a higher risk of excessive (low) inflation than that of low (excessive) inflation.

$$
BoR_{\alpha,\beta,a}(\underline{\pi}, \overline{\pi}, T) = (1-a)\beta EIR_{\beta-1}(\overline{\pi}, T) - a\alpha DR_{\alpha-1}(\underline{\pi}, T) \tag{17}
$$

In Figure 15, we present the evolution of a particular case of this Balance of Risk measure (i.e., $BoR_{\alpha=\beta=2,a=0.5}$), where we have given the same weight ($a = 0.5$), and the same risk aversion ($\alpha = \beta = 2$) for both excessive and low inflation. The thresholds for excessive inflation has been set in the ECB's target ($\overline{\pi} = 2\%$), while for the low inflation we have opted for 1.5% ($\underline{\pi} = 1.5\%$).

For the longer horizon (5y5y), the balance of risk was positive for most of the sample period (since 2010 until the third quarter of 2014), indicating that investors were more worried about excessive inflation than their were about deflation. However, since the end of 2014, the balance of risks becomes closer to zero (and negative in 2016), indicating a shift to more concerns about lower inflation. This outcome is in line with the assessment by the ECB that prompted the start of the PSPP.

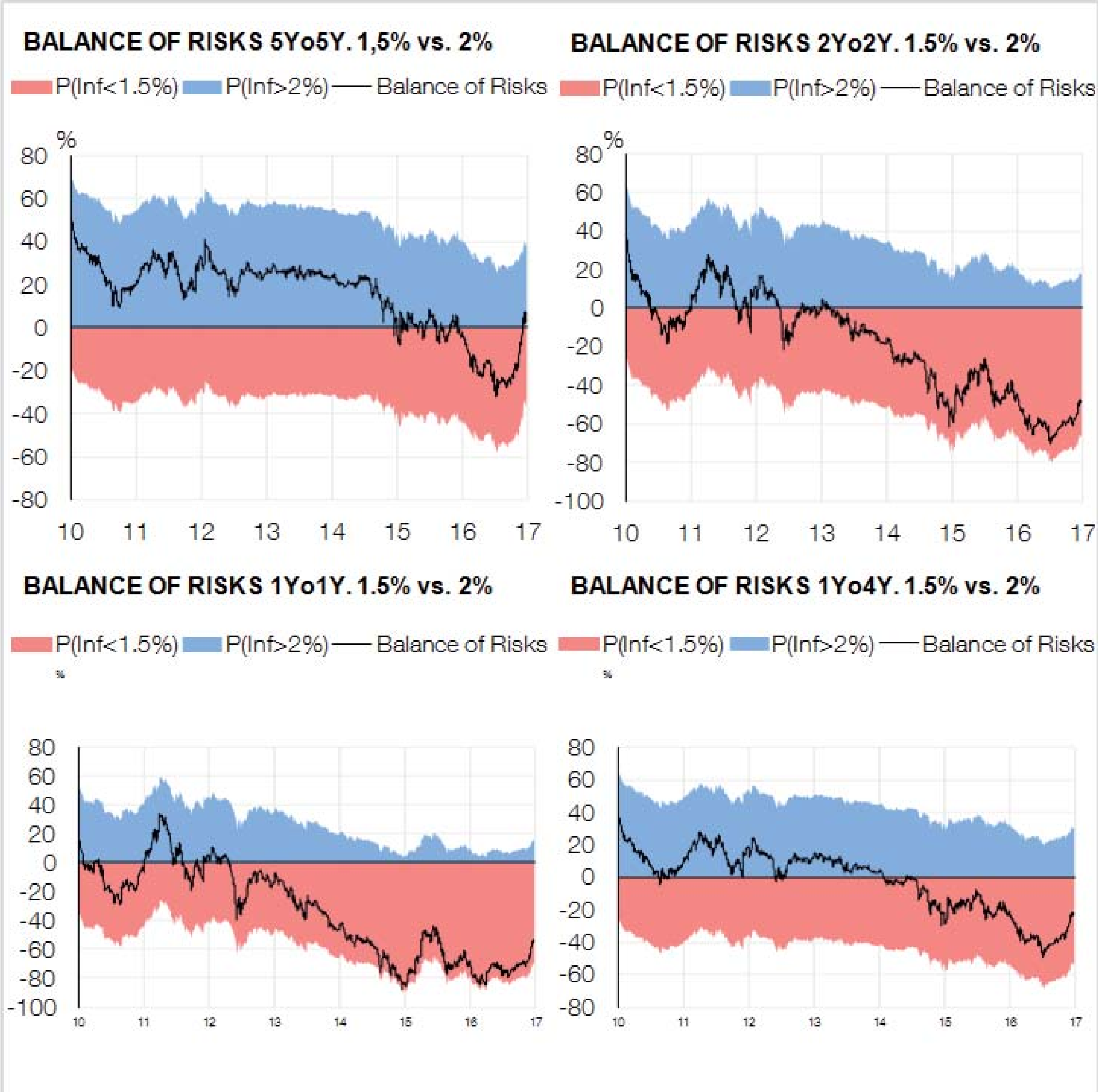

Figure 15: Balance of Risk $BoR_{\alpha=1}$ (see, Killian and Manganelli, 2007) computed with the risk-neutral densities for maturities of 2yo2y (blue) and 5yo5y (red) inflation forward estimated using 0% (right) and 1.8% (left) for the deflation risk and 4% (right) and 2% (left) for the excesive inflation risk.

4.4 Event study

All the above indicators obtained from RNDs give a better picture of the market of inflation compensations than restricting just to the means. In fact, although we have been commenting the effect of the announcement of the PSPP in previous subsections, we are going to use a different approach here, using an event study to look at movements in all those indicators along the days where important monetary policy announcement have been made, in a similar way as used by Kilponen et al. (2015) and Altavilla et al. (2017) for bond markets, or Speck (2016) for ILS.

The days considered in the analysis include the announcement of the Securities Market Programme (SMP) on may 10th, 2010, the Longer Term Refinincing Operations (LTRO, TLTRO and TLTRO II), the Outright Monetary Transactions (OMT), and the different segments of the Asset Purchase Programme (APP: ABSPP, CBPP, PSPP and CSPP). Overall, we are considering 12 dates along the past 7 years. For each date and indicator, we have run a linear regression where the dependent variable is the daily change in the indicator, and the independent variable is a dummy variable that it is equal to one on the day of the corresponding monetary policy decision. We will consider that the decision has produced a significant effect on the corresponding measure of market inflation compensation if the coefficient is statistically significant. Results of this analysis are presented in tables 1, 2 and 3.

