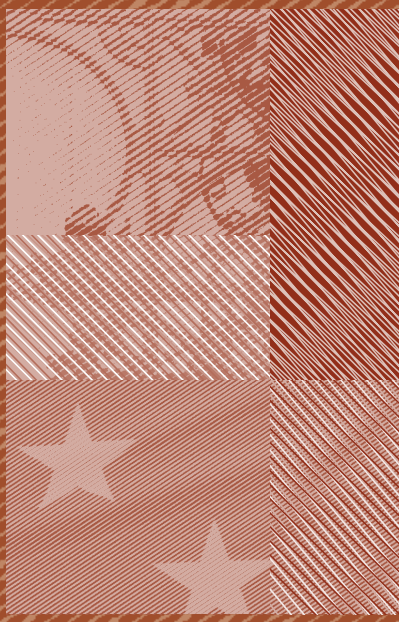
SOLVING PORTFOLIO PROBLEMS WITH THE SMOLYAK-PARAMETERIZED EXPECTATIONS ALGORITHM

2008

Ángel Gavilán and Juan A. Rojas



Documentos de Trabajo
N.º 0838

BANCO DE **ESPAÑA**
Eurosistema

SOLVING PORTFOLIO PROBLEMS WITH THE SMOLYAK-PARAMETERIZED EXPECTATIONS ALGORITHM

SOLVING PORTFOLIO PROBLEMS WITH THE SMOLYAK- PARAMETERIZED EXPECTATIONS ALGORITHM (*)

Ángel Gavilán and Juan A. Rojas (**) (***)

BANCO DE ESPAÑA

(*) The opinions expressed here are solely those of the authors and do not necessarily reflect the views of the Bank of Spain or the Eurosystem.

(**) Bank of Spain, Research Department. E-mail: angel.gavilan@bde.es.

(***) Bank of Spain, Research Department. E-mail: juan.rojas@bde.es.

Documentos de Trabajo. N.º 0838

2008

ISSN: 0213-2710 (print)
ISSN: 1579-8666 (on line)
Depósito legal: M. 6998-2009
Unidad de Publicaciones, Banco de España

Abstract

We propose a new numerical method to solve stochastic models that combines the parameterized expectations (PEA) and the Smolyak algorithms. This method is especially convenient to address problems with occasionally binding constraints (a feature inherited from PEA) and/or a large number of state variables (a feature inherited from Smolyak), i.e. DSGE models that incorporate portfolio problems and incomplete markets. We describe the proposed Smolyak-PEA algorithm in the context of a one-country stochastic neoclassical growth model and compare its accuracy with that of a standard PEA collocation algorithm. Despite estimating fewer parameters, the former is able to reach the high accuracy levels of the latter. We further illustrate the working of this algorithm in a two-country neoclassical model with incomplete markets and portfolio choice. Again, the Smolyak-PEA algorithm approximates the solution of the problem with a high degree of accuracy. Finally, we show how this algorithm can efficiently incorporate both occasionally binding constraints and a partial information approach.

Keywords: Portfolio Choice, Dynamic Macroeconomics, Computational Methods

JEL classification: E2, C68.

1 Introduction

Macroeconomists are becoming increasingly interested in DSGE open economy models that incorporate portfolio problems and incomplete markets. These models are needed, for instance, to understand what is behind gross and net international capital flows, whose magnitudes have substantially increased over the last decades, and to analyze the macroeconomic impact that shocks may have in an economy as a function of its international portfolio structure (i.e. in terms of valuation effects).

Solving this kind of models, however, it is not an easy task. In fact, only recently few methods have been proposed in the literature which can deal with these models. Devereux and Sutherland (2007) and van Wincoop and Tille (2007) are examples of this literature. They have both developed perturbation methods where, put simply, the solution is based on approximations of the problem's equilibrium conditions around the steady state, with a special emphasis on the portfolio equations.[1] These methods are then convenient to analyze the effect of different shocks on a country's equilibrium portfolio and on prices *around the steady state*. However, since these are local methods, they can not be used to study the optimal path of adjustment of an economy hit by permanent shocks that change its steady state. One example of this kind of shocks would be the worldwide demographic change. The literature has extensively studied the impact of this shock on net international capital flows using one-asset multi-country models.[2] However, if one were to analyze the effect of this shock on gross capital flows or asset prices with a model incorporating portfolio decisions, then a global solution method would be needed.

In this paper we propose a global solution method that is suitable for solving stochastic models characterized by a large number of state variables and, in particular, DSGE open economy models with portfolio problems and incomplete markets. We combine two existing algorithms in the literature. On the one hand, we consider the parameterized expectation algorithm (PEA), a well-known projection method.[3] In short, this method's approach to solve stochastic problems consists on parameterizing the conditional expectations appearing in the problems' first-order conditions. More specifically, we consider a variant of PEA called *PEA collocation*. This approach basically approximates those conditional expectations using polynomials whose unknown coefficients are determined by imposing that the approximation exactly holds at the so-called *collocation points*.

On the other hand, we make use of the Smolyak algorithm, first introduced in

[1] Evans and Hnatkovska (2005) also develop a numerical method to solve problems with portfolio decisions and incomplete markets using perturbation methods, although in combination with projection methods and continuous time approximations techniques.

[2] See, for instance, Domeij and Floden (2006) and Krueger and Ludwig (2007).

[3] See, for instance, Den Haan and Marcet (1990), Marcet and Lorenzoni (1999), Marcet and Singleton (1999), Christiano and Fisher (2000) and Maliar and Maliar (2003).

the economic literature by Krueger and Kubler (2004). This is a sparse grid method that is particularly convenient to address high-dimensional problems. An undesirable feature of the standard PEA collocation algorithm is that it becomes computationally unfeasible in problems with a large number of state variables. The underlying reason is that the number of collocation points and unknown coefficients to be found with this method grow exponentially with the dimension of the problem. The Smolyak algorithm breaks this *curse of dimensionality* basically by choosing carefully both the approximating function and the collocation points.

By combining the PEA collocation and the Smolyak algorithms, in what we call the *Smolyak-PEA* algorithm, we thus have a global projection method that is suitable to address high-dimensional problems, as those involving portfolio decisions in a multi-country set up. We illustrate the working of this method in the context of a stochastic two-country neoclassical model with incomplete markets and portfolio decisions and find that this method's accuracy easily meets usual standards in the literature.

In addition, we apply the algorithm to two extensions of the basic set up. First, we consider a partial information scenario where individuals, in order to take their portfolio decisions, do not know the whole asset distribution in the economy but only some moments of it. The partial information approach, made popular by Krusell and Smith (1998), it is commonly adopted in the literature to solve models in which some kind of distribution enters into the individuals informational set. The advantage of this approach is that it reduces the dimensionality of these problems substantially. However, it obviously entails an accuracy loss relative to the full information case. With the exception of Krueger and Kubler (2004), this accuracy loss has not been quantified. In this sense, we show how the Smolyak-PEA algorithm can be implemented in a partial information scenario and provide a measure of the accuracy loss caused by the departure from full information.

Second, we add occasionally binding constraints to the portfolio problem faced by individuals. Such restrictions on portfolio decisions are common both in financial markets are in theoretical models. We show how the Smolyak-PEA algorithm can easily deal with this type of constraints. In fact, as argued by Christiano and Fisher (2000), it is precisely in the presence of these constraints when parameterizing conditional expectations is clearly more convenient than parameterizing policy functions, since the latter approach must undertake a cumbersome direct search for both the policy and multiplier functions. In addition, we study how the optimal policy functions change, in our set up, depending on the severity of portfolio constraints.

