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2012

Giancarlo Corsetti, Luca Dedola
and Francesca Viani

Documentos de Trabajo
N.º 1212

BANCO DE ESPAÑA
Eurosistema

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Giancarlo Corsetti

CAMBRIDGE UNIVERSITY, EUI, AND CEPR

Luca Dedola

EUROPEAN CENTRAL BANK AND CEPR

Francesca Viani

BANCO DE ESPAÑA

(*) We thank an anonymous referee, our discussant Alessandro Rebucci, Marianne Baxter, Mario Crucini, Mick Devereux, Robert Kollmann and participants in the Conference "International Risk Sharing" on October 22-23 2010 in Brussels, for useful comments; we are grateful to Domenico Giannone for many helpful discussions. Charles Gottlieb provided excellent research assistance. The work on this paper is part of PEGGED (Politics, Economics and Global Governance: The European Dimensions), Contract no. SSH7-CT-2008-217559 within the 7th Framework Programme for Research and Technological Development. Support from the Pierre Werner Chair Programme at the European University Institute is also gratefully acknowledged. The views expressed in this paper do not necessarily reflect those of the ECB, the Banco de España, or any of the institutions to which the authors are affiliated.

Documentos de Trabajo. N.º 1212

2012

ISSN: 1579-8666 (on line)

Abstract

We decompose the correlation between relative consumption and the real exchange rate into its dynamic components at different frequencies. Using multivariate spectral analysis techniques we show that, at odds with a high degree of risk-sharing, in most OECD countries the dynamic correlation tends to be quite negative, and significantly so, at frequencies lower than two years —the appropriate frequencies for assessing the performance of international business cycle models. Theoretically, we show that the dynamic correlation over different frequencies predicted by standard open-economy models is the sum of two terms: a term constant across frequencies, which can be negative when uninsurable risk is large; and a term variable across frequencies, which in bond economies is necessarily positive, reflecting the insurance that intertemporal trade provides against forecastable contingencies. Numerical analysis suggests that leading mechanisms proposed by the literature to account for the puzzle are consistent with the evidence across the spectrum.

Keywords: Consumption-exchange rate anomaly, incomplete markets, frequency domain analysis.

JEL classification: F41, F42.

Resumen

Descomponemos la correlación entre el consumo relativo y el tipo de cambio real en sus componentes dinámicos a diferentes frecuencias. Utilizando técnicas de análisis espectral multivariado mostramos que, en contradicción con un alto grado de diversificación del riesgo, en la mayoría de los países de la OCDE la correlación dinámica tiende a ser bastante negativa, y significativamente negativa a frecuencias inferiores a dos años —las frecuencias apropiadas para evaluar el desempeño de los modelos internacionales del ciclo económico—. En teoría mostramos que la correlación dinámica a diferentes frecuencias predicha por modelos estándar de economía abierta, es la suma de dos términos: un término constante en cada frecuencia, que puede ser negativo cuando el riesgo no asegurable es grande; y un término que varia con la frecuencia, que en economías con bonos es necesariamente positivo y que refleja la cobertura de riesgo contra contingencias predecibles proporcionada por el comercio intertemporal. El análisis numérico sugiere que los mecanismos principales propuestos por la literatura para dar cuenta de la anomalía, son consistentes con la evidencia empírica a diferentes frecuencias del espectro.

Palabras claves: Anomalía consumo-tipo de cambio, mercados incompletos, análisis de frecuencia de dominio.

Códigos JEL: F41, F42.

1 Introduction

Understanding the role that frictions and distortions in capital markets play in shaping the international business cycle has long been a key objective of international macroeconomic research — more so in recent years, in light of the disruptive aggregate consequences of the financial market imperfections commonly placed at the root of the global crisis. The notion that international financial markets are efficiently integrated, for instance, appears at odds with a number of stylized facts, pointing to apparent violations of the most basic conditions for efficient risk-sharing across countries. Not only portfolios remain poorly diversified internationally, despite capital account liberalization and financial globalization. Most crucially, systematic cross-country evidence suggests that domestic consumption demand hardly falls (as it would be efficient) when the price of domestic consumption is high relative to the price abroad, i.e. when the real exchange rate appreciates — a fact detailed by Backus and Smith (1993) and many subsequent studies.

The pervasiveness of the theoretical 'puzzle' or 'anomaly' emphasized by Backus Smith (1993), lies in the fact that standard international business cycle models have a hard time matching this evidence, even when they explicitly eschew the assumption that markets are complete.[1] Addressing the BS puzzle requires more than allowing for financial imperfections. It calls for a thorough reconsideration of the received wisdom on the international transmission mechanism. In this sense, risk sharing defines a key hurdle for international open macro models — a point emphasized by Obstfeld and Rogoff (2001) and Chari Kehoe and McGrattan (2002) among others.

In this paper, we re-examine the Backus-Smith (henceforth BS) puzzle from an empirical and a theoretical perspective, by decomposing the BS correlation — between relative consumption and real exchange rates — in its dynamic components at different frequencies. We make two contributions to the literature. First, we show that, among the OECD countries, the dynamic correlation between relative consumption and real depreciation tends to be more negative, and significantly so, at frequencies lower than two years, that is, at business cycle and lower frequencies. This finding casts doubts on a possible interpretation of the BS evidence, as a manifestation of the so-called 'exchange rate disconnect' puzzle. On the contrary, the evidence of a negative correlation is pervasive at the appropriate frequencies for confronting the international business cycle models with the data, strengthening the case for placing international risk-sharing (and lack thereof) centerstage in the development of international economics. Furthermore, in some cases in which an overall BS statistic is close to zero, we find that the dynamic correlation is positive at high frequencies, but negative at lower frequencies. Spectral analysis indeed underscores the point that the BS correlation (especially when computed with first-differenced data) may give a somehow distorted picture, as it places too much weight on high

[1] See Obstfeld and Rogoff (2001) and references therein for the link between the Backus-Smith statistic and other indicators of lack of of international risk sharing, such as the Feldstein-Horioka puzzle and the consumption correlation puzzle.

frequencies. In light of these results, the BS puzzle is 'worse than you think', as it cannot be attributed to the weak connection between the exchange rate and macroeconomic variables.

Second, we show that the dynamic BS correlation predicted by standard open-economy models can be decomposed into the sum of two terms. One term is related to the risk insured via intertemporal trade. In bond economies, this terms is always positive, as a function of the amount of insurance that trade in bond provides against forecastable contingencies, and will generally vary across frequencies, depending on the mechanism by which shocks propagate according to the model. The other term, related to uninsurable risk, can be positive or negative, but is constant across frequencies. Drawing on this theoretical analysis, in the last part of the paper we carry out an exercise applying spectral decomposition to simulated data from a small set of open economy models that the literature has recently proposed as possible resolution of the BS anomaly. These models feature incomplete markets, but envision different transmission mechanisms that amplify the amount of uninsurable risk generated by business cycle impulses, enough to make the constant term in our decomposition negative and large.

One mechanism hinges on productivity shocks in the traded goods sector being the prevailing source of macroeconomic fluctuations, and, via Balassa Samuelson effects, a dominant role of nontradable prices as drivers of real exchange rate movements: positive output gains in domestic tradables simultaneously raise relative consumption and the relative price of nontradables, while international tradable prices barely move or even fall (as in Benigno and Thoenissen 2006). A different mechanism emphasizes strong endogenous income effects from output shocks to both tradables and nontradables: irrespective of the sectoral origin of the shock, output (productivity) gains cause all international relative prices of a country to strengthen, together with a rise in relative consumption (as in Corsetti, Dedola and Leduc 2008a or Ghironi and Melitz 2004).[2] We will show that these different mechanisms are able to deliver not only an overall negative Backus-Smith correlation, but also a negative dynamic correlation at business cycle and lower frequencies, broadly in line with the evidence. There are nonetheless some notable quantitative differences across models.[3]

The paper is organized as follows. Section 2 works out an analytical framework for analyzing the prediction by open-economy models, regarding the BS correlation at different frequencies. Section 3 carries out our empirical analysis. Using simulations, section 4 compares the performance of open economy models with the data. The appendices include a description of the data and the model, together with a robustness analysis of our empirical results.

