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#### Abstract

For reasons of empirical tractability, analysis of cointegrated economic time series is often developed in a partial setting, in which a subset of variables is explictly modeled conditional on the rest. This approach yields valid inference only if the conditioning variables are weakly exogenous for the parameters of interest. This paper proposes a new test of weak exogeneity in panel cointegration models. The test has a limiting Gumbel distribution that is obtained by first letting T  $\rightarrow \infty$  and then letting N  $\rightarrow \infty$ . We evaluate the accuracy of the asymptotic approximation in finite samples via simulation experiments. Finally, as an empirical illustration, we test weak exogeneity of disposable income and wealth in aggregate consumption.

Keywords: panel data, cointegration, weak exogeneity, Monte Carlo methods.

JEL classification: C23, C32.

#### Resumen

El análisis empírico de series temporales bajo cointegración se desarrolla habitualmente en un contexto parcial en el que un subconjunto de las variables se analiza condicionado al resto. Este enfoque produce inferencia válida solo si las variables condicionantes son débilmente exógenas respecto a los parámetros de interés. Este trabajo propone un nuevo test de exogenidad débil en modelos de cointegración con datos de panel. El test resultante se distribuye bajo la hipótesis nula como una distribución Gumbel a medida que T (número de períodos) y N (número de unidades) van a infinito de forma secuencial (primero T y después N). Además, se evalúa la precisión de esta aproximación asintótica en muestras finitas mediante experimentos de simulación. Por último, como ilustración empírica utilizamos el test propuesto para comprobar la validez del supuesto de exogenidad débil de la renta disponible y la riqueza en ecuaciones de consumo agregado.

Palabras clave: datos de panel, cointegración, exogenidad débil, Monte Carlo.

Códigos JEL: C23, C32.

#### 1 Introduction

Analysis of economic time series under cointegration is often developed using their autoregressive—error correction model (VAR-ECM) representation. In many cases, however, the number of variables involved makes impractical the full-system approach in which all of them are jointly modeled (e.g. Johansen, 1988). Thus, researchers often resort to conditional (or partial) VAR-ECM analysis, which models a smaller set of variables conditional on the rest. Conditional modeling relates naturally to economic theory, which typically specifies relations involving endogenous variables along with a (potentially large) number of exogenous variables. Thus, the conditional approach may be a logical choice if theory makes clear predictions regarding the relations among a subset of variables, but offers little guidance on how the remaining variables are determined. Alternatively, the choice may be dictated by empirical tractability, when the large dimensionality of the full system is likely to make its empirical analysis overly cumbersome.

However, the conditional approach yields efficient inference only if the variables in the conditioning set are weakly exogenous with respect to the parameters of interest. If this is not the case, efficient inference generally requires a full-system approach. In the context of cointegrated time series, the parameters of interest typically are those of the cointegrating vector(s), in which case the conditioning variables are said to be weakly exogenous if they do not display error-correcting behavior. This topic has been discussed at length by Johansen (1992) and Boswijk (1995) among others. This literature has developed variable-addition tests to verify the validity of the weak exogeneity asumptions underlying the conditional approach; see Urbain (1995) and Boswijk and Urbain (1997) for further details and empirical applications.

To date, the focus of this literature has been confined to the time-series context. But application of the VAR-ECM approach to panel settings has become increasingly popular (see, e.g., Breitung and Pesaran, 2008). This is partly due to the power deficiencies of pure time-series approaches to cointegration (especially with relatively short time samples, as is commonly the case), which under appropriate conditions can be mitigated by combining time-series information with cross-sectional information. It is also motivated by the increasing availability of both micro and macroeconomic panel data sets.

Modelling panel data within the VAR-ECM framework is arguably more demanding than in the case of pure time series data. For instance, the researcher must elicit certain assumptions regarding parameter heterogeneity and interactions among units in the panel. The laxer the assumptions, the larger the number of parameters to be estimated. Conditional analysis can help reduce that number drastically, and this makes it very appealing for the practitioner in the panel setting.

Validity of the inference obtained from the conditional ECM in a panel context depends on weak exogeneity requirements similar to those that apply in the time-series case, and this naturally poses the need for weak exogeneity testing in panel implementations of the conditional approach. However, although the tests from the time-series literature cited above can shed light on the assumption of weak exogeneity for each cross-sectional unit in the panel, it is not obvious how they should be combined in order to jointly test weak exogeneity for the panel as a whole. Possible options range from testing whether weak exogeneity holds on average across all units,<sup>1</sup> to simply joining all individual-unit tests into a panel-wide Wald test.<sup>2</sup>

In this paper we present a new panel weak exogeneity test. Like most of the time-series literature, we focus on the case in which the parameters of interest are those of the cointegrating vector(s). Weak exogeneity then amounts to the requirement that the cointegrating vector(s) not enter the marginal model. Following Westerlund and Hess (2011), we propose a test of this hypothesis based on the maximum of the individual Wald test statistics over all the cross-sectional units. We show that this maximal test has a limiting Gumbel distribution as both T (the time series sample size) and N (the number of units) go to infinity. We also discuss how parameter heterogeneity and cross-sectional dependence of the type considered by Bai and Kao (2006) can be accommodated within our testing strategy. Simulation experiments investigate the size and power properties of the test in finite samples. Overall, our simulation results suggest that the test performs well under sample sizes commonly encountered in applied research. In particular, the testing procedure has good small sample properties in situations where the number of units and time periods are roughly similar, under which the conventional joint Wald test can be expected to behave poortly. Also, the test proposed here has power against alternatives under which the test of "average weak exogeneity" has zero power.

Finally, we illustrate the usefulness of the approach in practice by testing weak exogeneity of disposable income and wealth in an aggregate consumption function.

The rest of the paper is organized as follows. Section 2 lays out the analytical framework and presents the weak exogeneity test. Section 3 reports the simulation experiments. Section 4 presents an empirical application of the test. Finally, section 5 offers some concluding observations.

<sup>&</sup>lt;sup>1</sup>See, e.g., Gemmell, Kneller and Sanz (2012) and Acosta-Hormaechea and Yoo (2012) for recent examples of this approach.

<sup>&</sup>lt;sup>2</sup>Canning and Pedroni (2008) examine alternative panel testing strategies in a related setting, in which the concern is what they call 'long-run causality' rather than the validity of conditional inference.