Before analyzing the performance of the Smolyak-PEA algorithm in a two-country model with portfolio decisions, the following section applies it to the standard one-country stochastic neoclassical growth model. This model, that allows us to introduce

in a simple way both the PEA and the Smolyak algorithms, has been traditionally used in the literature as the benchmark model in which to illustrate the working of different computational methods and to set comparisons among them. We follow this line and compare the performance of the Smolyak-PEA algorithm with that of a standard PEA collocation. We find that, for this simple model, the Smolyak-PEA algorithm achieves accuracy levels similar to those of a standard PEA collocation, despite the fact that it uses a substantially smaller number of collocation points and of coefficients in the approximating function. This is precisely the great advantage of this method and what makes it suitable to deal with high-dimensional problems, as those involving multi-country portfolio decisions.

2 One-country stochastic model

Consider the standard one-country stochastic neoclassical growth model:

$$\max \; E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

subject to

$$c_t + k_{t+1} = (1-\delta)k_t + \theta_t k_t^{\alpha} \; \forall t \tag{1}$$

where $\log \theta_t = \rho \log \theta_{t-1} + \epsilon_t$, $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ and the initial conditions (k_0, θ_0) are given.

The first order condition of this problem is:

$$c_t^{-\sigma} = \beta E_t \left[c_{t+1}^{-\sigma} (\theta_{t+1} \alpha k_{t+1}^{\alpha-1} + 1 - \delta) \right] \tag{2}$$

If the utility function is logarithmic ($\sigma = 1$) and capital completely depreciates in one period ($\delta = 1$), this problem has an analytical solution. Namely, $c_t = (1 - \alpha\beta)\, \theta_t k_t^{\alpha}$. For general parameter values, however, a closed-form solution to this problem does not exist and numerical methods are needed to solve it.

2.1 PEA collocation

The parameterized expectation algorithm (PEA) is one of the available methods in the literature that can handle this type of stochastic problems. Its approach is simple. It notes that the conditional expectation in equation (2) is a function of the state variables, (k_t, θ_t), and replaces it with a function $\Psi(k_t, \theta_t; \Phi)$, where both the functional form (Ψ) and the parameters (Φ) are chosen so that this approximation

is as accurate as possible.[4] Different ways of implementing this idea depend on the functional form chosen for the approximating function and/or on the procedure used to find the best set of parameters of that function. Christiano and Fisher (2000) describe several of these PEA algorithms in detail and compare them with other non-PEA algorithms in the context of the standard one-country stochastic neoclassical growth model. The comparison favors the so-called PEA *collocation* algorithm, a special case of Chebyshev PEA. On what follows, we focus on this method, that works in the following fashion:

A1.- Form a two-dimensional grid (since there are two state variables in this example), H^2, combining n_k different values of k and n_θ different values of θ.[5] The n_k (n_θ) different values of k (θ) are chosen to be the n_k (n_θ) zeros of a Chebyshev polynomial of order n_k (n_θ). The n zeros of a Chebyshev polynomial of order n, $\zeta^n = \{\zeta_1, ..., \zeta_n\}$, are defined by:

$$\zeta_j = \cos\left(\frac{2j-1}{2n}\pi\right), \ j = 1, ..., n$$

The different values in each dimension are then combined to create the grid using the tensor product. Namely,

$$H^2 = \zeta^{n_k} \times \zeta^{n_\theta}$$

A2.- Construct an approximating function combining Chebyshev polynomials in the following fashion:

$$\Psi(k, \theta; \Phi) = \sum_{i=0}^{n_k} \sum_{j=0}^{n_\theta} \phi_{ij} T_i(\varphi(k)) T_j(\varphi(\theta)) \tag{3}$$

where $T_n(x)$ denotes a Chebyshev polynomial of order n evaluated at $x \in [-1, 1]$.[6] Chebyshev polynomials of different orders are defined in the following way:

$$\begin{aligned} T_0(x) &= 1 \\ T_1(x) &= x \\ T_n(x) &= 2xT_{n-1}(x) - T_{n-2}(x) \ \forall n \geq 2 \end{aligned}$$

[4] In principle, choosing carefully the approximating function Ψ it should be possible to attain high levels of accuracy. For instance, if Ψ is a polynomial, arbitrary levels of accuracy could be obtained in the approximation by progressively increasing the order of the polynomial.

[5] In general, form a d-dimensional grid, H^d, of the d state variables of the problem.

[6] State variables do not usually lie on the interval $[-1, 1]$, but it is possible to set upper and lower bounds for them. The function $\varphi(\cdot)$ is then used to change the scale of those variables. For instance, in the case of k, $\varphi : [\underline{k}, \overline{k}] \to [-1, 1]$. An example of such an scaling function would be $\varphi(k) = 2\frac{k-\underline{k}}{\overline{k}-\underline{k}} - 1$.

A3.- In the approximating function (3) there are $n_k \times n_\theta$ unknown coefficients ϕ_{ij} that need to be determined. The approach used by this algorithm to find those coefficients is simple. It imposes that, at all grid points, also known as collocation points, $\Psi(k, \theta; \Phi)$ approximates *exactly* the conditional expectation in (2), where the latter is computed using standard quadrature techniques (i.e., Gauss-Hermite quadrature). The fact that there are exactly as many unknown coefficients as grid points makes it possible to reach such exact approximation. In this sense, since there are as many equations $\Psi(\cdot, \Phi) = E_t[\cdot]$ (one for each grid point) as unknowns (the ϕ_{ij}'s), it would be possible to find those unknown coefficients using any standard non-linear equations solver. This approach, however, may be computationally cumbersome, especially if the number of grid points or unknowns is large. This is why, in practice, researchers have vastly prefer an iterative procedure of the following type:

A3i.- Make an initial assumption, Φ_0, about the unknown coefficients.

A3ii.- For each grid point solve for c_t such that

$$c_t^{-\sigma} = \beta \Psi(k, \theta; \Phi_0)$$

and use the budget constraint (1) to find k_{t+1}.

A3iii.- For each grid point compute $E_t\left[c_{t+1}^{-\sigma}(\theta_{t+1}\alpha k_{t+1}^{\alpha-1} + 1 - \delta)\right]$ using standard quadrature techniques. The value to k_{t+1} has been obtained in step A3ii and c_{t+1} is computed so that $c_{t+1}^{-\sigma} = \beta\Psi(k_{t+1}, \theta_{t+1}; \Phi_0)$ for the different values of θ_{t+1} considered in the quadrature procedure.

A3iv.- Obtain a new set of coefficients, Φ_1, regressing $E_t\left[c_{t+1}^{-\sigma}(\theta_{t+1}\alpha k_{t+1}^{\alpha-1} + 1 - \delta)\right]$ on the right hand side of equation (3).

A3v.- If Φ_0 and Φ_1 are close enough (for a given degree of precision), stop the iterative procedure and approximate the conditional expectation in (2) using the approximating function (3) with these coefficients. Otherwise, update the coefficients and repeat steps A3ii-A3iv until convergence between Φ_i and Φ_{i+1} is achieved.

2.2 Smolyak-PEA

The PEA collocation algorithm described in section 2.1 works fairly well for low dimensional problems. However, as the dimension of the problem increases the implementation of this algorithm becomes exponentially more complicated. To get a sense of the *curse of dimensionality* faced by this algorithm simply note that, while in a 2-dimensional problem using polynomials of order 4 in the approximation implies finding 25 unknown coefficients, in problems with 6 or 9 state variables it would imply solving for 15,625 and 1,953,125 unknowns, respectively. The latter is clearly unfeasible and it is where the Smolyak algorithm becomes extremely convenient.