[2] To match the BS correlation, some contributions emphasize marginal utility shifts, weakening the stark prediction of complete-market models of a strict link between relative consumption and real depreciation (see, e.g., Stockman and Tesar 1995, Mandelman et al. 2011, Raffo 2010 among others).

[3] In a companion paper (Corsetti et al. 2011) we analyze, both theoretically and empirically, a different decomposition of the BS correlation, distinguishing the real exchange rate for tradables and nontradables. In our findings, either mechanism appears to be present in the data, although the evidence is stronger for the mechanism hinged upon strong wealth effects.

2 Risk sharing at different frequencies: a theoretical framework

Consider a standard open economy model with a domestic and a foreign country. In this framework, the equations pricing Arrow-Debreu securities and the law of one price in the asset market imply that the growth of marginal utility of consumption, expressed in the same units, is equalized across countries state by state:

$$\beta \frac{U_C\left(C_t\right)}{U_C\left(C_{t-1}\right)} = \beta \frac{U_C^*\left(C_t^*\right)}{U_C^*\left(C_{t-1}^*\right)} \frac{RER_{t-1}}{RER_t} \tag{1}$$

where β denotes the discount rate (for simplicity assumed to be identical across borders), U_C and U_C^* denote the marginal utility of consumption, C and C^* denote consumption, in the domestic and the foreign economy respectively; RER is the real exchange rate, defined as the relative price of foreign consumption P^* in terms of domestic consumption P, i.e., $RER = P^*/P$. Under a symmetric specification of the two countries, perfect risk-sharing further implies that marginal utilities of consumption, again expressed in the same units, are equalized in levels, i.e.

$$U_{c,t} = U_{c^*,t}^* \frac{1}{RER_t} \tag{2}$$

Intuitively, the consumption allocation across countries should be such that the marginal benefit from an extra unit of domestic consumption equals its marginal cost, given by the foreign marginal utility of domestic consumption, times the relative price of C_t in terms of C_t^* (the inverse of the domestic real exchange rate). If a complete set of state-contingent securities is available, the above condition holds in a decentralized equilibrium independently of trade frictions and goods market imperfections (including shipping and trade costs, as well as sticky prices or wages), even when these frictions and imperfections cause deviations from the law of one price and failure of purchasing power parity (PPP).

2.1 Uninsurable risk and the consumption-real exchange rate correlation

Under the assumption that, in each country, the national representative agent has preferences represented by a time-separable, constant-relative-risk-aversion utility function of the form $\frac{C^{1-\sigma}-1}{1-\sigma}$, with $\sigma > 0$, the expressions (1) and (2) become

$$\frac{C_t^{-\sigma}}{C_{t-1}^{-\sigma}} = \frac{\left(C_t^*\right)^{-\sigma}}{\left(C_{t-1}^*\right)^{-\sigma}} \frac{RER_{t-1}}{RER_t}$$

$$RER_t = \left(\frac{C_t}{C_t^*}\right)^{\sigma}$$

which translate into conditions on the correlation between the (logarithm of the) ratio of domestic to foreign consumption and the (logarithm of the) real exchange rate RER, respectively, in growth rates and levels.[4] Against the hypothesis of perfect risk-sharing, many empirical studies have found these correlations to be significantly below one, or negative — in addition to the seminal paper by Backus and Smith (1993), see e.g. Kollmann (1995), and Ravn (2001) among others.[5] It is worth stressing that similar empirical results are obtained even

emphconditional on identified shocks to productivity (see Corsetti, Dedola and Leduc 2008c), casting doubts on the notion that the model marginal utility shifters (i.e., taste shocks) could readily provide a solution to the puzzle.

Indeed, a key question addressed by the literature is under what conditions, if any, the empirical evidence can be reconciled with models of the international business cycle which do not assume complete markets. In models in which only an uncontingent bond is traded across borders, for instance, (1) will generally hold not state by state, but only in expectations:

$$E_{t-1}\left[\beta\frac{C_t^{-\sigma}}{C_{t-1}^{-\sigma}}\right] = E_{t-1}\left[\beta\frac{\left(C_t^*\right)^{-\sigma}}{\left(C_{t-1}^*\right)^{-\sigma}}\frac{RER_{t-1}}{RER_t}\right] \tag{3}$$

It follows that, relative to the case of complete markets, trade in one bond ensures that the ex-post differential in expected utility growth (measured in the same unit) is equalized only up to an i.i.d. stochastic variable, itself a function of fundamental shocks (see e.g. Obstfeld 1994, Cochrane 2004):

$$\frac{C_t^{-\sigma}}{C_{t-1}^{-\sigma}} + \zeta_t = \frac{\left(C_t^*\right)^{-\sigma}}{\left(C_{t-1}^*\right)^{-\sigma}}\frac{RER_{t-1}}{RER_t} \tag{4}$$

whereas $E_{t-1}(\zeta_t) = 0$. Uninsurable risk breaks, ex-post, the tight link between the growth rates of C/C^* and RER, which intertemporal trade can ensure only ex-ante.

By virtue of uninsurable risk, standard models with incomplete markets may be able to predict an overall negative correlation between these variables, matching the BS evidence. But for this to be the case, the amount of uninsurable risk in response to shocks must be sufficiently large — calling attention on the specific properties of the international transmission mechanism embedded in the model economy. Theoretically, the challenge is demanding in two respects. First, it is well understood that, if shocks are purely transitory, restricting international trade in assets may not prevent the market allocation from being

[4] Lewis (1996) rejects non-separability of preferences between consumption and leisure as an empirical explanation of the low correlation of consumption across countries.

[5] Under the (unrealistic) assumption of PPP (i.e., $RER = 1$), efficient risk-sharing implies perfect equalization of the ex-post marginal utility of consumption. Thus, under PPP (and ruling out shocks to preferences), complete markets would imply perfect cross-country correlation of consumption — a strict condition also rejected by the data.

close to the complete-market one. Consider economies in which trade is in one non-contingent bond only. Intuitively, when agents in one country get a temporary positive output shock, they will want to lend to the rest of the world, so that consumption increases both at home and abroad (see e.g., Baxter and Crucini [1995]). Second, the amount of uninsurable risk is also a function of relative price movements. If higher output is associated, in equilibrium, to lower international prices of domestic goods, higher output in one country benefits foreign consumers by boosting their income in real terms — a mechanism which automatically contributes to production risk-sharing. Under some restrictions on the model's parameters, relative price movements can actually ensure complete sharing of production risk, independently of trade in financial assets — a point underscored by Cole and Obstfeld (1991), Corsetti and Pesenti (2001, 2005) and Corsetti, Dedola and Leduc (2010).

These considerations nonetheless suggest a useful insight: otherwise standard open-economy models with imperfect asset markets (and stable preferences) may predict a low or negative correlation between relative consumption and real depreciation in response to persistent shocks, and/or if they embed a transmission mechanism mutes, or reverses, the role of relative price adjustments in providing risk-sharing. Several recent contributions indeed exploit this insight, to reconsider the international transmission mechanism, emphasizing channels through which business cycle disturbances translate into large ex-post wedges between marginal utilities.