<sup>&</sup>lt;sup>3</sup>Such scenario is common, for example, in the case of international macroeconomic panel data. Of course, in an alternative situation in which the time dimension of the panel greatly exceeds its cross-sectional dimension, the testing strategy could be based instead on the joint Wald test. Further, such situation could allow better accommodating flexible forms of cross-unit dependence, for example through SURE estimation of the parameters of the conditional model allowing for an unrestricted covariance matrix.

## 2 The Model

Consider the  $m \times 1$  vector  $X_{it}$ , which corresponds to the observation for unit i (i = 1, ..., N) in period t (t = 1, ..., T), and define the panel VAR( $p_i$ ) model

$$X_{it} = \sum_{j=1}^{p_i} \Pi_{ij} X_{it-j} + \epsilon_{it} \tag{1}$$

where  $\epsilon_{it}$  are independent identically distributed  $\epsilon_{it} \sim N_m(0, \Omega_i)$ . Note that this assumption rules out dependence across units in the panel, but below we shall relax this restriction and discuss how cross-sectional dependence can be accommodated in our testing procedure. The model admits an error-correction representation (see Engle and Granger, 1987):<sup>4</sup>

$$\Delta X_{it} = A_i X_{it-1} + \sum_{j=1}^{p_i - 1} A_{ij} \Delta X_{it-j} + \epsilon_{it}$$

$$\tag{2}$$

where we assume cointegration by imposing  $rank(A_i) = r_i$  with  $0 < r_i < m \ \forall i$ . We can therefore decompose  $A_i$  as  $\alpha_i \beta_i'$ , where  $\alpha_i$  and  $\beta_i$  are  $m \times r$  matrices of full column rank for all i. While  $\alpha_i$  contains the adjustment parameters, the columns of  $\beta_i$  represent the cointegrating vectors. In what follows we assume that the latter are the parameters of interest.

Note that all parameters are allowed to be individual-specific. Therefore, our testing procedure can handle situations in which both short-run dynamics and long-run relationships differ across the units in the panel. However, we do need to assume that the number of cointegration relations is the same for all units, i.e.,  $r_i = r \,\forall i$  with 0 < r < m.<sup>5,6</sup>

## 2.1 Conditional Analysis and Weak Exogeneity

Based on economic theory, or just to reduce the dimensionality of the system for computational tractability, a researcher might be interested in modelling the equations for a subset of variables in the above formulation, taking the rest as given. For this purpuse, we can partition the vector  $X_{it}$  as  $(y'_{it}, z'_{it})'$ , where  $y_{it}$  represents a g-vector containing the variables of interest and the k-vector  $z_{it}$  contains the set of conditioning variables (m = g + k).

<sup>&</sup>lt;sup>4</sup>To avoid notational clutter, the model omits unit-specific constants and time-trends.

<sup>&</sup>lt;sup>5</sup>Larsson and Lyhagen (2000) discuss in detail how to test for a common cointegrating rank across units in the panel.

<sup>&</sup>lt;sup>6</sup>Note that the formulation in (2) rules out cointegration across units in the panel, and excludes also the possibility that deviations from the long-run equilibrium in a given unit could impact the behavior of other units.

Against this background, the model in (2) can be decomposed into two components. First, the conditional model for  $y_{it}$  given  $z_{it}$ :

$$\Delta y_{it} = \Pi_{0i} \Delta z_{it} + \alpha_{i,yz} \beta_i' X_{it-1} + \sum_{j=1}^{p_i - 1} A_{ij,yz} \Delta X_{it-j} + \epsilon_{it,yz}$$

$$\tag{3}$$

where  $\alpha_i$ ,  $A_{ij}$ , and  $\Omega_i$  are partitioned so that  $\Pi_{0i} = \Omega_{i,yz}\Omega_{i,zz}^{-1}$ ,  $\alpha_{i,yz} = \alpha_{i,y} - \Pi_{0i}\alpha_{i,z}$ ,  $A_{ij,yz} = A_{ij,y} - \Pi_{0i}A_{ij,z}$  for  $j = 1, ..., p_i - 1$ , and  $\epsilon_{it,yz} = \epsilon_{it,y} - \Pi_{0i}\epsilon_{it,z}$ . And, second, the marginal model of  $z_{it}$ , which consists of the last k equations of (2):

$$\Delta z_{it} = \alpha_{i,z} \beta_i' X_{it-1} + \sum_{j=1}^{p_i - 1} A_{ij,z} \Delta X_{it-j} + \epsilon_{it,z}$$

$$\tag{4}$$

Since the parameters of the marginal and conditional models are interrelated, analysis of the full system in (2) is generally required for drawing efficient inference about  $\beta_i$ . However, when the vector  $z_{it}$  is weakly exogenous for  $\beta_i$  in the sense of Engle et al. (1983), analysis of the conditional system is efficient and equivalent to full-model analysis.

Formally, the conditioning variables  $z_{it}$  are said to be weakly exogenous for the parameters of interest if, (i) the parameters in the conditional model and the parameters in the marginal model are variation free (i.e. they are not subject to any joint restrictions), and (ii) the parameters of interest are functions of the parameters in the conditional model only. If the parameters of interest are those corresponding to the cointegrating vectors  $\beta_i$ , a necessary and sufficient condition for  $z_{it}$  to be weakly exogenous is that  $\alpha_{i,z} = 0$  (see Johansen, 1992 Theorem 1). In this case, efficient inference on  $\beta_i$  can be conducted by analyzing (3) alone.

As suggested by Johansen (1992), if  $g \geq r$  one can easily test weak exogeneity using variable addition tests. In particular, we can estimate the cointegrating vectors from the conditional system (3), and insert the superconsistent estimates  $\hat{\beta}_i$  into the marginal equations (4). Hence, for fixed  $\beta_i = \hat{\beta}_i$  we can test the hypothesis that the coefficients  $\alpha_{i,z}$  are jointly zero by, for instance, a Wald test.