The Smolyak algorithm was first introduced in the economic literature by Krueger and Kubler (2004) and it is particularly indicated to deal with high-dimensional problems. The reason is simple. By choosing carefully both the approximating function and the collocation points, the number of unknown coefficients to be found in this algorithm does not grow exponentially with the dimension of the problem. More interestingly, as we will show, despite employing less coefficients in the approximations relative to other methods, there is no substantial accuracy loss in using this approach.

Krueger and Kubler (2004) study an overlapping generations economy with stochastic aggregate production and, by using the Smolyak algorithm, manage to solve problems with 20-30 state variables. In a different application of this algorithm, Malin, Krueger and Kubler (2007) consider a multi-country stochastic neoclassical growth model and are able to solve problems with up to 12 state variables. In both applications, however, the Smolyak algorithm is used to approximate the policy functions.[7] The approach in this paper, instead, is to use the Smolyak algorithm to approximate the conditional expectations that appear in stochastic models, that is, in combination with the PEA algorithm. As argued by Christiano and Fisher (2000), this alternative route is particularly indicated if the problem being analyzed features occasionally binding constraints, as it is usually the case, for instance, in portfolio problems. In those cases, approximating the conditional expectations is more convenient than approximating the policy functions, since the latter approach needs to jointly parameterize the policy and the multiplier functions, what may be a cumbersome and complicated task.

We now describe how to combine the Smolyak and the PEA collocation algorithms. Incorporating the former into the procedure described in section 2.1 basically entails two changes:

B1.- *The grid*. There are two basic differences between the grid in PEA collocation and the grid in Smolyak. On the one hand, in Smolyak, the grid points in each dimension are not the *zeros* of Chebyshev polynomials but the *extrema* of those polynomials. The n extrema of a Chebyshev polynomial of order n, $G^n = \{G_1, ..., G_n\}$, are defined by:

$$G_j = -\cos\left(\frac{j-1}{n-1}\pi\right), \; j = 1, ..., n$$

This difference is important because the extrema of a Chebyshev polynomial of a given order are nested in the extrema of higher order polynomials, i.e. $G^n \subset G^{n+1}$ $\forall n = 1, 2, ...$ This is crucial in reducing the dimensionality of the problem.

On the other hand, in Smolyak, the multi-dimensional grid is not created by simply

[7] Both papers provide a detailed description of the Smolyak algorithm.

applying the tensor product to the set of grid points in each dimension. Instead, it is constructed according to the following expression:

$$H^{d,\lambda} = \bigcup_{\mathbf{i}:\ |\mathbf{i}|=d+\lambda} G^{m(i_1)} \times ... \times G^{m(i_d)}$$

where $\mathbf{i} = (i_1, ..., i_d) \in Z^d_{++}$ is a vector of positive integers, $|\mathbf{i}| = i_1 + ... + i_d$, d denotes the dimension of the problem (the number of state variables) and λ is a measure of the desired dimension of the grid (and, by extension, of the accuracy pursued in the whole approximation procedure). Finally, the $m(\cdot)$'s are integers that define the number of Chebyshev extrema to consider in each possible dimension. They are constructed in the following way:

$$\begin{aligned} m(1) &= 1 \\ m(i) &= 2^{i-1} + 1 \text{ for } i = 2, 3, ... \end{aligned}$$

B2.- *The approximating function.* In the Smolyak algorithm, the approximating function is constructed in the following fashion:

$$\Psi(V; \Phi)^{d,\lambda} = \sum_{q \leq |\mathbf{i}| \leq d+\lambda} (-1)^{d+\lambda-|\mathbf{i}|} \begin{pmatrix} d-1 \\ d+\lambda-|\mathbf{i}| \end{pmatrix} p^{\mathbf{i}}(V) \tag{4}$$

where

$$p^{\mathbf{i}}(V) = \sum_{l_1=1}^{m(i_1)} ... \sum_{l_d=1}^{m(i_d)} \phi_{l_1 ... l_d} T_{l_1}(\varphi(v_1)) ... T_{l_d}(\varphi(v_d))$$

and $q = \max(d, \lambda + 1)$. $V = (v_1, ..., v_d)$ is a d-dimensional vector of the state variables and the scaling function φ is used to place the values these variables on the interval $[-1, 1]$.

Thus, while in the standard collocation method the approximating function is just the tensor product of Chebyshev polynomials in each dimension, in Smolyak this function is constructed using linear combinations of tensor products of Chebyshev polynomials in certain dimensions. In any case, note that in this algorithm, as in the standard PEA collocation, the number of polynomial coefficients to be estimated is exactly equal to the number of grid points in $H^{d,\lambda}$. In this sense, the number of grid points in $H^{d,\lambda}$ (equal to the number of unknown coefficients in (4)) is given by equation (5) for an accuracy level $\lambda = 2$ and by equation (6) for an accuracy level $\lambda = 3$.

$$\dim(H^{d,2}) = 1 + 4d + 2d(d-1) \tag{5}$$

$$\dim(H^{d,3}) = 1 + 8d + 6d(d-1) + 8\frac{d(d-1)(d-2)}{6} \tag{6}$$

A simple exploration of these formulas makes evident how the Smolyak algorithm is able to avoid the *curse of dimensionality* faced by the PEA collocation algorithm. For instance, in a problem with 9 state variables, using Smolyak with an accuracy level $\lambda = 2$ ($\lambda = 3$) would imply solving for 181 (1,177) unknowns. This is feasible and it contrasts with the number of unknowns that a PEA collocation algorithm would need to estimate in a problem with the same dimension. Moreover, we will show below (and this has also been shown by Krueger and Kubler (2004) and Malin, Krueger and Kubler (2007)) that using Smolyak with accuracy levels $\lambda = 2$ or $\lambda = 3$ is enough to reach the precision levels in the approximations typically considered in the literature.

2.3 Quantitative evaluation

This section evaluates, in the context of the one-country stochastic neoclassical growth model, the performance of the Smolyak-PEA algorithm presented in section 2.2 and compares it with that of the standard PEA collocation. The purpose of this comparison is to show that applying the Smolyak method to the parameterization of conditional expectations delivers similar or even better results in terms of the quality of approximations than the standard PEA collocation algorithm. And this is despite the fact that the former requires the computation of a much lower number of polynomial coefficients.

To begin with, we consider the following standard parameterization of the model:

Table 1. Parameter values

α	β	σ	δ	ρ
0.33	0.95	1.5	0.02	0.9

Regarding volatility, we follow Malin, Krueger and Kubler (2007) and evaluate the performance of the two algorithms in two different scenarios. A low volatility scenario, where $\sigma_\epsilon = 0.001$, and a high volatility one where $\sigma_\epsilon = 0.01$.

The comparison between algorithms is established in terms of two commonly used measures of accuracy: an *economic* measure in line with Judd (1992) and a *statistical* measure in line with Den Haan and Marcet (1994). The *economic* measure evaluates how large are (in economic terms) the deviations from the problem's equilibrium conditions generated by the approximation procedure. In this particular example, the deviation is just:

$$u_t = c_t - \left[\beta E_t \left[c_{t+1}^{-\sigma}(\theta_{t+1}\alpha k_{t+1}^{\alpha-1} + 1 - \delta)\right]\right]^{\frac{-1}{\sigma}} \tag{7}$$

Expressing that deviation in consumption terms, one may consider two different measures of accuracy after evaluating (7) for a particular set of points:[8]

$$E_{mean} = \log_{10}\left(E\left|\frac{u_t}{c_t}\right|\right) \tag{8}$$

$$E_{\max} = \log_{10}\left(\max\left|\frac{u_t}{c_t}\right|\right) \tag{9}$$

The interpretation of these measures is straightforward. For instance, $E_{mean} = -5$ ($E_{\max} = -5$) means that, for the set of points considered, individuals using the approximation rule make, on average (at most), an error of \$1 when consuming \$100,000.