In the two model specifications discussed by Corsetti et al. (2008a), for instance, a negative unconditional BS correlation is generated by strong wealth effects of productivity shocks which, irrespective of their sectoral origin, drive up domestic consumption demand (relative to foreign) and simultaneously cause all international prices of the country, but especially the price of tradables, to rise. Strong wealth effects follow either the assumption of a relatively low trade elasticity, amplifying income effects of price changes, or the assumption of persistent shocks which, with a high trade elasticity, significantly raise the present discounted value of expected future income — see also Ghironi and Melitz (2004), for a model with firm entry and endogenous nontradability, and Nam and Wang (2010) and Opazo (2006) stressing shock persistence in the form of 'news shocks'. In Benigno and Thoenissen (2006), instead, productivity gains are concentrated in the tradable sectors. Under the appropriate calibration of trade elasticities, these gains lead to higher consumption, and appreciate considerably the domestic price of nontradables, while moving the international price of tradables very little — see also Cova, Pisani, Batini and Rebucci (2008).[6]

[6] Asset market imperfections may also imply that the response to shock create heterogeneous wealth effects within a country. In this case, the maintained assumption of a national representative agent would not be justifiable: with agents' heterogeneity, uninsured risk is not symmetric across individuals, and aggregate consumption is no longer the appropriate variable to focus on theoretically and empirically. For a discussion of within and across border risk sharing, see Crucini (1999) and Pistaferri and Kocherlakota (2007) among others.

2.2 A spectral characterization of the BS correlation in DSGE models

In a generic DSGE model, the solutions for the differential in consumption growth and the exchange rate growth (up to a first order of approximation), can be written in terms of their state-space representation:

$$\begin{aligned} \sigma\Delta(\widehat{C}_t-\widehat{C}_t^*) &= \pi_1^C\mathbf{S}_{t-1}+\pi_2^C\boldsymbol{\varepsilon}_t\equiv x_t \\ \Delta\left(\widehat{RER}_t\right) &= \pi_1^R\mathbf{S}_{t-1}+\pi_2^R\boldsymbol{\varepsilon}_t\equiv y_t \end{aligned}$$

where S_{t-1} is the vector of state variables (endogenous and exogenous) predetermined at t, including lagged variables, and thus orthogonal to the vector of fundamental *i.i.d.* shocks ε_t.[7] By combining these expressions, as to derive the covariance and variances below:

$$\begin{aligned} Cov(x_t,y_t) &= \pi_1^C Var(\mathbf{S}_{t-1})\pi_1^{R\prime}+\pi_2^C Var\left(\boldsymbol{\varepsilon}_t\right)\pi_2^{R\prime} \\ Var\left(x_t\right) &= \pi_1^C Var(\mathbf{S}_{t-1})\pi_1^{C\prime}+\pi_2^C Var\left(\boldsymbol{\varepsilon}_t\right)\pi_2^{C\prime} \\ Var\left(y_t\right) &= \pi_1^R Var(\mathbf{S}_{t-1})\pi_1^{R\prime}+\pi_2^{R\prime} Var\left(\boldsymbol{\varepsilon}_t\right)\pi_2^{R\prime} \end{aligned} \tag{5}$$

we can provide a crucial insight on the properties of the BS correlation predicted under different model specifications. Consider first the case of complete markets. It is straightforward to verify that, under the simplifying assumption of CRRA and separable preferences, i.e., $\sigma\Delta(\widehat{C}_t-\widehat{C}_t^*)=\Delta\left(\widehat{RER}_t\right)$, ex-post equalization of the RER-weighted growth rates in marginal utilities implies

$$\begin{aligned} \pi_1^C &= \pi_1^R=\pi_1 \\ \pi_2^C &= \pi_2^R=\pi_2. \end{aligned} \tag{6}$$

Holding these restrictions, the covariance $Cov(x_t,y_t)$ is the sum of two quadratic expressions, hence always positive; moreover, it is identically equal to each of the variances,

$$Cov(x_t,y_t)=\pi_1 Var(\mathbf{S}_{t-1})\pi_1'+\pi_2 Var\left(\boldsymbol{\varepsilon}_t\right)\pi_2'=Var\left(x_t\right)=Var\left(y_t\right)$$

ensuring that the BS correlation coefficient is equal to one.

The restrictions (6) do not necessarily hold under incomplete markets, however. Namely, in a bond economy, the equalization of the growth rates of the RER-weigthed marginal utilities only holds in expectations:

$$\begin{aligned} E_{t-1}\sigma\Delta(\widehat{C}_t-\widehat{C}_t^*) &= \pi_1^C\mathbf{S}_{t-1}= \\ E_t\Delta\left(RER_t\right) &= \pi_1^{RER}\mathbf{S}_{t-1} \end{aligned}$$

[7] If the variable $\widehat{X}=\widehat{C}_t,\widehat{C}_t^*$, and $\widehat{RER}_t$ in levels, have a state-space representation

$$\widehat{X}_t=\nu\mathbf{Z}_{t-1}+\eta\varepsilon_t,$$

then their growth rates will have the representation assumed in the text with

$$\mathbf{S}_{t-1}=\begin{bmatrix}\mathbf{Z}_{t-1}\\ \widehat{\mathbf{X}}_{t-1}\end{bmatrix}$$

and corresponding coefficient matrices.

thus implying

$$\pi_1^C = \pi_1^R = \pi_1$$

without restricting π_2^C and π_2^R to be identical. Rewriting (5) under the relevant restrictions,

$$\begin{aligned} Cov(x_t, y_t) &= \pi_1 Var(\mathbf{S}_{t-1})\pi_1' + \pi_2^C Var\,(\boldsymbol{\varepsilon}_t)\,\pi_2^{R\prime} \\ Var\,(x_t) &= \pi_1 Var(\mathbf{S}_{t-1})\pi_1' + \pi_2^C Var\,(\boldsymbol{\varepsilon}_t)\,\pi_2^{C\prime} \\ Var\,(y_t) &= \pi_1 Var(\mathbf{S}_{t-1})\pi_1' + \pi_2^{R\prime} Var\,(\boldsymbol{\varepsilon}_t)\,\pi_2^{R\prime} \end{aligned}$$

makes it clear that, in a bond economy, only the first term of the covariance is a quadratic form, thus always positive. As discussed further below, this term is related to the insurance intertemporal trade provides against predictable contingencies. The second term will instead be positive or negative, depending on the magnitude of the uninsurable risk predicted by the model. In Dedola and Leduc (2008a), Ghironi and Melitz (2004) and Benigno and Thoenissen (2006), for instance, relative price movements magnify the uninsurable component of fundamental risk, up to moving the real exchange rate and the consumption differential in opposite directions. This means that the term $\pi_2^C Var\,(\boldsymbol{\varepsilon}_t)\,\pi_2^{R\prime}$ is not only negative, but actually larger, in absolute value, than the quadratic form in the variance of S.

For a bond economy, the above expressions have crucial implications for the analysis in the frequency domain. Recall that the spectrum of i.i.d. vectors is constant across frequencies (see e.g. Hamilton 1994). So, whether or not the second term in the covariance (that is, $\pi_2^C Var\,(\boldsymbol{\varepsilon}_t)\,\pi_2^{R\prime}$) is negative and large in the model, its value will be constant across the spectrum. Conversely, the spectrum of the quadratic form in the variance of the state vector (that is, $\pi_1 Var(\mathbf{S}_{t-1})\pi_1'$) will generally not be constant, reflecting the fact that S can also be represented as a VAR.

Our theoretical decomposition so establishes a key result: standard incomplete-market models will typically predict the cospectrum of the BS correlation to vary across frequencies, as a function of the component of fundamental risk that is insurable via intertemporal trade. The key point is that cross-border trade in one bond does allow agents to insure consumption against *predictable* dynamics of income — which are themselves a function of the endogenous dynamics of the state variables and thus of the specific mechanism through which exogenous shocks propagate according to the model. It follows that in a bond economy the strength and even the sign of the correlation can be expected to vary across frequencies, depending on the dynamics of the variance of the vector of state variables.