This testing procedure is straightforward in the time-series framework discussed in Johansen (1992). However, in the panel setting considered here, the choice of testing procedure is less obvious. One might be tempted to construct a Wald test for the null that  $\alpha_{i,z} = 0$  for all i simultaneously, but such test will be poorly behaved when N and T are roughly similar, as commonly encountered in practice. Alternatively, one might consider a test of the null that the average of  $\alpha_{i,z}$  across i equals zero, but this would be informative about a somewhat different hypothesis, namely that weak exogeneity holds 'on average' among the cross-sectional units. There are situations in which such hypothesis would hold even though unit-specific weak exogeneity does not; for instance, if the  $\alpha_{i,z}$ 

are distributed symmetrically around zero, so that some units exhibit  $\alpha_{i,z} > 0$  and others  $\alpha_{i,z} < 0$ . In such setting, a test of weak exogeneity based on whether the average  $\alpha_{i,z}$  equals zero would have zero power. We next suggest an alternative test based on the maximum of the individual Wald test statistics.

#### 2.2 A Panel Test of Weak Exogeneity

Stacking the observations for all units in the panel, we can write the marginal system in (4) as follows:

$$\Delta z_i = [I_k \otimes \hat{\xi}_{i(-1)}] \alpha_{i,z} + [I_k \otimes \Delta X_{i(-i)}] A_{i,z} + \epsilon_{i,z}$$

$$\tag{5}$$

where  $\Delta z_i = (\Delta z_{i1}^1, ..., \Delta z_{iT}^1, \Delta z_{i1}^2, ..., \Delta z_{iT}^2, ..., \Delta z_{i1}^k, ..., \Delta z_{iT}^k)'$  is a  $kT \times 1$  vector,  $\hat{\xi}_{i(-1)}$  is a  $T \times 1$ 

vector given by  $(\hat{\xi}_{i0},...,\hat{\xi}_{iT-1})'$  with  $\hat{\xi}_{it-1} = \hat{\beta}'_i X_{it-1}$ . Finally,  $\Delta X_{i(-j)}$  is the  $T \times m(p_i - 1)$  matrix of data corresponding to all lags of  $\Delta X_{it}$  included in the model with  $A_{i,z} = (A'_{i1,z}, A'_{i2,z}, ..., A'_{ip-1,z})'$  the corresponding  $km(p_i - 1) \times 1$  vector of coefficients.

We can estimate the coefficients in (5) by ordinary least squares (OLS) and obtain:<sup>7</sup>

$$\begin{pmatrix} \hat{\alpha}_{i,z} \\ \hat{A}_{i,z} \end{pmatrix} = \begin{pmatrix} I_k \otimes \hat{\xi}'_{i(-1)} \hat{\xi}_{i(-1)} & I_k \otimes \hat{\xi}'_{i(-1)} \Delta X_{i(-j)} \\ I_k \otimes \Delta X'_{i(-j)} \hat{\xi}_{i(-1)} & I_k \otimes \Delta X'_{i(-j)} \Delta X_{i(-j)} \end{pmatrix}^{-1} \begin{pmatrix} (I_k \otimes \hat{\xi}'_{i(-1)}) \Delta z_i \\ (I_k \otimes \Delta X'_{i(-j)}) \Delta z_i \end{pmatrix}$$
(6)

with covariance matrix estimated by:

$$\hat{V}\begin{pmatrix} \hat{\alpha}_{i,z} \\ \hat{A}_{i,z} \end{pmatrix} = \hat{\sigma}_i^2 \begin{pmatrix} I_k \otimes \hat{\xi}'_{i(-1)} \hat{\xi}_{i(-1)} & I_k \otimes \hat{\xi}'_{i(-1)} \Delta X_{i(-j)} \\ I_k \otimes \Delta X'_{i(-j)} \hat{\xi}_{i(-1)} & I_k \otimes \Delta X'_{i(-j)} \Delta X_{i(-j)} \end{pmatrix}^{-1}$$
(7)

where  $\hat{\sigma}_i^2$  is the least squares variance estimator.

For a given unit in the panel we can consider the following Wald test of the null of weak exogeneity:

$$\hat{W}_i = \hat{\alpha}'_{i,z} \left[ \hat{V}(\hat{\alpha}_{i,z}) \right]^{-1} \hat{\alpha}_{i,z} \tag{8}$$

which is asymptotically distributed as a  $\chi_k^2$  as  $T \to \infty$ .

In the panel setting, we formulate the null hypothesis to be tested as

$$H_0: \quad \alpha_{i,z} = 0 \quad \text{ for all } i$$
 (9)

against the alternative

$$H_a: \quad \alpha_{i,z} \neq 0 \quad \text{ for at least one } i$$
 (10)

<sup>&</sup>lt;sup>7</sup>In this particular case, these estimates coincide with equation-by-equation estimates because right-hand-side variables are the same in all equations and there are no cross-equation restrictions.

In parallel with the poolability test of Westerlund and Hess (2011),<sup>8</sup> consider using the maximum of the individual Wald test statistics (8) for testing  $H_0$  versus  $H_a$ :

$$\hat{W}_{max} = \max_{1 \le i \le N} \hat{W}_i \tag{11}$$

To derive its asymptotic distribution, we define the normalized test statistic:

$$\hat{W}_{Zmax} = \frac{1}{c_N} \left( \hat{W}_{max} - d_N \right) \tag{12}$$

where  $c_N = 2$  and  $d_N = F^{-1} \left(1 - \frac{1}{N}\right)$  with F(x) being the chi-squared distribution function with k degrees of freedom evaluated at x.

**Theorem 1** Under 
$$H_0$$
, as  $T \to \infty$ ,  $N \to \infty$ :  
 $P(\hat{W}_{Zmax} \le x) \longrightarrow exp(-e^{-x})$ 

**Proof** See Appendix.

Theorem 1 indicates that the  $W_{Zmax}$  test has a limiting Gumbel distribution as  $T \to \infty$ ,  $N \to \infty$ . Because the asymptotic distribution of the test is obtained by first letting  $T \to \infty$  and then letting  $N \to \infty$ , this implies that the test is appropriate in cases where N is moderate and T is large (see Phillips and Moon, 1999); this type of data configuration can be expected for instance in multy-country macroeconomic data. In the Monte Carlo experiments we investigate the accuracy of the asymptotic approximation under different (N, T) configurations.