As can be seen from the event study outcomes, monetary policy decisions have had different impacts over the inflation compensation measures. Firstly, we observe that the impact of the latter measures has been milder than the ones obtained from the first decisions. This is partially a consequence of the starting situation when the measures where announced, as well as their cumulative effect, but also a result of the relative relevance of each of them. For instance, the magnitude of the PSPP is much higher than the later CSPP, and this can explain why the effect of the later is negligible over market inflation compensations. An additional source of divergences in their impact is that not all of these non-conventional measures were designed to influence inflation expectations, but were considered, instead, taking into account their impact on financial stability.

Another relevant result from the analysis is that if we only look at mean values (the ones we could get from the ILS), we might loose part of the impact of the corresponding monetary policy decision. For instance, the LTRO, TLTRO and the ABSPP & CBPP3 announcements effects where significant in reducing variances (uncertainty) and the correlations, but had no significant effects on the means. By contrast, some other measures, like the PSPP, affected the means, but neither variances nor correlations. Thus, we can conclude that the use of the overall distribution helps to better understand the effect of monetary policy decisions.

Table 1: Event study

	SMP announcement 10/05/2010		SMP extension 07/08/2011		LTRO announcement 08/12/2011		OMT announcement 02/08/2012	
Variance 1Y	-0.016		-0.001		-0.048		-0.014	
Variance 10Y	0.247	***	-0.026		-0.084		0.055	
Correlation 1Y	0.024	*	-0.002		0.006		0.008	
Correlation 9Y	0.012		-0.002		0.004		0.014	
Mean 1y1y	0.135	***	-0.068	**	-0.014		0.055	**
Mean 1y4y	0.103	***	-0.009		-0.018		0.058	**
Mean 2y2y	0.120	***	-0.022		-0.008		-0.024	
Mean 5y5y	-0.037	*	-0.016		-0.013		0.000	
Variance 1y1y	0.004		-0.004		-0.075	*	-0.018	
Variance 1y4y	0.122	***	-0.014		-0.096	**	-0.001	
Variance 2y2y	0.089	***	-0.010		-0.070	**	-0.003	
Variance 5y5y	0.060		-0.017		0.022		-0.027	
Probability deflation 1y1y	-0.020	***	0.008		-0.002		-0.011	
Probability deflation 1y4y	-0.008	**	0.000		-0.001		-0.008	*
Probability deflation 2y2y	-0.012	**	0.002		-0.002		0.003	
Probability deflation 5y5y	0.005		0.001		0.002		-0.001	
Probability high inflation 1y1y	0.011	***	-0.005	**	-0.005	**	0.002	
Probability high inflation 1y4y	0.017	***	-0.002		-0.006	*	0.006	**
Probability high inflation 2y2y	0.016	***	-0.003		-0.004		-0.002	
Probability high inflation 5y5y	-0.002		-0.003		-0.001		-0.001	
Balance of Risk 1y1y	0.042	***	-0.026	*	-0.004		0.021	
Balance of Risk 1y4y	0.019	***	-0.002		-0.002		0.013	*
Balance of Risk 2y2y	0.032	***	-0.006		-0.001		-0.007	
Balance of Risk 5y5y	0.011		0.000		-0.007		0.002	