To shed light on this result, it is useful to work out the simplest example, assuming one state and one shock, i.e. $\mathbf{S}_t = \rho\mathbf{S}_{t-1} + \mu\varepsilon_t$. In this case, the covariance terms is:

$$Cov(x, y) = \left(\frac{\pi_1^2\mu^2}{1-\rho^2} + \pi_2^C\pi_2^{RER}\right) Var(\varepsilon),$$

implying that the cospectrum evolves across frequencies according to the following expression:

$$\left[\frac{\pi_1^2\mu^2}{1+\rho^2-2\rho Cos\,(\omega)}+\pi_2^C\pi_2^{RER}\right]\frac{var(\varepsilon)}{2\pi}.$$

In this example, for the covariance to be negative, it must be the case that:

$$\frac{\pi_1^2}{1-\rho^2}<-\frac{\pi_2^C\pi_2^{RER}}{\mu^2}.$$

In this univariate case, a negative covariance necessarily implies a negative cospectrum at any frequency if $\pi_1^2=0$ (the random walk case) or, more generally, if:

$$\begin{aligned}\frac{1}{1-\rho^2} &\geq \frac{1}{1+\rho^2-2\rho Cos\,(\omega)}\\ &\iff 2\rho\frac{\rho-Cos\,(\omega)}{1+\rho^2-2\rho Cos\,(\omega)}\geq 0\\ &\iff \rho\geq Cos\,(\omega)\,.\end{aligned}$$

It is apparent that, for a stationary process with $|\rho|<1$, such a condition would be impossible to satisfy at low frequencies, for $\omega\to 0$. Moreover, depending on whether ρ is above or below zero, the term $\frac{\pi_1^2\mu^2}{1+\rho^2-2\rho Cos\,(\omega)}$ in the cospectrum will become more positive at lower or higher frequencies.[8]

Multivariate examples are more complex to treat analytically, precluding the derivation of general conditions on the sign of the cospectrum. Yet, the main lesson from the analysis will carry through. In general, an overall negative covariance between relative consumption and the real exchange rate does not necessarily imply a negative cospectrum and thus a negative dynamic correlation at all frequencies.

On theoretical and empirical grounds, these results raise a key question, regarding the extent to which intertemporal trade efficiently insures risk at different frequencies. From a theoretical perspectives, it might be possible that in the one-bond economies proposed by the literature to address the BS puzzle, the amount of insurance provided by trade in bonds become large enough at lower frequency, to switch the sign of the BS correlation over the spectrum, from negative to positive. Matching the evidence at business cycle and lower frequencies would then appear to be crucial for these models — at least as matching the simple correlation on first-differenced or HP filtered data. Indeed, as our analytical framework clarifies, in one-bond economies the properties of the dynamic

[8] The condition for the the cospectrum to be negative at any frequency is

$$\left|\frac{1}{1+\rho^2-2\rho Cos\,(\omega)}\right|<-\frac{\pi_2^C\pi_2^{RER}}{\pi_1^2\mu^2},$$

In general, conditions like the above cannot be assessed without the knowledge of the structural parameters of the model.

Backus-Smith correlation at different frequencies map into the component of fundamental risk that can be insured through intertemporal trade.[9]

3 Spectral analysis of the consumption-exchange rate correlation

In this section, we reconsider the BS evidence using spectral analysis. Namely, focusing on a sample of OECD countries, we estimate a measure of the contributions of cycles of different frequency to the Backus-Smith correlation.

As is well known (see e.g. Hamilton, Chapter 10.4), given any two covariance-stationary series x and y, the cospectrum is the frequency-domain equivalent of the covariance between them. Specifically, the cospectrum measures the portion of the covariance between x and y that is attributable to cycles of a given frequency ω. The correlation between relative consumption and real exchange rate at frequency ω is then measured by the dynamic correlation (see Croux et al. 2001)

$$\rho_{C,RER}(\omega) = \frac{C_{C,RER}(\omega)}{\sqrt{S_C(\omega) \cdot S_{RER}(\omega)}},$$

where $C_{C,RER}$ denotes the cospectrum between the two series and S their spectra at frequency ω.

Our sample consists of 20 OECD countries, for which we have quarterly data over the period 1971:1 and 2009:2. For each country, as customary in the literature, we calculate the correlation between the ratio of domestic to foreign consumption and the real exchange rate, vis-à-vis both the US and a trade-weigthed aggregate of the other countries in the sample — an aggregate dubbed 'Rest of the World', or ROW. Results are shown in the panels A and B of Table 1, respectively. In each panel, the first two columns report the standard Backus-Smith statistics, i.e. the correlations for first-differenced data and HP-filtered data series. The remaining three columns instead display the dynamic correlations for first-differenced series. Business-cycle frequencies refer to a time horizon of 8 to 32 quarters, low frequencies to more than 32 quarters,

[9] Insurable risk, as a function of the evolution of state variables, depends on the theoretical predictability of either the rate of real depreciation, or (equivalently) the differential in consumption growth. To wit: consider a bond economy such that, in line with a widespread view in the literature, the equilibrium real exchange rate follows a random walk:

$$E_t \Delta (RER_t) = \pi_1 \mathbf{S}_{t-1} = 0.$$

This implies that the expected consumption growth differential is also constant and equal to zero

$$E_{t-1} \sigma \Delta (\widehat{C}_t - \widehat{C}_t^*) = \pi_1^C \mathbf{S}_{t-1} = 0.$$

In such an economy, trade in bonds is not useful to smooth consumption. Hence, the contemporaneous covariance with the cross-country differential in the growth rate of consumption will generall be non-zero, because of uninsured risk (consistent with the well-established but often misunderstood point, that a random walk for the exchange rate is not a dimension of the exchange rate 'disconnect' puzzle), but the dynamic correlation should be expected to be constant across all frequencies.

and high frequencies to less than 8 quarters. A star indicates that the estimate is significantly lower (or larger, if positive) than zero at the 5 percent confidence level— technical details on the estimation are given in the appendix, which also include two tables with the confidence intervals for the dynamic correlation (Tables 3A and 3B).

In Table 1A, the first two columns reproduce the well-known result, that the BS correlation for the OECD countries against the US is low or negative, whether it is measured using differenced data or HP-filtered data. In line with earlier studies, nearly all entries in the first two columns have a negative sign — even when positive, the correlation coefficient is typically close to zero.

TABLE 1A

Correlation between real exchange rate and relative consumption vis-à-vis the US

COUNTRY	Differenced	HP-filtered	Dynamic Correlation		
			Low freq	BC freq	High freq
Australia	-0,07	-0,24	-0,33	-0,10	-0,01
Austria	0,00	0,10	-0,16	-0,11	0,04
Belgium	-0,08	0,03	-0,20	-0,09	-0,04
Canada	-0,16	-0,10	-0,21	-0,24	-0,12
Switzerland	-0,06	-0,04	-0,12	0,02	-0,09
Denmark	-0,20	-0,26	-0,30	-0,25	-0,16
Spain	-0,21	-0,27	-0,53	-0,37	0,01
Finland	-0,10	-0,41	-0,47	-0,33	0,05
France	-0,07	-0,03	-0,22	-0,06	-0,04
Germany	-0,12	-0,06	-0,21	-0,18	-0,10
Ireland	-0,05	-0,18	-0,16	-0,01	-0,04
Italy	-0,01	-0,23	-0,25	-0,10	0,10
Japan	0,02	0,25	-0,14	0,01	0,05
Korea	-0,46	-0,48	-0,47	-0,55	-0,44
Netherlands	-0,12	-0,09	-0,22	-0,22	-0,08
Norway	-0,06	-0,16	-0,13	-0,13	-0,03
New Zealand	-0,04	-0,36	-0,44	-0,27	0,10
Sweden	-0,17	-0,30	-0,45	-0,20	-0,10
UK	-0,08	-0,41	-0,42	-0,34	0,02
US	NA	NA	NA	NA	NA
Median	-0,08	-0,18	-0,22	-0,18	-0,04

NOTE: See the data appendix for a description of the data.