#### 2.3 Cross-Sectional Dependence

The recent panel time-series literature has paid considerable atention to the possibility that the individual units in a panel dataset may be interdependent. We next discuss how cross-sectional dependence can be accommodated in our testing procedure. Specifically, we consider a setting along the lines of Bai and Kao (2006), in which the long-run disequilibrium error  $\xi_{it}$  has a stationary common component  $(f_t)$  and a stationary idiosyncratic component  $(\eta_{it})$ :

$$\beta_i' X_{it} = \xi_{it} = \lambda_i' f_t + \eta_{it} \tag{13}$$

<sup>&</sup>lt;sup>8</sup>The Westerlund and Hess (2011) test is based on the maximum across panel units of their individual Hausman test statistics for the null that their respective cointegrating vector parameters equal those of all the other cross seciton units.

where  $f_t$  is a  $q \times 1$  vector of latent common factors,  $\lambda_i$  is a  $q \times r$  matrix of factor loadings and  $\eta_{it}$  is a vector of idiosyncratic errors.

Importantly, by assuming that the factors  $f_t$  are stationary we rule out the presence of common stochastic trends (see, e.g., Bai et al., 2009), and thus the possibility of cointegration between the variables and the factors (see, e.g., Gengenbach et al., 2012) and, in particular, cointegration across units. From the practical perspective, whether stationarity of  $f_t$  represents a restrictive assumption depends on the application at hand. In the international business cycle literature, for example, global technology shocks – widely assumed the primary force driving world business cycles – are typically viewed as persistent but stationary (see, e.g., Kehoe and Perri, 2002); the same applies to global demand (spending) shocks (e.g., Boileau et al., 2010). In other situations, it may be more difficult to rule out common stochastic trends; for example, panel tests of purchasing power parity usually involve country-specific I(1) variables defined relative to a reference country, and this tends to introduce common stochastic trends in the analysis (e.g., Urbain and Westerlund, 2011).

Our testing procedure can accommodate cross-sectional dependence of the assumed form without modification. In practice, the test statistic can be computed as described above but replacing the original variables  $y_{it}$  and  $z_{it}$  with the residuals of a regression of these variables on their cross-sectional averages, i.e., the common correlated effects (CCE) approach proposed in Pesaran (2006). Furthermore, it would be straightforward to allow also for stationary common factors in the conditioning variables (as noted in Remark 1.1 in Bai and Kao, 2006), for example by also employing in the variable-addition tests the CCE approach.

A particular (and more restrictive) form of cross-sectional dependence worth mentioning arises from further imposing that  $\lambda_i = \lambda \, \forall i$  together with q = 1. In this case, one can remove the common effect prior to the test just by cross-sectional de-meaning of the data.

In the simulations below, we investigate the impact of these types of cross-sectional dependence on the finite-sample behavior of the test.

<sup>&</sup>lt;sup>9</sup>Strictly speaking, if the common factors are I(1), the CCE approach in Pesaran (2006), whose use is suggested below in the text, would still remove the common components (Kapetianos et al., 2011) and allow performing the panel weak exogeneity test. However, such procedure would not take proper account of the possible presence of cross-unit cointegration.

## 3 Monte Carlo Evidence

To evaluate the finite-sample behavior of the proposed testing procedure, we build upon the simulation designs in Phillips and Hansen (1990), Phillips and Loretan (1991), and Boswijk (1995). For each unit in the panel (i = 1, ..., N) we consider the case of two cointegrated variables under the following data generating process (DGP):<sup>10</sup>

$$y_{it} = \theta_i z_{it} + u_{y,it} \tag{14}$$

$$z_{it} = z_{it-1} + u_{z,it} (15)$$

$$u_{it} = \epsilon_{it} + \Gamma_i \epsilon_{it-1}$$

where  $u_{it} = (u_{y,it}, u_{z,it})'$  and  $\epsilon_{it} = (\epsilon_{1,it}, \epsilon_{2,it})'$ . Moreover,  $\epsilon_{it} \sim N(0, \Omega)$  with

$$\Omega = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \quad \text{and} \quad \Gamma_i = \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \gamma_i & \Gamma_{22} \end{pmatrix}$$
 (16)

The  $\gamma_i$  parameter is the error correction coefficient in the marginal model for  $z_{it}$  and thus determines whether or not  $z_{it}$  is weakly exogenous for  $\theta_i$ . In our simulations, we consider a fraction  $\delta$  of units not satisfying the weak exogeneity of  $z_{it}$ ; for these units,  $\gamma_i = -0.8$ , as in Phillips and Loretan (1991). The remaining  $(1 - \delta)N$  units do satisfy the weak exogeneity condition (i.e.  $\gamma_i = 0.0$ ). With respect the remaining parameter values to be chosen, in our baseline design we also follow Phillips and Loretan (1991) and fix  $\rho = 0.5$ ,  $\Gamma_{11} = 0.3$ ,  $\Gamma_{12} = 0.4$ , and  $\Gamma_{22} = 0.6$  for i = 1, ..., N. The cointegrating vector is also common to all units in the baseline design with  $\theta_i = 2 \,\forall i$ .

Given the simulated samples for each of the N units over T periods, our simulation exercises consist of 2 steps. First, for fixed  $\theta_i$  we compute the individual Wald statistics of the form (8) for the null  $H_0: \gamma_i = 0$ . Second, from the N individual Wald statistics thus obtained we compute the normalized  $\hat{W}_{Zmax}$  statistic and evaluate its size and power computing the rejection frequencies under different values of  $\delta$ , the fraction of units in which  $z_{it}$  is not weakly exogenous.

(8) employ a generated regressor. In the time-series context (i.e., N=1), this should be of little consequence for the behavior of the variable-addition test, owing to the super-consistency of  $\hat{\theta}_i$  with respect to T; see, e.g., Boswijk and Urbain (1997). In the panel context, however, the situation is somewhat less clearcut, because  $\hat{\theta}$  based on the pooled approach converges at just the standard rate  $\sqrt{N}$  with respect to the cross-sectional sample size. This might raise concerns about the performance of our testing procedure unless T is very large. Nevertheless, the results shown in the table reveal that performance is little affected by the use of the pooled estimator of the long-run relationship.