Regarding the *statistical* measure, we consider the test proposed by Den Haan and Marcet (1994), which basically checks whether simulated series generated by a given numerical solution method satisfy the rational expectations assumption or not. According to this assumption, errors made by individuals at a given period when forecasting the future must be orthogonal to that period information set, Ω_t. More specifically, it must happen that:

$$E\left[e_t \otimes h\left(x_t\right)\right] = 0$$

for any k-dimensional vector x_t belonging to Ω_t and any function $h : R^k \to R^q$, where e_t denotes the forecasting errors at period t. In our particular example,[9]

$$\begin{aligned} e_t &= \beta E_t c_{t+1}^{-\sigma}(\theta_{t+1}\alpha k_{t+1}^{\alpha-1} + 1 - \delta) - c_t^{-\sigma} \\ h_t &= \left[\begin{array}{ccccccc} 1 & k_t & k_{t-1} & k_{t-2} & \theta_t & \theta_{t-1} & \theta_{t-2} \end{array}\right]' \end{aligned}$$

In practice, this test is typically implemented in the following way:

(i) Simulate the model a large number of times and, for each of them, compute $TB_T' A_T^{-1} B_T$, where

$$\begin{aligned} B_T &= \frac{1}{T}\sum^{T} e_t \otimes h_t \\ A_T &= \frac{1}{T}\sum^{T} e_t^2 h_t h_t' \end{aligned}$$

[8] Note that, *by construction*, deviations in (7) are exactly zero at the collocation points, both in the Smolyak-PEA and in the PEA collocation algorithms. Thus, it only makes sense to compute this accuracy measure at points different from the collocation points used in the estimation of the approximating function.

[9] We use the same function h as Den Haan and Marcet (1994) but, in principle, a different one could be used.

and T is the number of periods considered in each simulation.

(ii) Den Haan and Marcet (1994) show that, under some conditions, the statistic $TB_T' A_T^{-1} B_T$ is distributed according to a χ_q^2. Then, observe the percentage of the simulations in which the statistic lies in the lower ($P_{.05}$) and upper ($P_{.95}$) critical 5% of that distribution. The closer those two percentages are to 5%, the better.

Tables 2 and 3 show the performance of the standard PEA collocation and the Smolyak-PEA algorithms for the low volatility and high-volatility scenarios, respectively, in terms of the accuracy measures described above. We stop both algorithms when, in the curse of the iterative procedure they use to find the unknown coefficients of their approximating functions, it happens that $\| \Phi_i - \Phi_{i+1} \| < 1e-6$. For the PEA collocation exercise we consider $n_k = n_\theta = 10$. We compute the *economic* accuracy measures for two set of points. On the one hand, we simulate the model for 1200 periods and compute (8) and (9) using the last 1000 periods (*Simulation*). We start the simulation in the non-stochastic steady state and drop the first 200 periods so as to get rid of the effect of initial conditions. The sequence of shocks is the same for both algorithms, what makes the accuracy measures computed in this exercise fully comparable. On the other hand, we also form a new grid of the state variables and compute (8) and (9) using those grid points (*Grid*). Finally, to implement the Den Haan-Marcet test, we run 1000 simulations of the model of 1000 periods each.

Table 2. Accuracy evaluation: Low volatility scenario

	Economic measure				*Statistical* measure	
	Simulation		Grid		Den Haan-Marcet	
	E_{mean}	$E_{\max}$	E_{mean}	$E_{\max}$	$P_{.05}$	$P_{.95}$
PEA collocation	-7.62	-7.12	-6.45	-6.03	3.6%	5.2%
Smolyak-PEA ($\lambda = 2$)	-7.05	-6.57	-6.08	-5.64	4.0%	3.8%
Smolyak-PEA ($\lambda = 3$)	-7.63	-7.12	-6.45	-6.04	4.7%	4.9%

Table 3. Accuracy evaluation: High volatility scenario

	Economic measure				*Statistical* measure	
	Simulation		Grid		Den Haan-Marcet	
	E_{mean}	$E_{\max}$	E_{mean}	$E_{\max}$	$P_{.05}$	$P_{.95}$
PEA collocation	-6.24	-6.21	-6.24	-6.03	4.3%	4.6%
Smolyak-PEA ($\lambda = 2$)	-4.22	-3.77	-3.91	-3.32	3.8%	6.6%
Smolyak-PEA ($\lambda = 3$)	-6.29	-5.54	-5.46	-4.71	4.9%	4.3%

Looking at Tables 2 and 3, it is clear that both the Smolyak-PEA and the PEA collocation algorithms reach very high levels of accuracy, both in terms of the *economic* accuracy measures and in terms of the Den Haan-Marcet test. The fact that, overall, the Smolyak-PEA algorithm exhibits a similar degree of accuracy as the standard

collocation approach is remarkable and it is a very promising result prior to applying this algorithm to problems with a large number of state variables, where methods like PEA collocation face the *curse of dimensionality* and can not be used.[10] Note also that this result holds despite the fact that the Smolyak-PEA algorithm employs fewer coefficients than the PEA collocation one. Namely, in the latter we are estimating 100 coefficients while in the former only 13 (29) are estimated with $\lambda = 2$ ($\lambda = 3$).

Obviously, within the Smolyak-PEA algorithm, the higher λ the better the approximation. Moreover, as in Malin, Krueger and Kubler (2007), it is reasonable to expect that accuracy in the high volatility case is lower than in the low volatility one. Finally, the *economic* accuracy measure is higher when computed at the simulated points rather than at the grid points. The reason is that the grid points selected for doing these accuracy tests are evenly spaced in the state space and, consequently, they include some combinations of the state variables that are extreme and where non-linearities are potentially important. In contrast, the simulated points are more centered around the realizations of the shocks that are more likely and where the model behaves more smoothly.

3 Two-country model with portfolio decisions and incomplete markets

This section shows how to apply the Smolyak-PEA algorithm presented in section 2.2 to a problem with portfolio decisions and incomplete markets. Namely, consider a one-sector stochastic neoclassical growth model *with two countries*. A representative agent in each country solves the following intertemporal maximization problem:

$$Max\ E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_{i,t}^{1-\sigma}}{1-\sigma}$$

subject to

$$c_{i,t} + \sum_{j=1}^{2} a_{i,j,t+1} + \frac{\pi}{2} \sum_{j=1}^{2} a_{i,j,t+1}^2 = \sum_{j=1}^{2} (1 + r_{j,t}) a_{i,j,t} + w_{i,t} L_{i,t}\ \forall t \tag{10}$$

where $c_{i,t}$ denotes consumption in country i at period t, $i = \{1, 2\}$, $a_{i,j,t}$ is the capital of country j in hands of country i's representative agent at period t, $r_{j,t}$ is the net rate of return of country j's capital at period t, and w denotes wages. Each country's representative agent is endowed one unit of time in each period, which

[10] The purpose of this exercise is not to reach the highest accuracy level with each algorithm, but to see if these methods' accuracy is in the same order of magnitude. In this sense, it is clear that one could increase the accuracy of the PEA collocation algorithm by considering larger values of n_k and n_θ, and of the Smolyak-PEA by increasing λ.

he inelastically supplies to his domestic labor market (i.e., labor is immobile across countries).[11] The discount parameter, β, is assumed to be the same for all agents.