Linear regressions where the dependent variables are daily changes in each of the moments estimated by inflation options (in rows), while the independent variables are dummy variables for the days where an unconventional monetary policy announcement was made (in columns). SMP is the Securities Markets Programme; LTRO stands for Longer Term Refinincing Operations; ans OMT are the Outright Monetary Transactions. Only the beta estimator for the corresponding variable in each regression is reported. ***; **; * stars represents 1%, 5% and 10% significant values respectively.

Table 2: Event study (cont.)

	TLTRO announcement 05/06/2014		ABSPP & CBPP3 announcement 04/09/2014		PSPP announcement 22/01/2015		PSPP details 05/03/2015	
Variance 1Y	0.041		0.036		0.011		0.011	
Variance 10Y	-0.226	***	-0.129	**	0.032		0.053	
Correlation 1Y	-0.046	***	-0.035	**	0.000		0.002	
Correlation 9Y	-0.053	***	-0.043	***	0.000		0.001	
Mean 1y1y	0.031		0.019		0.086	***	0.049	*
Mean 1y4y	0.030		0.013		0.091	***	0.027	
Mean 2y2y	0.036		0.008		0.094	***	0.035	
Mean 5y5y	0.014		-0.008		0.041	**	0.052	***
Variance 1y1y	0.044		0.043		0.019		0.021	
Variance 1y4y	-0.049		-0.009		0.029		0.040	
Variance 2y2y	-0.031		-0.004		0.021		0.029	
Variance 5y5y	0.020		0.105		-0.072		-0.048	
Probability deflation 1y1y	-0.003		-0.001		-0.027	***	-0.013	**
Probability deflation 1y4y	-0.007	*	-0.002		-0.013	***	-0.002	
Probability deflation 2y2y	-0.009	*	-0.002		-0.020	***	-0.005	
Probability deflation 5y5y	0.000		0.006	*	-0.010	***	-0.009	**
Probability high inflation 1y1y	0.001		0.000		0.000		0.000	
Probability high inflation 1y4y	0.000		0.000		0.007	**	0.003	
Probability high inflation 2y2y	0.000		0.000		0.002		0.001	
Probability high inflation 5y5y	0.003		0.006		-0.001		0.001	
Balance of Risk 1y1y	0.028	*	0.027	*	0.036	**	0.028	*
Balance of Risk 1y4y	0.011		0.005		0.031	***	0.012	*
Balance of Risk 2y2y	0.014		0.004		0.042	***	0.020	*
Balance of Risk 5y5y	0.026	***	0.022	***	0.040	***	0.010	

Linear regressions where the dependent variables are daily changes in each of the moments estimated by inflation options (in rows), while the independent variables are dummy variables for the days where an unconventional monetary policy announcement was made (in columns). TLTRO stands for Targeted Longer Term Refinincing Operations; ABSPP is the Asset Backed Securities Purchase Programme; CBPP3 is the 3rd Covered Bonds Purchase Programme; and PSPP is the Public Sector Purchase Programme. Only the beta estimator for the corresponding variable in each regression is reported. ***; **; * stars represents 1%, 5% and 10% significant values respectively.

Table 3: Event study (cont.)