 $^{10}$ In a recent paper, Gengenbach et al. (2013) derive a Granger-type representation theorem given the triangular representation of a panel cointegration model as in (14)-(15) but including common factors in both the long-run relationship and the conditioning variables. In particular, they show that such a panel cointegration model also admits an error-correction representation. It can thus be shown that a marginal model for  $\Delta z_{it}$  as in equation (4) can be derived from the error-correction representation with common factors entering in both the long-run relationship and the short-run dynamics (see Remark 2 in Gengenbach et al., 2013).

This first exercise assumes a known  $\theta_i$ , common across units, i.e.,  $\theta_i = \theta \, \forall i$ . In real applications, however,  $\theta$  needs to be estimated. Hence we perform a second set of simulations involving an additional step estimating  $\theta$  at the beginning of each replication. In particular, we consider two different estimation strategies, one based on pooling the data for all the units, and another based on unit-by-unit estimation. Under homogeneity of the cointegrating vector, the pooled approach is more efficient than the unit-by-unit approach. However, we also consider the unit-by-unit strategy because the speed of adjustment in (3) might vary among cross-sectional units even in the case of homogeneous  $\theta$ .<sup>11</sup> For both the pooled and the unit-by-unit approaches we follow Boswijk (1995) and employ OLS including three lags of  $\Delta z_{it}$  and two lags of  $\Delta y_{it}$  as suggested by Phillips and Hansen (1990). Holding the resulting  $\hat{\theta}$  fixed, we repeat the two steps above.

Panel A of Table 1 presents the simulation results based on 50,000 replications. For sample sizes commonly found in practice, we see that the test is appropriately sized given the nominal 5% size considered. However, in cases when N is not substantially smaller than T, the test appears slightly oversized, which is not surprising in view of the sequential limit theory underlying its asymptotic distribution. Power rises steadily with both N and T, as well as with  $\delta$ , the number of units violating weak exogeneity in the simulated samples. Indeed, as sample sizes grow, the power of the test becomes considerable even when only 20% of units violate weak exogeneity.

In Panel B of Table 1, we explore the performance of the test when using the estimated  $\hat{\theta}$  based on the pooled approach instead of the true  $\theta$ .<sup>12</sup> While power is uniformly lower than when using  $\theta$ , the difference is almost negligible.<sup>13</sup> Next, in Panel C of Table 1 we present the results based on the unit-by-unit estimates of the common cointegrating vector. Power is always slightly smaller than in the pooled approach reported in Panel B, as expected from the superior efficiency of the pooled estimator under homogeneity. However, the differences are in all cases fairly modest.

In turn, in Table 2 we evaluate the performance of the test in situations in which cross-sectional units are not independent from each other. For this purpose, we consider the DGP in (14) but allowing cross-section dependence through the error term  $u_{it}$ . In particular, we allow for cross-section dependence in the long-run relationship by re-specifying equation (14) as

$$y_{it} = \theta_i z_{it} + \lambda_i' f_t + u_{y,it} \tag{17}$$

<sup>&</sup>lt;sup>11</sup>An alternative approach could be the pooled mean group —PMG— estimator discussed in Pesaran et al. (1999). The PMG estimator can accommodate homogeneous long-run coefficients together with heterogeneous speed of adjustment and short-run coefficients.

<sup>&</sup>lt;sup>12</sup>In practical terms, this means that the subsequent estimation of  $\gamma_i$  and the construction of the Wald statistics <sup>13</sup>Further, closer comparison of panels A and B of Table 1 reveals that the decline in power from using  $\hat{\theta}$  instead of  $\theta$  is inversely related to T/N, as should be expected in light of the theoretical argument mentioned in the previous footnote.

In Panel A we consider the simplest case of a common time effect by setting  $f_t \sim N(0,1)$  and  $\lambda_i = 1 \,\forall i$ . As explained above, in this case we deal with dependence across units in our panel by employing in the testing procedure the cross-sectionally demeaned variables. In Panel B we consider a more general form of cross-section dependence as discussed in Bai and Kao (2006). Specifically, the common factors are allowed to have an heterogeneous impact on the individual units by setting  $f_t \sim N(0,1)$  and  $\lambda_i \sim N(1,1)$  as in Westerlund and Hess (2011).

The Monte Carlo simulations reported in Table 2 show that cross-sectional dependence of these forms has a fairly modest effect on the size and power properties of our testing procedure. While power is generally smaller than in the case of cross-sectional independence, it is still considerable in all cases.

In Table 3, we explore the effects of introducing heterogeneity in the long-run relationship and considering an alternative type of serial correlation in the idiosyncratic errors. In particular, in Panel A of Table 3 we allow the long-run parameter  $\theta_i$  to vary across units. Overall, the size and power properties of the test for different values of  $\delta$  are very similar to those found in Table 1 under homogeneity. In a majority of cases power is slightly smaller than in the homogeneous case, but the differences are just in the third decimal. Therefore, we conclude that cross-section heterogeneity can be appropriately handled by our testing procedure.

Table 1: Size and Power under Cross-section Homogeneity

Parameters	N	Т	$\delta = 0.0$	$\delta = 0.2$	$\delta = 0.4$	$\delta = 0.6$	$\delta = 0.8$	$\delta = 1.0$
	Panel A: Using $\theta_i$							
$\theta_i = 2.0 \ \forall i$	10	50	0.073	0.323	0.504	0.636	0.736	0.806
$\rho = 0.5$	10	100	0.059	0.639	0.862	0.947	0.981	0.993
$\Gamma_{11} = 0.3$	10	150	0.056	0.863	0.980	0.997	1.000	1.000
$\Gamma_{12} = 0.4$	20	50	0.090	0.431	0.639	0.775	0.858	0.912
$\Gamma_{22} = 0.6$	20	100	0.069	0.796	0.955	0.990	0.998	0.999
	20	150	0.065	0.961	0.998	1.000	1.000	1.000
				Pa	nel B: Us	ing poole	$\hat{\theta}$	
	10	50	0.074	0.321	0.501	0.632	0.731	0.801
	10	100	0.059	0.636	0.860	0.946	0.980	0.992
	10	150	0.056	0.861	0.980	0.997	1.000	1.000
	20	50	0.091	0.429	0.636	0.771	0.855	0.909
	20	100	0.069	0.795	0.955	0.989	0.998	0.999
	20	150	0.065	0.960	0.998	1.000	1.000	1.000
				Panel	C: Using	unit-by-u	$\inf \hat{\theta}_i$	
	10	50	0.074	0.286	0.445	0.572	0.671	0.744
	10	100	0.058	0.612	0.838	0.934	0.974	0.989
	10	150	0.055	0.850	0.977	0.996	0.999	1.000
	20	50	0.090	0.380	0.573	0.709	0.800	0.861
	20	100	0.068	0.766	0.944	0.985	0.997	0.999
	20	150	0.065	0.954	0.998	1.000	1.000	1.000