In this set up, the portfolio problem faced by individuals in both countries is clear. In order to smooth their consumption over time, to transfer wealth across periods, individuals can use different mixtures of two assets, capital in country 1 and capital in country 2, being the return of both investments uncertain.[12] Note that these portfolio decisions are affected by the presence of quadratic portfolio costs on each type of asset holdings, the third term on the left hand side of equation (10). The reason to include these costs is the following. It is well-known that open economy models with incomplete markets face stationarity problems. Using small quadratic portfolio costs is one of the routes pursued in the literature to get rid of those problems.[13] In particular, with very small portfolio costs, governed by the parameter π, one can induce stationarity but still get portfolio decisions close to those resulting if these costs were completely absent.

The two first order conditions of country i's representative agent problem are:

$$\begin{aligned} c_{i,t}^{-\sigma} &= \frac{\beta}{1+\pi a_{i,1,t+1}} E_t\left[c_{i,t+1}^{-\sigma}(1+r_{1,t+1})\right] \\ c_{i,t}^{-\sigma} &= \frac{\beta}{1+\pi a_{i,2,t+1}} E_t\left[c_{i,t+1}^{-\sigma}(1+r_{2,t+1})\right] \end{aligned}$$

A representative firm in each country hires labor and capital in each period in a competitive fashion. Namely, the representative firm in country i solves the following problem at period t:

$$\max\ \theta_{i,t} K_{i,t}^{\alpha} L_{i,t}^{1-\alpha} - (r_{i,t}+\delta) K_{i,t} - w_{i,t} L_{i,t}$$

where each firm TFP (i.e., each country TFP) evolves according to the following vector autoregressive process:

$$\begin{pmatrix} \log\theta_{1,t} \\ \log\theta_{2,t} \end{pmatrix} = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix} \begin{pmatrix} \log\theta_{1,t-1} \\ \log\theta_{2,t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \tag{11}$$

or in compact form $\log(\theta_t) = A\log(\theta_{t-1}) + \varepsilon_t$. The innovations ε_t are serially independent and they are distributed according to a bivariate normal distribution

[11] The Smolyak-PEA algorithm presented in this paper may also solve models with endogenous labor supply. We consider an inelastic labor supplied here just for simplicity.

[12] Adding a risk-free asset to this problem would not imply any significant change in the solution method described here.

[13] This is the approach followed, for instance, by Heathcote and Perri (2002). For a discussion on different alternative ways of inducing stationarity in this type of models see Schmitt-Grohe and Uribe (2003).

with mean zero and variance-covariance matrix Σ. By making the off-diagonal terms of matrix A equal to zero we are assuming, as most of the literature, that there are no spillovers in the transmission of shocks (although shocks may be correlated, as determined by matrix Σ). However, everything that follows would perfectly apply to any arbitrary matrix A.

The first order conditions of firms' problems define the economy prices. Namely, for $i = \{1, 2\}$ and $\forall t$

$$\begin{aligned} r_{i,t} &= \theta_{i,t}\alpha\left(\frac{K_{i,t}}{L_{i,t}}\right)^{\alpha-1} - \delta \\ w_{i,t} &= \theta_{i,t}\left(1-\alpha\right)\left(\frac{K_{i,t}}{L_{i,t}}\right)^{\alpha} \end{aligned}$$

The model is closed with the market clearing conditions in the labor and capital markets:

$$\begin{aligned} L_{i,t} &= 1 \ \forall i, t \\ K_{i,t} &= \sum_{j=1}^{2} a_{j,i,t} \ \forall i, t \end{aligned}$$

3.1 Full information vs. partial information

The state variables of the problem presented above are the TFP level in each country and the whole distribution of assets in the economy. More specifically, the vector of state variables is:[14]

$$[a_{1,1} \ a_{1,2} \ a_{2,1} \ a_{2,2} \ \theta_1 \ \theta_2 \ ] \tag{12}$$

We are going to consider two different cases: a *full information* and a *partial information* case. In the *full information* case, we consider that individuals, when solving their consumption-portfolio problems, know the whole state of the economy given by vector (12). Instead, in the *partial information* case, we consider that individuals do not know the whole distribution of assets in the economy, and they only know their own asset holdings and the aggregate amount of capital in each country. More specifically, the representative agent in country i, $i = \{1, 2\}$, has the following information:

$$[a_{i,1} \ a_{i,2} \ K_1 \ K_2 \ \theta_1 \ \theta_2 \ ] \tag{13}$$

[14] We drop time subscripts for simplicity.

In this two-country set up, the full and the partial information approaches have the same dimension, 6.[15] However, to see the usefulness of the partial information approach, note that in a 6-country set up, for instance, the dimension of the problem in the full information case would be 42, while in the partial information case it would only be 18. Thus, even if the Smolyak algorithm is able to handle high-dimensional problems, if the dimension of the problem is too high, it may be useful to combine the Smolyak algorithm and the partial information approach. We argue that this is specially relevant for the study of international portfolio problems, which become very high-dimensional problems very easily.

3.1.1 Full information. Computation strategy

The full information problem can be solved with the Smolyak-PEA algorithm basically following the steps described in section 2 for the case of a one-country neoclassical growth model. One just need to realize that, in this case, the model's equilibrium conditions contain four conditional expectations that need to be parameterized. These are the steps to follow:

C1.- Form a grid of the state vector (12) according to the procedure B1.

C2.- Construct an approximating function *for each of the 4* conditional expectations of the model according to B2. Namely,

$$\Psi_{11}(a_{1,1}, a_{1,2}, K_1, K_2, \theta_1, \theta_2; \Phi_{11}) \simeq E_t \left[c_{1,t+1}^{-\sigma} (\theta_{1,t+1} \alpha \left(\frac{K_{1,t+1}}{L_{1,t+1}} \right)^{\alpha-1} + 1 - \delta) \right]$$

$$\Psi_{12}(a_{1,1}, a_{1,2}, K_1, K_2, \theta_1, \theta_2; \Phi_{12}) \simeq E_t \left[c_{1,t+1}^{-\sigma} (\theta_{2,t+1} \alpha \left(\frac{K_{2,t+1}}{L_{2,t+1}} \right)^{\alpha-1} + 1 - \delta) \right]$$

$$\Psi_{21}(a_{2,1}, a_{2,2}, K_1, K_2, \theta_1, \theta_2; \Phi_{21}) \simeq E_t \left[c_{2,t+1}^{-\sigma} (\theta_{1,t+1} \alpha \left(\frac{K_{1,t+1}}{L_{1,t+1}} \right)^{\alpha-1} + 1 - \delta) \right]$$

$$\Psi_{22}(a_{2,1}, a_{2,2}, K_1, K_2, \theta_1, \theta_2; \Phi_{22}) \simeq E_t \left[c_{2,t+1}^{-\sigma} (\theta_{2,t+1} \alpha \left(\frac{K_{2,t+1}}{L_{2,t+1}} \right)^{\alpha-1} + 1 - \delta) \right]$$

[15] The fact that in this partial information set up country i's representative agent could, in principle, know the asset holdings of country j's representative agent simply using the market clearing condition in capital markets, $K_{i,t} = \sum_{j=1}^{2} a_{j,i,t}$, could be misleading. In particular, it could induce someone to think that the information in hands of individuals is the same in the full and in the partial information set ups. This is not correct. As it will be clearer below, in the partial information approach one country's representative agent behaves as if he could not really know the other agent's capital holdings (in fact, in a multi-country setting with more than 2 countries, he wouldn't be able to know that even after using the market clearing conditions).

C3.- For each conditional expectation, follow the iterative procedure A3i-A3v. Just note that one needs to solve simultaneously for $c_{i,t}$, $a_{i,1,t+1}$and $a_{i,2,t+1}$, $i = \{1, 2\}$. In the computations, $K_{i,t+1} = \sum_{j=1}^{2} a_{j,i,t+1}$ $\forall i$.