TABLE 1B

Correlation between real exchange rate and relative consumption vis-à-vis the rest-of-the-world

COUNTRY	Differenced	HP-filtered	Dynamic Correlation		
			Low freq	BC freq	High freq
Australia	-0,05	-0,19	-0,32	-0,01	-0,03
Austria	0,07	0,00	-0,27	-0,14	0,12
Belgium	0,12	0,51	0,21	0,25	0,04
Canada	-0,13	-0,05	-0,12	-0,21	-0,10
Switzerland	0,19	0,16	0,29	0,26	0,15
Denmark	-0,06	-0,17	-0,15	-0,05	-0,05
Spain	-0,05	-0,17	-0,35	-0,21	0,11
Finland	-0,07	-0,45	-0,55	-0,36	0,14
France	0,11	0,21	0,13	0,12	0,10
Germany	-0,02	0,07	-0,07	0,10	-0,05
Ireland	0,19	0,30	0,32	0,32	0,12
Italy	-0,08	-0,27	-0,32	-0,12	-0,03
Japan	0,09	0,32	-0,07	0,12	0,10
Korea	-0,41	-0,51	-0,49	-0,53	-0,38
Netherlands	-0,04	-0,07	-0,17	-0,18	0,01
Norway	0,03	0,00	-0,01	-0,04	0,05
New Zealand	0,05	-0,32	-0,44	-0,17	0,16
Sweden	-0,15	-0,23	-0,28	-0,24	-0,10
UK	0,06	0,07	0,02	0,02	0,09
US	-0,20	-0,15	-0,36	-0,26	-0,14
Median	-0,03	-0,06	-0,16	-0,09	0,05

NOTE: The rest-of-the-world is a trade-weighted aggregate of all the countries in the sample (excluding the base country). See the data appendix for a description of the weights.

Spectral analysis unveils an important new dimension of the evidence. The dynamic correlation is mostly negative, but not constant across frequencies. At lower frequencies, the inverse correlation tends to be stronger. The average across countries goes from -.04 for high frequencies (not significantly different from zero), to -.18 and -.22 for business-cycle and low frequencies, respectively. The correlation is significantly negative for 8 countries at high frequencies, and for 11 countries at business-cycle and lower frequencies. The highest values of the inverse correlation are recorded for Korea (-.55) at business-cycle frequencies, and Spain (-.53), Finland (-.47), Korea (-.48), and Sweden (-.47) at low frequencies. Observe also that, for the two countries displaying an overall positive correlation (Austria and Japan), the correlation at business-cycle and/or low frequencies is actually negative. By way of example, the estimates for Austria are 0.4, -.11, and -.16, at high, business-cycle and low frequencies, respectively. This example illustrates the theoretical result, that standard correlations in the time domain with differenced data tend to boost the high frequency components of time series (see e.g. Croux et al. 2001).

Because of the different economic structure in the US relative to our aggregate of OECD countries (reflecting, e.g., economic size, openness, financial structure and policy regimes), it is reasonable to expect some variability in results when countries are assessed against the ROW. Comparing the first two columns of Table 1B with Table 1A, indeed, the point estimates of the standard BS statistic calculated against the ROW are negative for a smaller set of countries. Even so the BS correlation never exceeds +.20 (for the first-differenced series). The minimum value is -.41, again for Korea.

Yet, also when each country is assessed relative to the ROW, the Backus-Smith puzzle is more pronounced at business-cycle and lower frequencies. In table 1B, the sample average ranges from .05 (high frequencies) to -.16 (low frequencies). The correlation is significantly negative for only 4 countries (out of 20) at high frequencies, but for 8 and 9 countries at business cycle and lower frequencies, respectively. Moreover, for four countries (Austria, Japan, Norway and New Zealand), a small positive value of the overall statistic in the first column of the table appears to be an average of negative and positive values at low and high frequencies.

Our main result is illustrated most clearly by Figure 1A and 1B, which plot the dynamic correlation for the median across countries (vis-à-vis the US and the ROW, respectively) together with a 5 percent and 95 percent confidence bands around the point estimate, against the frequency ω. Vertical broken lines are drawn corresponding to 8 and 32 quarters, the boundaries identifying business cycle frequencies in the tables — the values in the tables are indeed weighted averages of the points shown in these figures over the relevant intervals. In either graph, the dynamic correlation is always significantly lower than unity — it is actually significantly lower than .2 at all frequencies. Most crucially, the dynamic correlation becomes significantly negative only at business cycle and lower frequencies.[10]

[10] Plots of the dynamic correlation for each country are available in the web appendix.

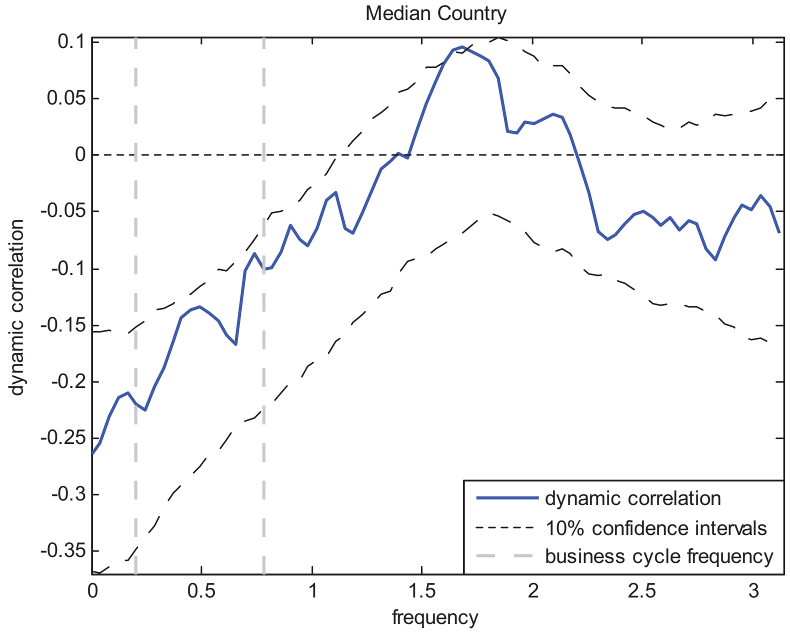

FIGURE 1A – Dynamic correlation vis-à-vis the US for the median country

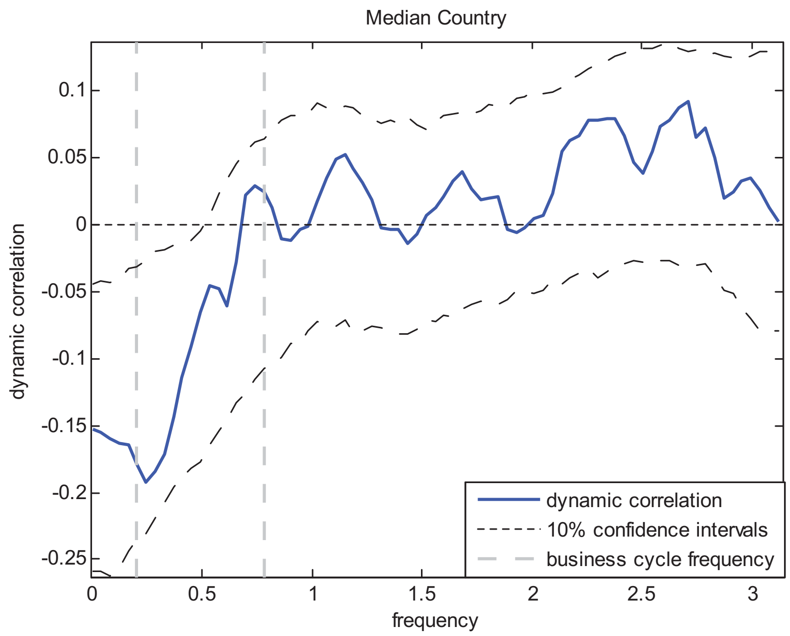

FIGURE 1B – Dynamic correlation vis-à-vis the rest-of-the-world for the median country

The importance of this result is best appreciated in light of a recent contribution by Rubio-Ramirez and Rabanal (2010), showing that the business-cycle and lower frequencies account for the bulk of the variability of the real exchange rate. Our evidence further suggests that it is precisely at the frequencies at which the real exchange rate is more volatile (but arguably also more predictable), that it is also significantly 'connected' to key macroeconomic variables, like domestic and foreign consumption demand.