Notes: 50,000 replications;  $\delta$  refers to the fraction of units for which weak exogeneity does not hold. For the case  $\delta=0.0$  the numbers represent the size of the  $\hat{W}_{Zmax}$  test. For the remaining cases with  $\delta>0.0$ , the numbers in the table refer to power. The nominal level test is fixed at 5%. We fix  $\gamma_i=-0.8$  in  $N\delta$  units and  $\gamma_i=0.0$  in the remaining  $N(1-\delta)$  units.

In Panel B of Table 3 we consider autoregressive idiosyncratic errors instead of moving average errors as in the simulations above. For this purpose, we define:

$$u_{it} = \Psi_i u_{it-1} + \epsilon_{it} \tag{18}$$

where  $\epsilon_{it}$  is defined above and

$$\Psi_i = \begin{pmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21,i} & \Psi_{22} \end{pmatrix} \tag{19}$$

Table 2: Size and Power under Cross-section Dependence

Parameters	N	Т	$\delta = 0.0$	$\delta = 0.2$	$\delta = 0.4$	$\delta = 0.6$	$\delta = 0.8$	$\delta = 1.0$
	Panel A: Cross-Section Dependence with a Common Time Effect							
$\lambda_i = 1.0 \ \forall i$	10	50	0.076	0.240	0.400	0.557	0.689	0.795
$\theta_i = 2.0 \ \forall i$	10	100	0.058	0.471	0.749	0.898	0.965	0.991
$\rho = 0.5$	10	150	0.055	0.701	0.931	0.988	0.999	1.000
$\Gamma_{11} = 0.3$	20	50	0.093	0.372	0.587	0.741	0.843	0.908
$\Gamma_{12} = 0.4$	20	100	0.068	0.716	0.924	0.983	0.997	0.999
$\Gamma_{22} = 0.6$	20	150	0.065	0.921	0.996	1.000	1.000	1.000
	Panel B: Cross-Section Dependence as in Bai and Kao (2006)							
$\lambda_i \sim N(1,1)$	10	50	0.077	0.240	0.403	0.557	0.689	0.797
$\theta_i = 2.0 \ \forall i$	10	100	0.056	0.471	0.749	0.897	0.966	0.990
$\rho = 0.5$	10	150	0.057	0.706	0.932	0.988	0.998	1.000
$\Gamma_{11} = 0.3$	20	50	0.093	0.370	0.580	0.736	0.843	0.908
$\Gamma_{12} = 0.4$	20	100	0.068	0.714	0.923	0.983	0.997	0.999
$\Gamma_{22} = 0.6$	20	150	0.064	0.921	0.995	1.000	1.000	1.000

Notes: 50,000 replications;  $\delta$  refers to the fraction of units for which weak exogeneity does not hold. For the case  $\delta = 0.0$  the numbers represent the size of the  $\hat{W}_{Zmax}$  test. For the remaining cases with  $\delta > 0.0$ , the numbers in the table refer to power. The nominal level test is fixed at 5%. We fix  $\gamma_i = -0.8$  in  $N\delta$  units and  $\gamma_i = 0.0$  in the remaining  $N(1 - \delta)$  units.

In order to ensure comparability with the DGP based on moving average errors and guarantee stationarity of the resulting autoregressive errors, we set  $\Psi_{11} = 0.3$ ,  $\Psi_{12} = 0.2$ ,  $\Psi_{22} = 0.6$ ; moreover, we choose a value of  $\Psi_{21,i}$  that produces  $\alpha_{i,z} = \gamma_i = -0.8$  as in the simulations above. For this purpose, we make use of the results in Cappuccio and Lubian (1996) and Gengenbach et al. (2013) who derive the conditional and marginal ECM representation for a cointegration model as in (14)-(15). In particular, when the idiosyncratic errors are AR(1) it can be shown that the error correction coefficient in the marginal model has the form:

$$\alpha_{i,z} = \frac{\Psi_{21,i}}{1 - \Psi_{11} - \Psi_{22} - \Psi_{21,i}\Psi_{12} + \Psi_{11}\Psi_{22}}$$
 (20)

Given the values above for  $\Psi_{11}$ ,  $\Psi_{12}$  and  $\Psi_{22}$ , we set  $\Psi_{21,i}$  so that  $\alpha_{i,z} = -0.8 = \gamma_i$ . By doing this we ensure that the error correction behavior of the conditioning variables is the same under both the autoregressive and the moving average errors for the  $\delta N$  units not satisfying the weak exogeneity condition. For the remaining  $(1 - \delta)N$  units we set  $\Psi_{21,i} = 0$ .

Table 3: Size and Power under Cross-section Heterogeneity and AR(1) Errors

Parameters	N	Т	$\delta = 0.0$	$\delta = 0.2$	$\delta = 0.4$	$\delta = 0.6$	$\delta = 0.8$	$\delta = 1.0$
	Panel A: Cross-Section Heterogeneity							
$\theta_i \sim U(1,3)$	10	50	0.074	0.324	0.506	0.638	0.734	0.803
$\rho = 0.5$	10	100	0.058	0.641	0.866	0.948	0.979	0.993
$\Gamma_{11} = 0.3$	10	150	0.057	0.862	0.981	0.998	0.999	1.000
$\Gamma_{12} = 0.4$	20	50	0.092	0.431	0.638	0.773	0.858	0.910
$\Gamma_{22} = 0.6$	20	100	0.067	0.795	0.956	0.990	0.998	1.000
	20	150	0.066	0.962	0.998	1.000	1.000	1.000
	Panel B: AR(1) Errors							
$\theta_i = 2.0 \ \forall i$	10	50	0.072	0.346	0.545	0.683	0.781	0.841
$\rho = 0.5$	10	100	0.047	0.695	0.900	0.968	0.990	0.996
$\Psi_{11} = 0.3$	10	150	0.041	0.901	0.989	0.999	1.000	1.000
$\Psi_{12} = 0.2$	20	50	0.087	0.466	0.686	0.817	0.893	0.937
$\Psi_{22} = 0.6$	20	100	0.056	0.843	0.975	0.996	0.999	1.000
	20	150	0.048	0.978	0.999	1.000	1.000	1.000