	APP Extension 03/12/2015		TLTRO II & CSPP announcement 10/03/2016		CSPP details 21/04/2016		APP extension 08/12/2016	
Variance 1Y	0.003		-0.016		0.008		0.045	
Variance 10Y	-0.058		0.031		-0.056		-0.026	
Correlation 1Y	-0.009		0.013		-0.014		-0.043	***
Correlation 9Y	-0.007		0.010		-0.007		0.000	
Mean 1y1y	-0.030		0.003		0.028		0.026	
Mean 1y4y	-0.044	*	-0.006		0.019		0.018	
Mean 2y2y	-0.041		-0.008		0.027		0.023	
Mean 5y5y	-0.024		-0.035		-0.012		0.011	
Variance 1y1y	0.000		-0.020		0.005		0.030	
Variance 1y4y	-0.026		-0.002		-0.023		-0.015	
Variance 2y2y	-0.018		-0.002		-0.017		-0.027	
Variance 5y5y	0.006		-0.149		0.030		-0.047	
Probability deflation 1y1y	0.009		-0.002		-0.009		-0.004	
Probability deflation 1y4y	0.006		0.001		-0.006		-0.004	
Probability deflation 2y2y	0.007		0.002		-0.009	*	-0.009	*
Probability deflation 5y5y	0.003		-0.007		0.005		-0.003	
Probability high inflation 1y1y	0.000		0.000		0.000		0.000	
Probability high inflation 1y4y	-0.003		0.000		0.000		0.000	
Probability high inflation 2y2y	-0.001		0.000		0.000		0.000	
Probability high inflation 5y5y	-0.001		-0.007		0.001		0.000	
Balance of Risk 1y1y	-0.015		0.004		0.014		0.022	
Balance of Risk 1y4y	-0.018	**	0.007		0.004		0.007	
Balance of Risk 2y2y	-0.022	**	0.007		0.007		0.005	
Balance of Risk 5y5y	-0.011		-0.010		-0.003		0.009	

Linear regressions where the dependent variables are daily changes in each of the moments estimated by inflation options (in rows), while the independent variables are dummy variables for the days where an unconventional monetary policy announcement was made (in columns). APP stands for the Asset Purchase Programme; TLTRO II is the 2nd Targeted Longer Term Refinincing Operations; CSPP is the Corporate Sector Purchase Programme. Only the beta estimator for the corresponding variable in each regression is reported. ***; **; * stars represents 1%, 5% and 10% significant values respectively.

5 Inflation under the objective measure, $\mathcal{P}$

Previous estimates are risk neutral approaches to actual inflation expectations. Although, those densities are linked to actual inflation (i.e., if densities diverge from expectations, traders would profit from exploiting such differences), derivatives also imply a transfer of inflation risk from one part of the contract to the other. Such risk transfers can be seen as an insurance against inflation, and priced accordingly (i.e., there is an inflation risk premium). This will imply that actual (objective) probabilities can diverge from the ones obtained from derivatives (e.g., Evans, 1998; Gürkaynak et al., 2010; Pflueger and Viceira, 2011).

There are several approaches to build the objective $\mathcal{P}$–measure and then address the inflation risk-premium. We can directly estimate a parametric model: e.g., Kitsul and Wright (2013) use Stock and Watson (2007) and Primiceri (2006) autoregresive models; while Fleckenstein et al. (2014) use a Duffie and Singleton (1997) swap-rates type model. Alternatively, we can use the survey of professional forecasters (SPF), who directly provide values of the expected inflation cumulative probability for different maturities (see, García, 2003). Hence we can fit the static $\mathcal{P}$–measure at different maturities. This is also the approach in Kitsul and Wright (2013) and Scharnagl and Stapf (2015).

In both cases, we obtain the whole $\mathcal{P}$ density function. Comparing the risk neutral (Q) and the objective measure ($\mathcal{P}$), we get the kernel that allows to go back and forth from the risk-neutral and objective world. Therefore, the $\frac{Q}{\mathcal{P}}$ ratio gives a measure of the risk premium associated to inflation derivatives (i.e., what is the number that multiply the objective probability to get the risk-neutral one).

Thus, the risk premium (RP) can be obtained as the difference between expected values of both types of measures,

$$RP_{t,T} = E_t^{\mathcal{P}}\left[\pi_{t,T}\right] - E_t^{Q}\left[\pi_{t,T}\right] = E_t^{\mathcal{P}}\left[\frac{\mathcal{P}\left(\pi_T\right) - Q\left(\pi_T\right)}{\mathcal{P}\left(\pi_T\right)} \times \pi_T^n\right], \tag{18}$$

or focusing simply in the tails, e.g., the deflation risk-premium (i.e. the risk premium for inflation below 0%),

$$E_t^{\mathcal{P}}\left[\frac{\mathcal{P}\left(\pi_T\right) - Q\left(\pi_T\right)}{\mathcal{P}\left(\pi_T\right)} \times 1_{\{\pi_T \leq 0\%\}}\right]. \tag{19}$$

or high inflation risk-premium (i.e. the risk premium for inflation above 0%),

$$E_t^{\mathcal{P}}\left[\frac{\mathcal{P}\left(\pi_T\right)-Q\left(\pi_T\right)}{\mathcal{P}\left(\pi_T\right)}\times 1_{\{\pi_T\geq 4\%\}}\right]. \tag{20}$$