Robustness is discussed in the appendix. While the use of first-differenced data is natural in the context of bond economies, empirically, it might boost the high frequency component of a series. For this reason, we re-do our analysis applying the bandpass filter described in Christiano and Fitzgerald (2003). The main conclusions from this robustness exercise are in line with the ones discussed above.

4 Models and evidence

In the previous section, we have produced new evidence at odds with a high degree of international risk insurance. The novel contribution of our analysis is that, in the data, the tendency of home consumption to rise (relative to foreign consumption) with a real appreciation, is clearer at business-cycle and lower frequencies, more so than at higher frequencies. Moreover, once high frequencies are eliminated, the BS correlation at business-cycle and lower frequencies is significantly different from zero at standard confidence levels. Hence, low values of the BS statistics should not be mistaken for a manifestation of the so-called exchange rate 'disconnect puzzle.'

These results clearly strengthens the case for placing the analysis of international risk-sharing centerstage in the development of international economics, as advocated e.g. by Obstfeld and Rogoff (2001) and Chari et al. (2002). If the BS correlation were to be driven by correlations at high frequencies, one could argue that market dynamics at high frequencies would not be appropriately captured by models designed to account for macro dynamics at business-cycle and lower frequencies. On the contrary, our evidence makes it clear that the BS puzzle is pervasive exactly at the frequencies over which open economy models appear to be relatively successful in matching the data in a number of dimensions.

In this section, we reconsider the contributions to the literature briefly discussed in Section 2, proposing alternative mechanisms by which the transmission of shocks can generate a negative correlation between relative consumption and real depreciation. These mechanisms create a variety of state dynamics, which may or may not fit our evidence from spectral analysis. In light of our analysis, a proper assessment of these models is to be carried out in the frequency domain.

In what follows, we focus on three incomplete-market models as case studies, two proposed by Corsetti et al. (2008a), and one by Benigno and Thoenissen (2006). To carry out our exercise, we set up a common two-country framework, in which each country produces tradables and nontradables, drawing on Corsetti et al. (2008b). We then simulate it under different calibrations of the shock processes and parameters specification, so as to make it broadly consistent with the key quantitative results in the contributions under consideration, and apply spectral analysis to the simulated time series. Since most features of the analysis are standard, a description of the general structure of model is provided in a web appendix. In the text and in the table, CDL refers to the two model specifications after Corsetti et al. (2008a), BT to that after Benigno and Thoenissen (2006).

In all our exercises, we calibrate the model for the US versus the rest of the OECD countries, with annual data. Since the dynamic correlations for the US in Table 1B and Figure 1B are computed using quarterly data, the comparison with the model is carried out at comparable frequencies by properly adjusting the parameter ω. Namely, business cycle frequencies are still assumed to include cycles from 2 to 8 years, for $\omega = [\pi/2, \pi/8]$. In our exercises, productivity shocks are both country- and sector-specific. Define $Z = [A_T, A_T^\star, A_{NT}, A_{NT}^\star]$, where A denotes TFP and the subscript T and NT refers to tradables and nontradables, respectively. We consider the process

$$Z\prime = \lambda Z + u,$$

where technology shocks are identified with Solow residuals in each sector.

The first specification under consideration is the low-trade-elasticity economy in Corsetti et al. (2008a). Correspondingly, we set the trade price elasticity equal to .375,[11] and take the CDL estimates of the productivity shocks based on annual data in manufacturing and services from the OECD STAN database. The second specification is the high-elasticity high-shock-persistence economy discussed in the same paper. The trade elasticity is set as high as 4, consistent with the estimates by Bernard et al. (2003). With a high elasticity, strong wealth effects require high persistence of the shocks: the autoregressive coefficient for the shocks hitting the tradable good sector is set close to one — while spillovers are set to zero to guarantee stationarity.[12]

The third model specification reproduces the results and the transmission mechanism in Benigno and Thoenissen (2006). These authors estimate the productivity process using a sample and a dataset — from the Groningen Growth and Development Centre — which differ from CDL. In their estimates, the shocks in the nontradable sector are less persistent and volatile than the shocks to tradables. For the mechanism envisioned by BT to be effective in addressing the BS puzzle, indeed, the shocks to the nontradable sector cannot be prominent: other things equal, positive shocks to nontradable productivity would tend to depreciate the price of nontradable goods, driving relative consumption and the real exchange rate in the same direction. In equilibrium, the unconditional Backus-Smith correlation could then have either sign.

[11] To get this value, we set the elasticity of substitution between domestic and foreign tradables in the consumption bundle equal to 0.74. With distribution costs accounting for 50 percent of the retail prices, the trade elasticity is half the elasticity of substitution, thus equal to .375 (see Corsetti and Dedola 2005). This value is slightly lower than the one used in the benchmark bond economy in CDL, depending on the specification of investment.

[12] The role of shock persistence is further explored by Opazo (2006) and Nam and Wang (2010), who focus on news shocks. Corsetti, Dedola and Leduc (2010) derive implications for the design of optimal policy in open economies.

Since the structure of our common model framework is marginally different from Benigno and Thoenissen (2006), we proceed by adjusting the CDL calibration as to reproduce the BT results. Specifically, starting from the baseline estimates of the shock process Z by CDL, we increase the autocorrelation of tradable shocks, and decrease the standard deviation of nontraded shocks. Moreover, we set the share of labour in tradables equal to 0.33 and the trade elasticity equal to 2, as in Benigno and Thoenissen (2006). With these adjustments, the model replicates the BT result, that is, it predicts a negative Backus-Smith correlation mostly driven by movements in the relative price of nontraded goods.[13]

The results from our exercises are summarized in Table 2, showing the unconditional correlation and the dynamic correlation arising at business cycle and low frequencies, both computed using first-differenced data. For the CDL models, we also report results assuming nominal rigidities, with an average duration of prices of 4.3 months (matching the evidence in Bils and Klenow 2004). We instead omit the simulations of the BT specification with nominal rigidities, as in this case results appear to be quite sensitive to price stickiness — the BS correlation turns positive and high (although not perfect) across the spectrum.

The table also includes results from a standard specification of the model, with a trade elasticity equal to 1.5. Under this specification, the BS correlation remains close to perfect across the spectrum — a well-known result which motivates the discussion of the BS findings as a 'puzzle' or 'anomaly' in the first place. For our purposes, this result is also helpful in reconciling our simulations, in which we assume non-separable preferences in consumption and leisure (as typical in the literature), with the analysis in Section 2, where we have assumed separability for clarity of exposition. The fact that, with an elasticity of 1.5, the BS correlation is close to one at all frequencies suggests that non-separability play no meaningful role in our experiments. The last lines in Table 2 reproduces the relevant empirical results from Table 2B, together with the extremes of the 5 percent to 95 percent confidence band.