Notes: 50,000 replications;  $\delta$  refers to the fraction of units for which weak exogeneity does not hold. For the case  $\delta = 0.0$  the numbers represent the size of the  $\hat{W}_Z$  test. For the remaining cases with  $\delta > 0.0$ , the numbers in the table refer to power. The nominal level test is fixed at 5%. We fix  $\gamma_i = -0.8$  in  $N\delta$  units and  $\gamma_i = 0.0$  in the remaining  $N(1 - \delta)$  units.

When considering autoregressive errors in Panel B of Table 3, power is slightly larger and size slightly smaller than in the case of moving average errors in Panel A of Table 1. We thus conclude that this alternative form of serial correlation in the errors can be appropriately handled by our testing procedure.

Finally, following Westerlund and Hess (2011), we also investigated the finite sample properties of an alternative panel weak exogeneity test based on the normalized sum (instead of the maximum) of the individual Wald test statistics:

$$\hat{\bar{W}}_Z = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \hat{W}_i - \sqrt{N}$$
 (21)

which is asymptotically distributed as a N(0,2) as  $T \to \infty$ ,  $N \to \infty$  (see Westerlund and Hess, 2011). Simulation results<sup>14</sup> indicate that, in finite samples, this test has similar power to that of the  $\hat{W}_{Zmax}$  test, but exhibits larger size distortions; hence the  $\hat{W}_{Zmax}$  test appears preferable.

<sup>&</sup>lt;sup>14</sup>These results are not reported to save space, but they are available upon request.

# 4 Empirical Illustration

In this section we apply the panel test of weak exogeneity to the estimation of a consumption function along the lines of Ludwig and Sløk (2004). They analyze the impact of changes in household wealth on consumption distinguishing between two different components of wealth, namely, housing wealth and stock market wealth, using quarterly data for a sample of 16 OECD countries over the period 1960-2000.

In analogy with equation (3) above, Ludwig and Sløk (2004) consider the following conditional error correction model:<sup>15</sup>

$$\Delta c_{it} = \alpha_i \left( c_{it-1} - \beta_0 - \beta_1 y_{it}^d - \beta_2 w_{it}^{sw} - \beta_3 w_{it}^{hw} \right) + a_1 \Delta y_{it}^d + a_2 \Delta w_{it}^{sw} + a_3 \Delta w_{it}^{hw} + \epsilon_{it}$$
 (22)

where  $c_{it}$  refers to log private per capita consumption in country i at quarter t,  $y_{it}^d$  is log per capita disposable income,  $w_{it}^{sw}$  is the log stock market wealth proxied by a stock market price index, and  $w_{it}^{hw}$  refers to log housing wealth proxied by a house price index (see Ludwig and Sløk (2004) for full details on the data used). In the notation of Section 2, we can define the vector  $X_{it} = (c_{it}, y_{it}^d, w_{it}^{sw}, w_{it}^{hw})'$ . Analysis of the conditional model in (22) provides efficient inference as long as the conditioning variables  $y^d$ ,  $w^{sw}$ , and  $w^{hw}$  are weakly exogenous for the long-run parameter vector  $\beta = (\beta_0, \beta_1, \beta_2, \beta_3)$ . Otherwise, analysis of the full model would be required.

We next use our panel test to assess the validity of the weak exogeneity assumption implicitly made by Ludwig and Sløk (2004) when estimating the conditional model in (22). Table 4 presents the country-specific Wald tests of weak exogeneity, together with the panel tests for each of the three conditioning variables – separately as well as jointly.

According to the individual tests in Panel A of Table 4, disposable income  $(y_{it}^d)$  and housing wealth  $(w_{it}^{hw})$  appear to be weakly exogenous with respect to the long-run relationship in equation (22) in each of the 16 countries in the sample. The panel tests reported in Panel B strongly support the assumption of weak exogeneity of disposable income and housing wealth with respect to the long-run consumption function, thus confirming the verdict of the individual tests. Regarding stock market wealth  $(w_{it}^{sw})$ , the individual tests in column 2 reject the null of weak exogeneity at the 5% level in 3 out of 16 countries. Overall, however, the panel test at the bottom of the table does not provide much evidence against the weak exogeneity of stock market wealth. Based on these findings, we conclude that none of the three conditioning variables – disposable income, stock market wealth and housing wealth – reacts significantly to deviations of aggregate consumption from its long-run equilibrium trajectory.

<sup>&</sup>lt;sup>15</sup>To avoid notational clutter we present here an ARDL(1,1,1) version of the model. However, in practice we allow for country-specific dynamics, with lag length determined by the Schwarz criterion.