5.1 Comparison with the Survey of Professional Forecasters

In order to quantify the relevance of potential bias in RNDs, as well as the $\frac{Q}{\mathcal{P}}$ ratio, we would need to know the true (objective) probabilities. Unfortunately, this *true* probabilities are not observed. The more straightforward approach is to use a survey, like the Survey of Professional Forecasters (SPF), that, being not based on prices, should be free of any potential investors sentiment bias. The SPF is a quarterly survey where a sample of professional forecasters is asked about euro area inflation (as well as about real GDP growth and unemployment rates) point estimates, and more importantly for our purpose, about probabilities assigned at different intervals (García, 2003). Thus, once in a quarter, we are able to compare the estimated risk neutral probabilities (option–implied ones) with objective probabilities (i.e., the SPF probabilities). This is the way in which SPF are used in the US by Gürkaynak et al. (2010), Croushore et al. (2010), Del Negro and Eusepi (2011), Adam and Padula (2011), Haubrich et al. (2012) or Chernov and Mueller (2012). However, some authors suggest that SPF has tended to overestimate the observed inflation (Ang et al., 2007; Chan et al., 2013). However, it is not possible to disentangle if this is a consequence of a failure of the survey to capture actual expectations, or a bias in forecasters (and investors) about those expectations. For instance, Gimeno and Marqués-Sevillano (2012) showed how in Spain during the 90s, the inflation decline was not captured neither by analysts nor even by the Spanish Central Bank own monetary policy targets. The short life of the euro area could have played a similar role for the whole area, like happen with the infraestimation of credit risk for euro area sovereign bonds. Therefore, although they might be imperfect, we are going to assume here that the SPFs are our best approach to the $\mathcal{P}$–measure.[6]

The left and central panels in figure 16 show the comparison of both density distributions (option implied in red and SPF in blue) for the 1 and 2 year horizons according to the December 2015 SPF wave. As can be seen, the option implied distributions are at the left of the SPF, what would imply that investors were willing to pay a premium to protect themselves from the risk of deflation, while they were not equally concerned about the high inflation scenario.

[6] A further discussion on the issue can be found on Del Negro and Eusepi (2011).

In fact, this comparison provides information on its own about were the biggest investors' aversion fall: deflation or high inflation. he right panel in figure 16 show the result of the $\frac{Q}{\mathcal{P}}$ ratio for different inflation values for the December 2015 SPF wave. In this ratio, values grater than one imply that investors are willing to pay a premium to protect themselves from that outcome, while values below one imply that investors are not even willing to pay the price derived from the objective probabilities of those outcomes. As can be seen, for inflation rates below 1%, investors were willing to pay a premium, but not for inflation rates about that threshold.

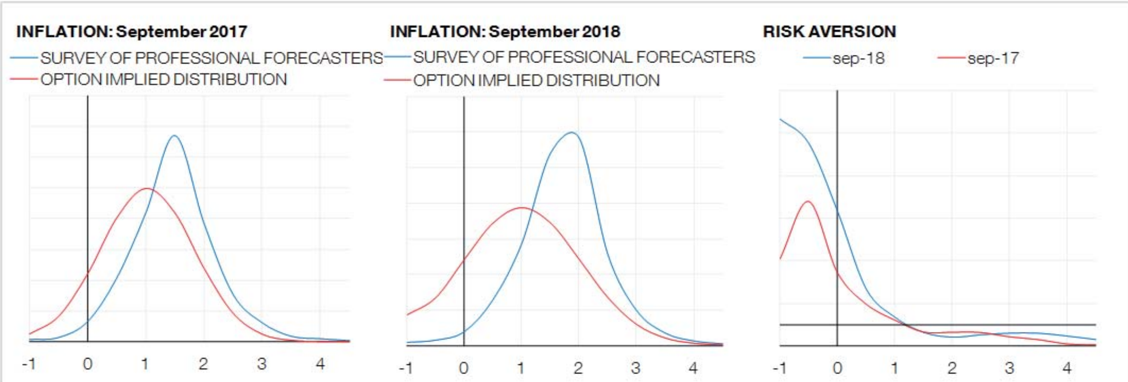

Figure 16: Risk-neutral Density forecast for the 1y (left) and 1y1y (center) inflation rates averaged between the 30th September and the 6th October 2016 (red) and the density distribution of the Survey of Professional Forecasters conducted the same days. In the left chart, $\frac{Q}{\mathcal{P}}$ kernel discount factor for both maturities.