As apparent from the table, each of the three models under consideration is able to generate low and even negative dynamic correlations at business-cycle and lower frequencies, at least under flexible prices. Against the three transmission mechanisms amplifying the amount of uninsurable risk in the economy due to productivity shocks, envisioned in these models, trade in one bond does not seem to provide an effective instrument to insure risks at any frequency. There are nonetheless a few notable differences. For some models our simulations produce correlations well within the confidence band of Figure 1B. This is

[13] It is worth stressing here that Balassa-Samuelson effects are not sufficient for the BT mechanism to work. Specifically, it turns out that the BT results are sensitive to the value of the trade elasticity. If this elasticity is set sufficiently above 2 — and, with more reason, if its calibration approximates the standard textbook Balassa-Samuelson model where cross-border trade is in one homogeneous good only —, the model no longer matches the BS correlation.

TABLE 2

Correlation between real exchange rate and relative consumption in the simulated model and in the data

Model specification	Differenced	Dynamic Correlation Low frequency	BC frequency
CDL low elasticity	-0.14	-0.20	-0.13
With nominal ridigities	-0.2	-0.14	-0.22
CDL high elasticity	-0.19	-0.20	-0.62
With nominal rigidities	-0.78	-0.60	-0.89
BT	-0.15	-0.51	-0.06
Standard model	0.99	0.99	1
Data (US vs ROW)	-0.20	-0.36	-0.26
Confidence int. Upper bound		-0.121	-0.125
Confidence int. Lower bound		-0.524	-0.522

NOTE: CDL refers to Corsetti, Dedola and Leduc (2008a); BT refers to Benigno and Thoenissen (2006); low elasticity = 0.74; high elasticity = 8. Nominal rigidities: average duration of price-stickiness = 4.3 months (from Bils and Klenow (2004)). Correlations computed over 5000 simulations. Frequency bands are defined as in Table 1A. Confidence intervals on US data are computed over 500 bootstrap simulations.

the case of the CDL specification with a low elasticity (first two rows of Table 2), which overall comes reasonably close to the evidence for the US. In other cases, the match is less satisfactory. The CDL specification with high elasticity and persistent shocks produces a correlation which is too negative at business cycle frequencies, relative to the evidence. At the same frequencies, the BT specification errs on the other side — as already mentioned, in our specification the BT model is also excessively sensitive to nominal rigidities.

5 Conclusions

Over two decades the Backus-Smith "puzzle" has been a challenge to the open-macro literature — its relevance going beyond pure academic research. Given that open macro models are increasingly used as tools for policy assessment in domestic and international institutions, a failure to match the correlation between important macro quantities (such as relative consumption across countries) and prices (the real exchange rate) raises fundamental issues in the extent to which standard model specifications can effectively account for financial and/or real distortions (and thus for trade-offs) relevant to the actual conduct of stabilization policies.

In this paper, we have provided theoretical and empirical arguments for refining the puzzle. First, by using spectral analysis, we have shown that the BS evidence is actually much starker at business cycle and lower frequencies, than suggested by contemporaneous correlation. In the data, the correlation between relative consumption and the real exchange rate for many countries is significantly negative, exactly at those frequencies which are most appropriate for assessing the performance of international business cycle models.

Second, we have shown that, in general, a model's prediction of a negative overall correlation does not necessarily imply a negative sign of the correlation at different frequencies. The dynamic correlation can be expected to vary as a function of structure of the model economy, determining the amount of insurance provided by intertemporal trade. It may well be possible that a model predicting a negative contemporaneous correlation, fails to match the evidence over the spectrum, if trade in bonds provides significant insurance against predictable contingencies. For a set of contributions proposing a solution to the BS puzzle in the framework of incomplete market models, we have nonetheless shown that the predicted correlation between relative consumption and the real exchange rate remains negative across the spectrum.

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A Appendix. Data sources

We collected quarterly data on real consumption from the OECD Economic Outlook for Australia, Austria, Belgium, Canada, Switzerland, Denmark, Spain, Finland, France, Germany, Ireland, Italy, Japan, Korea, the Netherlands, Norway, New Zealand, Sweden, the UK and the US. Consumer price indexes and nominal exchange rates are quarterly data from the IMF International Financial Statistics database for the period 1971:1-2009:2.

For each country in our dataset, the Foreign counterpart is either the US or a trade-weighted aggregate of all the other countries in the sample. In the latter case, S_t and $P_t^{\star}$ are trade-weighted averages of all the other countries in the dataset. Trade weights were built computing bilateral trade shares. Namely, we computed the trade share of country i from country j as

$$0.5 \cdot \frac{\exp_j^i}{\exp^i} + 0.5 \cdot \frac{\mathrm{imp}_j^i}{\mathrm{imp}^i}$$

where $\exp_j^i$ and imp_j^i are exports and imports from country j, and $\exp^i$ and imp^i denote total exports and imports of country i. Exports and imports are averages of annual data over the period 1980-2008, collected from the IMF Direction of Trade Statistics. For the median country trade weights account for roughly 73% of total imports and exports.

B Appendix. Spectral analysis

Spectra and cospectra are estimated non-parametrically using a smoothing window of length $m = (T)^{1/2}$, where T is the sample size. In particular, we use a Tukey window, which associates any linearly-spaced vector x to

$$w(x) = \left\{ \begin{array}{ll} \frac{1}{2} \cdot \left\{1 + \cos\left(\frac{2\pi}{r}\left[x - r/2\right]\right)\right\}, & 0 \leq x \leq r/2 \\ 1, & r/2 \leq x \leq 1 - r/2 \\ \frac{1}{2} \cdot \left\{1 + \cos\left(\frac{2\pi}{r}\left[x - 1 + r/2\right]\right)\right\}, & 1 - r/2 \leq x \leq 1 \end{array} \right\},$$

where r is the smoothing parameter indicating the ratio of taper to constant section in the window, and is assumed to be equal to 0.5.[14]

We build confidence intervals from 500 bootstrap replicates. For the dynamic correlation between relative consumption and real exchange rates, we use sigma-confidence intervals. More specifically, we apply the Fisher-z transformation to the simulated dynamic correlations in order for their distribution to get closer to a normal, compute sigma-intervals on the transformed series, and finally convert them into bands for the dynamic correlation.[15] For the median across countries we use percentile confidence bands — these are shown in the figure in the main text.

[14] The Tukey window collapses to a rectangular window for $r = 0$ and to a Hanning window for $r = 1$. Results obtained with these two alternative parametrizations are available upon request.

[15] See Croux et al. (2001).

The following two tables report confidence intervals for each country in our sample.