3.1.2 Partial information. Computation strategy

The partial information problem can be solved in the following fashion:

D1.- According to the procedure B1, form *two* grids, one for the state vector of country 1's representative agent ($i = 1$ in (13)) and another one for the state vector of country 2's representative agent ($i = 2$ in (13)).

D2.- Construct an approximating function *for each of the 4* conditional expectations of the model as in step C2.

D3.- Consider that individuals make forecasts about next period values of country 1 and country 2 aggregate capital, denoted by K_1' and K_2', combining current and past information in a particular fashion. Namely, we assume that all agents use the following forecasting rules:

$$\log K_1' = \eta_{10} + \eta_{11} \log K_1 + \eta_{12} \log K_2 + \eta_{13} \log \theta_1 + \eta_{14} \log \theta_2 \tag{14}$$

$$\log K_2' = \eta_{20} + \eta_{21} \log K_1 + \eta_{22} \log K_2 + \eta_{23} \log \theta_1 + \eta_{24} \log \theta_2 \tag{15}$$

or in compact form, $K_i' = F(K_1, K_2, \theta_1, \theta_2; \eta_i)$, $i = \{1, 2\}$, where the η's are parameters that need to be found optimally within the numerical procedure. Certainly, there are many alternative forecasting rules that could be used. They could include, for instance, more lags of the aggregate state variables. However, as we will show below, the forecasting rules (14) and (15), that are similar to others used in the literature that considers partial information set ups, work really well in our problem in terms of accuracy.

Forecasting rules are needed for an obvious reason. For exactly the same reason individuals need to know the law of motion of each country TFP, equation (11), in order to take their decisions, they also need to know how the other aggregate state variables in their information sets, K_1 and K_2, are expected to behave in the future. The forecasting rules provide that information.

D4.- Make an initial assumption, η_1^0 and η_2^0, about the values of the η's parameters in the forecasting rules and initiate the iterative procedure A3i-A3v. The difference with respect to the strategy described in section 3.1.1 for the full information case is that, in this case, the $K_{i,t+1}$'s that enter into the conditional expectations are

not constructed as $K_{i,t+1} = \sum_{j=1}^{2} a_{j,i,t+1}\ \forall i$, but as $K_{i,t+1} = F(K_{1,t}, K_{2,t}, \theta_{1,t}, \theta_{2,t}; \eta_i^0)$ $\forall i$. In this sense, note that individuals do not have enough information to infer other individuals' next period asset holdings and, therefore, can not use the former formula.

D5.- At the end of each iteration of the procedure A3i-A3v update, not only the coefficients in the approximating function, Φ, but also the coefficients in the forecasting rules, η. To update the η's, at the end of each iteration, simulate the model for a given sequence of technology shocks approximating the conditional expectations with the last Φ computed. Then, with the simulated data make the regressions implicit in (14) and (15) and obtain new η's.

D6.- Stop the iterative procedure when there is convergence both in Φ and in η.

3.1.3 Quantitative evaluation

This section evaluates how accurately the Smolyak-PEA algorithm is able to solve the full information and the partial information scenarios described above. For simplicity, we assume that the two countries are identical and use the same parameter values as in the one-country case (Table 1). In addition, we set $\pi = 0.0001$, which makes quadratic portfolio costs so small that they do not interfere significantly in the individuals' portfolio decisions. As in section 2.3, the quantitative evaluation is performed both in a low volatility scenario, with $\Sigma = \begin{pmatrix} 0.001^2 & 0 \\ 0 & 0.001^2 \end{pmatrix}$, and in a high volatility one with $\Sigma = \begin{pmatrix} 0.01^2 & 0 \\ 0 & 0.01^2 \end{pmatrix}$.

The unknown coefficients of the approximating functions and, in the case of the partial information scenario, of the forecasting rules are determined following the strategies described in sections 3.1.1 and 3.1.2. With them, we run 100 simulations of the model, of 1200 periods each, both in the low volatility and in the high-volatility scenarios. In each simulation, (8) and (9) are computed for the last 1000 periods (we drop the first 200 periods to eliminate the effect of initial conditions) and each of the 4 first order conditions of the problem. Figures 1-4 show these errors for the full and the partial information cases, in all simulations and in the two volatility scenarios. Table 4 summarizes the information in the figures by taking the average and the *max* across simulations.

Figure 1. E_{mean} - Low volatility

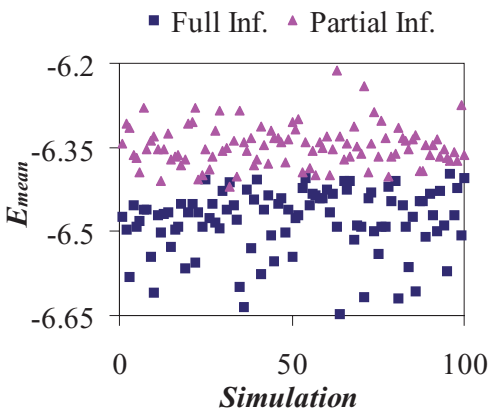


Figure 2. E_{max} - Low volatility

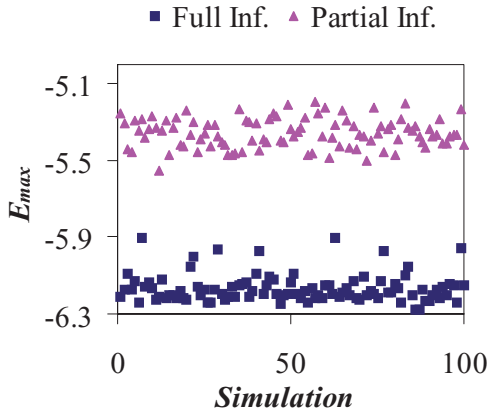


Figure 3. E_{mean} - High volatility

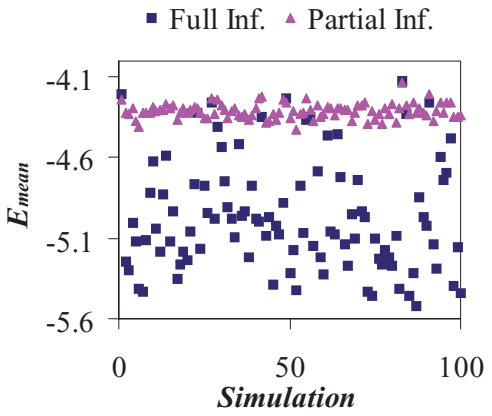
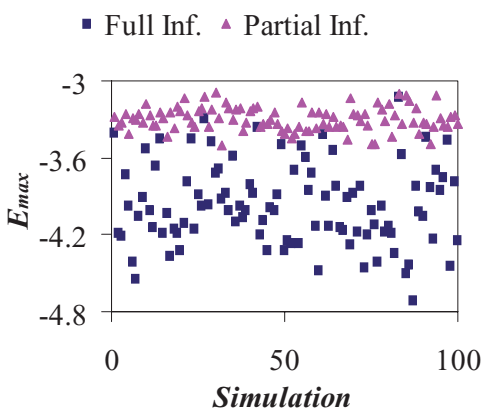


Figure 4. E_{max} - High volatility



As one could expect, the approximation is more accurate in the full information case than in the partial information case, and in the low volatility case than in the high volatility one. More importantly, (i) the algorithm's accuracy in all scenarios meets usual standards in the literature and (ii) it seems that the loss in accuracy associated to the use of the partial information approach is negligible. The latter result is especially interesting. Researchers have extensively used the Krusell-Smith partial information approach when solving the full information version of their models was difficult. Then, they usually report the R^2 obtained in the estimation of the forecasting rules, but do not compare their accuracy results with those of the full information scenario. We do this here and find that the accuracy loss of using a partial information approach is very small. One must note, however, that this is obviously a model-specific finding and it could not hold in more complex models. For completeness, Table 5 reports the estimated forecasting rules and their R^2.