Table 4: Weak Exogeneity Tests

	Panel A: Country-by-country tests							
Country	$y_{it}^d$	$w_{it}^{sw}$	$w_{it}^{hw}$	Joint				
Australia	0.214 (0.644)	0.666 (0.414)	0.000 (1.000)	0.880 (0.830)				
Belgium	0.019 (0.890)	$1.242 \ (0.265)$	0.000 (1.000)	$1.261 \ (0.738)$				
Canada	0.157 (0.692)	3.005 (0.083)	0.012 (0.913)	3.174 (0.366)				
Denmark	0.087 (0.768)	$1.452 \ (0.228)$	$0.001 \ (0.975)$	$1.541 \ (0.673)$				
Finland	$0.082 \ (0.775)$	0.008 (0.929)	0.014 (0.906)	0.104 (0.991)				
France	$0.050 \ (0.823)$	1.799 (0.180)	$0.001 \ (0.975)$	1.850 (0.604)				
Germany	0.020 (0.888)	2.668 (0.102)	$0.023\ (0.879)$	2.710 (0.439)				
Ireland	0.019 (0.890)	1.126 (0.289)	$0.001 \ (0.975)$	1.146 (0.766)				
Italy	$0.045 \ (0.832)$	$0.837 \ (0.360)$	0.088 (0.767)	0.969 (0.809)				
Japan	$0.226 \ (0.635)$	$0.400 \ (0.527)$	$0.004 \ (0.950)$	0.630 (0.890)				
Netherlands	0.005 (0.944)	6.572 (0.010)	0.011 (0.916)	6.588 (0.086)				
Norway	0.041 (0.840)	5.020 (0.025)	0.004 (0.950)	5.065 (0.167)				
Spain	$0.061\ (0.805)$	0.056 (0.813)	0.011 (0.916)	0.128 (0.988)				
Sweden	$0.082 \ (0.775)$	6.602 (0.010)	0.000 (1.000)	6.684 (0.083)				
United Kingdom	$0.221\ (0.638)$	$1.170 \ (0.279)$	0.077(0.781)	1.468 (0.690)				
United States	0.054 (0.816)	5.395 (0.020)	$0.007 \ (0.933)$	5.456 (0.141)				
	Panel B: Panel tests							
	$y_{it}^d$	$w_{it}^{sw}$	$w_{it}^{hw}$	Joint				
$\hat{W}_{Zmax}$ statistic	-1.622	1.566	-1.691	-0.316				
p-value	0.994	0.189	0.996	0.746				

Notes:  $y_{it}^d$  is log per capita disposable income,  $w_{it}^{sw}$  is the log stock market wealth, and  $w_{it}^{hw}$  refers to log housing wealth. Sample period is 1960:Q1-2000:Q4. In panel A, p-values are in parentheses. All p-values in the Table refer to the null hypothesis of weak exogeneity of the corresponding regressor(s).

The last column of Panel A in Table 4 reports joint tests of weak exogeneity of all three regressors for each individual country. The null cannot be rejected at the 5% level for any country, and only in 2 countries (the Netherlands and Sweden) out of 16 is it is rejected at the 10% level. In turn, the panel joint test at the bottom of the table, which considers all countries simultaneously, yields a p-value of 0.746, so that weak exogeneity of all three regressors for the panel as a whole cannot be rejected. All in all, these results indicate the the conditional analysis based on estimation of equation (22) alone, as done by Ludwig and Sløk (2004), is equivalent to full-model analysis in this context.

## 5 Concluding Remarks

In cointegrated panel settings, it is common for reasons of empirical tractability to model only a subset of variables conditional on another subset whose marginal processes are not modelled. This approach is as efficient as the full-model approach only when the conditioning variables are weakly exogenous for the parameters of interest.

The time series literature has proposed variable-addition tests of the null of weak exogeneity for the case of a single cross-section unit (e.g., Johansen 1992, Urbain 1995). In this paper, we extend this approach to panel settings. In particular, for the case in which the parameters of interest are those of the cointegrating vector(s), we propose a panel test based on the maximum of the individual Wald statistics for testing weak exogeneity, and we obtain the asymptotic Gumbel distribution of the resulting test statistic as both T—the number of time periods— and N—the number of cross-sectional units— tend to infinity.

Monte Carlo simulations indicate that the proposed panel test performs quite well in sample sizes commonly encountered in applied research. Moreover, the simulations also suggest that the test is robust to cross-sectional heterogeneity of the long-run parameters, as well as cross-sectional dependence of the type discussed in Bai and Kao (2006). Thus, the test should have wide applicability. Finally, we illustrate its use by assessing the weak exogeneity of disposable income and wealth for the estimation of an aggregate consumption function.

# A Appendix

#### A.1 Proofs

**Proof of Theorem 1** In order to derive the asymptotic distribution of the  $\hat{W}_{Zmax}$  test, we consider a sequential limit in which  $T \to \infty$  followed by  $N \to \infty$  as suggested by Phillips and Moon (1999). This sequential approach is based on fixing one of the indexes, say N, and allowing the other (i.e. T) to pass to infinity. By then letting N pass to infinity, a sequential limit is obtained.

We first use time series limit theory for any given unit i in the panel, i.e., we pass T to infinity for fixed N. In this case, it is a well-known result that:

$$\hat{W}_i \to \chi_k^2 \text{ as } T \to \infty$$
 (23)

for each unit in the panel i = 1, ..., N. Given this result, having passed T to infinity in equation (11) we obtain:

$$W_{max} = \max_{1 \le i \le N} W_i \tag{24}$$

where  $W_i \sim \chi_k^2$  for all i.

We next allow  $N \to \infty$  and apply limit theory to the statistic in (24). In particular we make use of a well-known result in extreme value theory which provides us with a set of limit laws for maxima. Let  $M_N$  be the maximum of a sequence of N iid chi-squared variables,  $M_n = max(X_1, ..., X_N)$ . Then, the Fisher-Tippett theorem states that:

$$c_N^{-1}(M_N - d_N) \longrightarrow \Lambda$$
 (25)

as  $N \to \infty$ . Where, for  $x \in \mathbb{R}$ ,  $\Lambda(x) = exp(-e^{-x})$  refers to the Gumbel distribution function, and the norming constants  $c_N > 0$  and  $d_N \in \mathbb{R}$  are the mean excess function and the empirical version of the  $(1 - n^{-1})$ -quantile of the underlying distribution function respectively (see Embrechts et al., 1997). Combining this result with the large T, fixed N asymptotic test in (24) we have that:

$$c_N^{-1}(W_{max} - d_N) \longrightarrow \Lambda \text{ as } N \to \infty$$
 (26)

where, as discussed in Westerlund and Hess (2011), the norming constants are given by  $d_N = F^{-1}(1 - \frac{1}{N})$  and  $c_N = F^{-1}(1 - \frac{1}{Ne}) - d_N$ , or  $c_N = 2$  as  $N \to \infty$ . Note that F(x) here refers to the chi-squared distribution function with k degrees of freedom.

Combining the results in (23) and (26):

$$\hat{W}_{Zmax} \longrightarrow \Lambda \text{ as } T \to \infty, N \to \infty$$
 (27)

which indicates that  $\hat{W}_{Zmax}$  has a limiting Gumbel distribution as  $T \to \infty, N \to \infty$ 

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