TABLE 3A

Correlation of real exchange rate and relative consumption vis-à-vis the US
Point estimates and 10 percent confidence intervals

COUNTRY	Spectral Decomposition								
	Low frequency			BC frequency			High frequency		
	LB	est	UB	LB	est	UB	LB	est	UB
Australia	-0,45	-0,33	-0,05	-0,29	-0,10	0,01	-0,11	-0,01	0,08
Austria	-0,36	-0,16	0,06	-0,24	-0,11	0,04	-0,05	0,04	0,13
Belgium	-0,39	-0,20	-0,05	-0,27	-0,09	0,03	-0,13	-0,04	0,05
Canada	-0,44	-0,21	0,00	-0,38	-0,24	-0,10	-0,20	-0,12	-0,03
Switzerland	-0,26	-0,12	0,17	-0,15	0,02	0,16	-0,18	-0,09	0,00
Denmark	-0,46	-0,30	-0,05	-0,41	-0,25	-0,14	-0,26	-0,16	-0,07
Spain	-0,66	-0,53	-0,33	-0,51	-0,37	-0,23	-0,09	0,01	0,09
Finland	-0,60	-0,47	-0,23	-0,46	-0,33	-0,21	-0,05	0,05	0,14
France	-0,36	-0,22	0,05	-0,22	-0,06	0,07	-0,12	-0,04	0,05
Germany	-0,41	-0,21	0,03	-0,35	-0,18	-0,07	-0,18	-0,10	0,00
Ireland	-0,31	-0,16	-0,10	-0,18	-0,01	0,10	-0,13	-0,04	0,06
Italy	-0,41	-0,25	0,01	-0,28	-0,10	0,01	0,01	0,10	0,19
Japan	-0,31	-0,14	0,13	-0,16	0,01	0,15	-0,04	0,05	0,15
Korea	-0,65	-0,47	-0,34	-0,62	-0,55	-0,43	-0,51	-0,44	-0,36
Netherlands	-0,44	-0,22	0,03	-0,37	-0,22	-0,09	-0,17	-0,08	0,01
Norway	-0,31	-0,13	0,13	-0,30	-0,13	0,00	-0,12	-0,03	0,06
New Zealand	-0,56	-0,44	-0,19	-0,38	-0,27	-0,12	0,00	0,10	0,17
Sweden	-0,56	-0,45	-0,17	-0,36	-0,20	-0,06	-0,19	-0,10	-0,01
UK	-0,57	-0,42	-0,20	-0,45	-0,34	-0,19	-0,07	0,02	0,10
US	NA	NA	NA	NA	NA	NA	NA	NA	NA
Median	-0,34	-0,22	-0,15	-0,27	-0,18	-0,13	-0,08	-0,04	0,01

NOTE: See note to Table 1A. Est denotes point estimates, LB and UB denote, respectively, lower and upper bound. See Appendix B for details on the computation of the intervals.

TABLE 3B

Correlation of real exchange rate and relative consumption vis-à-vis the rest-of-the-world

Point estimates and 10 percent confidence intervals

COUNTRY	Spectral Decomposition								
	Low frequency			BC frequency			High frequency		
	LB	est	UB	LB	est	UB	LB	est	UB
Australia	-0,42	-0,32	-0,03	-0,19	-0,01	0,08	-0,11	-0,03	0,07
Austria	-0,48	-0,27	-0,10	-0,26	-0,14	0,02	0,02	0,12	0,21
Belgium	0,01	0,21	0,42	0,10	0,25	0,38	-0,04	0,04	0,14
Canada	-0,37	-0,12	0,05	-0,35	-0,21	-0,07	-0,18	-0,10	-0,01
Switzerland	0,08	0,29	0,49	0,12	0,26	0,39	0,06	0,15	0,23
Denmark	-0,30	-0,15	0,10	-0,21	-0,05	0,07	-0,14	-0,05	0,04
Spain	-0,52	-0,35	-0,10	-0,37	-0,21	-0,06	0,01	0,11	0,18
Finland	-0,66	-0,55	-0,34	-0,51	-0,36	-0,24	0,03	0,14	0,21
France	-0,01	0,13	0,3	-0,01	0,12	0,27	0,01	0,10	0,19
Germany	-0,24	-0,07	0,19	-0,12	0,10	0,18	-0,13	-0,05	0,05
Ireland	0,12	0,22	0,49	0,18	0,32	0,43	0,04	0,12	0,21
Italy	-0,45	-0,32	-0,03	-0,30	-0,12	0,00	-0,11	-0,03	0,06
Japan	-0,25	-0,07	0,18	-0,05	0,12	0,23	0,01	0,10	0,20
Korea	-0,65	-0,49	-0,36	-0,60	-0,53	-0,41	-0,46	-0,38	-0,21
Netherlands	-0,39	-0,17	0,03	-0,29	-0,18	0,00	-0,08	0,01	0,09
Norway	-0,22	-0,01	0,20	-0,20	-0,04	0,08	-0,05	0,05	0,14
New Zealand	-0,54	-0,44	-0,18	-0,30	-0,17	-0,03	0,06	0,16	0,24
Sweden	-0,50	-0,28	-0,11	-0,38	-0,24	-0,10	-0,20	-0,10	-0,02
UK	-0,22	0,02	0,23	-0,12	0,02	0,18	0,00	0,09	0,17
US	-0,51	-0,36	-0,12	-0,40	-0,26	-0,14	-0,22	-0,14	-0,05
Median	-0,26	-0,16	-0,04	-0,17	-0,09	-0,01	-0,01	0,05	0,08

NOTE: See note to Table 1B and 6A

C Appendix. Bandpass filter approach

Since first-differencing might boost the low frequency component of a series, we check the robustness of our results using an alternative approach. Namely, we apply to the log-levels of relative consumption and real exchange rate, the bandpass filter described in Christiano and Fitzgerald (2003), and compute the correlation arising at different frequency bands. We used the following bands. High frequency corresponds to cycles of length between 2 and 5 quarters, business-cycle frequency to cycles between 6 and 32 quarters, low frequency to cycles between 33 and 70 quarters. The following tables report the main results.

TABLE 4A

Correlation between real exchange rate and relative consumption vis-à-vis the US
Bandpass-filtered data

COUNTRY	Correlation		
	Low frequency	BC frequency	High frequency
Australia	-0,14	-0,41	-0,04
Austria	-0,20	-0,10	0,04
Belgium	-0,29	-0,23	-0,10
Canada	-0,72	0,06	-0,10
Switzerland	0,35	-0,35	-0,14
Denmark	-0,41	-0,10	-0,23
Spain	-0,84	-0,32	-0,04
Finland	-0,14	-0,60	0,00
France	-0,33	-0,22	-0,06
Germany	-0,26	-0,22	-0,17
Ireland	-0,61	-0,07	-0,12
Italy	-0,20	-0,33	0,08
Japan	0,01	0,20	0,06
Korea	-0,71	-0,58	-0,46
Netherlands	-0,39	-0,19	-0,09
Norway	-0,01	-0,16	-0,15
New Zealand	-0,19	-0,57	0,12
Sweden	-0,74	-0,33	-0,08
UK	-0,34	-0,36	0,04
US	NA	NA	NA
Median	-0,29	-0,23	-0,08

NOTE: See the data appendix for a description of the data.
High frequency: 2-5 quarters; business cycle frequency: 6-32 quarters; low frequency: 33-70 quarters

TABLE 4B

Correlation between real exchange rate and relative consumption vis-à-vis the rest-of-the-world – Bandpass-filtered data

COUNTRY	Correlation		
	Low frequency	BC frequency	High frequency
Australia	-0,19	-0,27	0,00
Austria	-0,31	-0,01	0,12
Belgium	0,03	0,34	0,09
Canada	-0,59	0,10	-0,10
Switzerland	0,76	-0,03	0,16
Denmark	-0,28	0,16	-0,12
Spain	-0,79	-0,03	0,11
Finland	-0,59	-0,64	0,05
France	0,07	0,15	0,15
Germany	-0,07	0,15	-0,10
Ireland	0,35	0,37	0,04
Italy	-0,63	-0,17	-0,03
Japan	0,57	0,31	0,13
Korea	-0,69	-0,60	-0,41
Netherlands	-0,23	0,06	0,06
Norway	0,01	0,18	-0,07
New Zealand	-0,41	-0,49	0,17
Sweden	-0,59	-0,20	-0,11
UK	-0,14	0,32	0,11
US	-0.66	-0,21	-0,14
Median	-0,25	0,03	0,04

NOTE: The rest-of-the-world is a trade-weighted aggregate of all the countries in the sample (excluding the base country). See the data appendix for a description of the weights

High frequency: 2-5 quarters; business cycle frequency: 6-32 quarters; low frequency: 33-70 quarters

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Telephone +34 91 338 6363. Fax +34 91 338 6488
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