The higher concern over deflation than over high inflation is something that is more a consequence of the later economic outcomes than a structural situation. If we track historically this risk premium discount factor $(\frac{Q}{\mathcal{P}})$, as in figure 17, we observe that in 2010 and 2011 the high-inflation discount factor was higher than the low inflation one, but that the opposite situation only dominates the latter years of the sample.

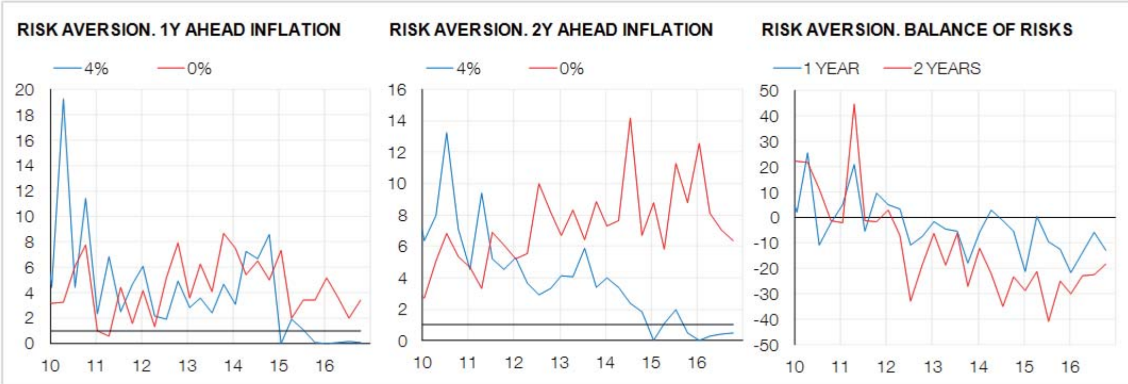

Figure 17: $\frac{Q}{\mathcal{P}}$ discount factor for the 4% (blue) and 0% (red) for the 1y (left) and 1y1y (center) inflation rates comparing the risk-neutral density and the contemporaneous SPF. The right panel show the difference between the 4% discount factor and the 0%. A positive value implies a higher risk aversion for the high inflation scenario while a negative value implies a higher risk aversion for the deflationary scenario.

6 Conclusions

This paper makes contributions both in methodological and policy dimensions. Although some other papers on the topic already exist, inflation options have been relatively recent addition to the inflation derivatives landscape, and their analysis remains limited to the estimation of RNDs for a few maturities (e.g. 5 or 10 years) or even the evolution of certain strike prices for regular policy briefing.

In this paper, we have presented a parsimonious model that allows to obtain risk-neutral multivariate density distributions of inflation compensations for several horizons using derivative prices. This very simple multivariate approach allows to extract information about forward inflation rates (e.g. five-on-five forward rates), that are especially relevant for monetary policy analysis, on a daily basis, in spite of the limited number of derivatives available.

The policy implications of this paper are also important. The estimation of the term structures of (spot and forward) inflation RNDs offers crucial additional dimensions to assess changes in inflation expectations, which may be an important addition for regular policy analysis. We have also shown, how this novel approach allows to get several indicators of inflation expectations and risk aversions, from probabilities of inflation bellow or above certain thresholds, to measures of inflation risk such as the implicit volatility, the balance of risks a la Killian and Manganelli (2007) or, comparing with the ECB's Survey of Professional Forecasters, to get a measure of of risk aversion at different levels of inflation. All of these measure are extremely relevant for analysts and policy makers concerned with the evolution of inflation expectations.

Although, this analysis has been made using euro-area inflation derivatives, the same approach could be used to extract US or UK inflation densities, as well as in other economic areas when their derivative markets develop enough for the option prices to be meaningful.

The methodological contribution of the paper is limited by the characteristics of available option data (sample size, limited number of strikes and lack of liquidity), and an extension in the liquidity and number of strikes traded will allow to explore more complex distributions that might include skewness and kurtosis properties.

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