Table 4. Full information vs. partial information: Accuracy

	Low volatility		High volatility	
	$mean(E_{mean})$	$\max(E_{\max})$	$mean(E_{mean})$	$\max(E_{\max})$
Full information	-6.4810	-5.9056	-4.9643	-3.1259
Partial information	-6.3475	-5.1964	-4.3108	-3.0841

Table 5. Partial information: Estimated forecasting rules

	Low volatility		High volatility	
Indep. var. \ *Dep. var*	$\log K_1'$	$\log K_2'$	$\log K_1'$	$\log K_2'$
Constant	0.1242	0.1241	0.1260	0.1265
$\log K_1$	0.47239	0.47239	0.47118	0.47103
$\log K_2$	0.47237	0.47238	0.47279	0.47271
$\log \theta_1$	0.73615	-0.59358	0.73717	-0.59239
$\log \theta_2$	-0.59359	0.73614	-0.59433	0.73533
R^2	0.999995	0.999995	0.999497	0.999664

3.2 Portfolio constraints

Both in financial markets are in theoretical models, portfolio decisions are often conditioned by occasionally binding constraints. This section shows how easily the Smolyak-PEA algorithm can accommodate this type of constraints. In fact, as pointed out by Christiano and Fisher (2000), the parameterized expectation approach has a relative advantage over other approaches focusing on policy functions precisely in the presence of these constraints. Thus, while the former method only needs to undertake slight modifications in the algorithm to deal with these constraints (as we will show below), the latter ones need to simultaneously parameterize the policy and the multiplier functions, what substantially complicates the search process. Therefore, we see this paper, that applies the Smolyak algorithm to the parameterization of conditional expectations, as complementary to the work in Krueger and Kubler (2004) and in Malin, Krueger and Kubler (2007), which apply the Smolyak algorithm to the parameterization of policy functions, being particularly useful in the presence of occasionally binding constraints.

To illustrate how the Smolyak-PEA algorithm works in the presence of occasionally binding constraints, we consider the two-country model described above and impose the following restriction on the portfolio decisions of country 1's representative agent:

$$a_{1,2,t+1} \leqslant \mu \left[a_{1,1,t+1} + a_{1,2,t+1} \right] \ \forall t \tag{16}$$

In words, this agent can not have more than a fraction $\mu \in [0,1]$ of his wealth invested abroad.[16] If $\mu = 0$, foreign investment is not allowed, while the constraint (16) never binds if $\mu = 1$. The equilibrium conditions at period t of this agent's maximization problem are now:[17]

$$c_{1,t}^{-\sigma}(1+\pi a_{1,1,t+1}) = \beta E_t\left[c_{1,t+1}^{-\sigma}(1+r_{1,t+1})\right] + \lambda_t \frac{\mu}{1-\mu}$$

$$c_{1,t}^{-\sigma}(1+\pi a_{1,2,t+1}) = \beta E_t\left[c_{1,t+1}^{-\sigma}(1+r_{2,t+1})\right] - \lambda_t$$

$$\lambda_t\left[\frac{\mu}{1-\mu}a_{1,1,t+1} - a_{1,2,t+1}\right] = 0$$

$$c_{1,t} + \sum_{j=1}^{2} a_{1,j,t+1} + \frac{\pi}{2}\sum_{j=1}^{2} a_{1,j,t+1}^2 = \sum_{j=1}^{2}(1+r_{j,t})a_{1,j,t} + w_{1,t}$$

where λ is the Lagrange multiplier associated to (16).

Using the Smolyak-PEA algorithm to solve this new problem is very easy. There is only one difference with respect to the procedure described in sections 3.1.1 and 3.1.2. In the curse of the iterative procedure A3i-A3v, when solving simultaneously for $c_{i,t}$, $a_{i,1,t+1}$and $a_{i,2,t+1}$, $i = \{1,2\}$, assume first that the portfolio constraints are not binding (in this case, $\lambda_t = 0$) and solve the problem. Then, check if optimal decisions found under that assumption actually satisfy the constraints. If they do, one can proceed. If they don't, solve the problem again but, at this time, looking also for a greater than zero Lagrange multiplier. This positive multiplier must also be taken into account when checking the accuracy of approximations.

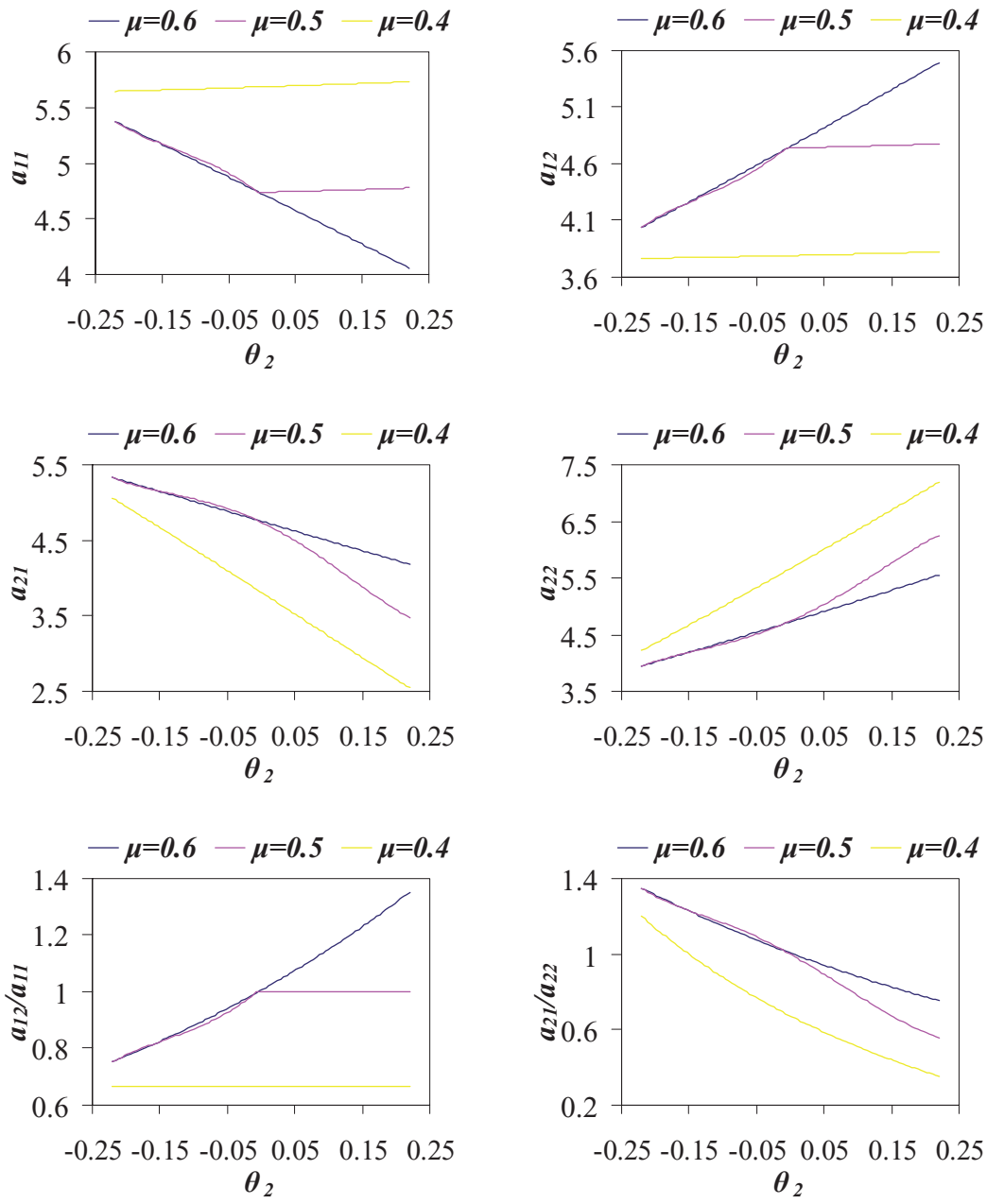
3.2.1 Policy functions

This section illustrates the effects of (16) on the policy functions of both countries' representative agents. In particular, we show how both individuals' optimal asset holdings, as a function of country 2's current technology level, change depending of the severity of country 1's portfolio constraint. We consider three values of μ. The first, $\mu = 0.6$, basically implies that (16) never binds. In the second case, $\mu = 0.5$, the constraint is occasionally binding. Finally, with $\mu = 0.4$, the constraint is binding almost always. The exercises are performed in a full information scenario and use the same parameter values considered in section 3.1.3. Furthermore, country 1's TFP level is always assumed to be at its mean value. Figure 5 shows the results.

[16] Similar restrictions on foreign investment are often encountered, for instance, by managers of pension funds in many countries.

[17] The problem of country 2's representative agent is identical to the one presented in the previous sections and we omit it here.

Figure 5: Policy functions



As expected, all agents want to invest more in country 2's capital the higher is country 2's TFP level today, since this signals a higher return to capital in that country in the following period due to the persistence in θ. However, the investment in country 2's capital of country 1's representative agent is restricted. When $\mu = 0.6$ this restriction does not bind. Instead, for $\mu = 0.5$ the restriction begins to bind

as θ_2 departs significantly from θ_1. Finally, for $\mu = 0.4$ the constraint always binds. Two results are worth-mentioning. First, note that domestic investment of country 1's representative agent increases with θ_2 when he is constrained. The reason behind this apparently counterintuitive result is obvious. Given the particular portfolio constraint we have considered, (16), constrained individuals may increase their foreign investment by increasing their domestic investment. That is why we find that domestic investment of constrained individuals increases even when that investment gets relatively less profitable. And second, note that country 2's representative agent, who does not face any portfolio constraint, still modifies his optimal decisions depending on the degree of country 1's portfolio constraint. This is simply due to the general equilibrium nature of our exercise.

4 Concluding remarks

In this paper we have proposed a global solution method that combines the parameterized expectations (PEA) and the Smolyak algorithms, and it is well-suited to deal with high-dimensional stochastic problems. We have shown how to apply this algorithm to multi-country models with portfolio decisions and incomplete markets, with and without occasionally binding constraints, and in a full and partial information schemes. In all these instances, the algorithm may be implemented at a relatively low cost and approximates the problems' solutions with a high degree of accuracy. We thus claim that the Smolyak-PEA algorithm is a powerful solution method that may be applied to a wide variety of relevant problems.

References

[1] CHRISTIANO, L. J., and J. FISHER (2000). "Algorithms for Solving Dynamic Models with Occasionally Binding Constraints", *Journal of Economic Dynamics and Control,* vol. 24, pp. 1179-1232.

[2] DEN HAAN, W., and A. MARCET (1990). "Solving the Stochastic Growth Model by Parameterizing Expectations", *Journal of Business and Economic Statistics,* vol. 8(1), pp. 31-34.

[3] – (1994). "Accuracy in Simulations", *Review of Economic Studies,* vol. 61, pp. 3-17.

[4] DEVEREUX, M. B., and A. SUTHERLAND (2007). "Solving for Country Portfolios in Open Economy Macro Models", *IMF Working Papers,* No. 07/284.

[5] DOMEIJ, D., and M. FLODEN (2006). "Population Aging and International Capital Flows", *International Economic Review,* vol. 47(3), pp. 1013-1032.

[6] EVANS, M. D., and V. HNATKOVSKA (2005). *Solving General Equilibrium Models with Incomplete Markets and Many Assets,* NBER Technical Working Papers, No. 0318.

[7] HEATHCOTE, J., and F. PERRI (2002). "Financial Autarky and International Business Cycles", *Journal of Monetary Economics,* vol. 49, pp. 601-627.

[8] JUDD, K. L. (1992). "Projection Methods for Solving Aggregate Growth Models", *Journal of Economic Theory,* vol. 58(2), pp. 410-452.

[9] KRUEGER, D., and F. KUBLER (2004). "Computing Equilibrium in OLG Models with Stochastic Production", *Journal of Economic Dynamics and Control,* vol. 28(7), pp. 1411-1436.

[10] KRUEGER, D., and A. LUDWIG (2007). "On the Consequences of Demographic Change for Rates of Returns to Capital, and the Distribution of Wealth and Welfare", *Journal of Monetary Economics,* vol. 54(1), pp. 49-87.

[11] KRUSELL, P., and A. A. SMITH (1998). "Income and Wealth Heterogeneity in the Macroeconomy", *Journal of Political Economy,* vol. 106(5), pp. 867-896.

[12] MALIAR, L., and S. MALIAR (2003). "Parameterized Expectations Algorithm and the Moving Bounds", *Journal of Business and Economic Statistics,* vol. 21(1), pp. 88-92.

[13] MALIN, B., D. KRUEGER and F. KUBLER (2007). *Computing Stochastic Dynamic Economic Models with a Large Number of State Variables: A Description*

and Application of a Smolyak-Collocation Method, NBER Working Papers, No. 13517.

[14] MARCET, A., and G. LORENZONI (1999). "The Parameterized Expectation Approach: Some Practical Issues", in R. Marimon and A. Scott (eds.), *Computational Methods for Study of Dynamic Economies,* New York, Oxford University Press, pp. 143-171.

[15] MARCET, A., and K. SINGLETON (1999). "Equilibrium Asset Prices and Savings of Heterogeneous Agents in the Presence of Incomplete Markets and Portfolio Constraints", *Macroeconomic Dynamics,* vol. 3, pp. 243-277.

[16] SCHMITT-GROHE, S., and M. URIBE (2003). "Closing Small Open Economy Models", *Journal of International Economics,* vol. 61(1), pp. 163-185.

[17] VAN WINCOOP, E., and C. TILLE (2007). I*nternational Capital Flows,* NBER Working Papers, No.12856.

BANCO DE ESPAÑA PUBLICATIONS

WORKING PAPERS[1]

0721 CLAUDIA CANALS, XAVIER GABAIX, JOSEP M. VILARRUBIA AND DAVID WEINSTEIN: Trade patterns, trade balances and idiosyncratic shocks.
0722 MARTÍN VALLCORBA AND JAVIER DELGADO: Determinantes de la morosidad bancaria en una economía dolarizada. El caso uruguayo.
0723 ANTÓN NÁKOV AND ANDREA PESCATORI: Inflation-output gap trade-off with a dominant oil supplier.
0724 JUAN AYUSO, JUAN F. JIMENO AND ERNESTO VILLANUEVA: The effects of the introduction of tax incentives on retirement savings.
0725 DONATO MASCIANDARO, MARÍA J. NIETO AND HENRIETTE PRAST: Financial governance of banking supervision.
0726 LUIS GUTIÉRREZ DE ROZAS: Testing for competition in the Spanish banking industry: The Panzar-Rosse approach revisited.
0727 LUCÍA CUADRO SÁEZ, MARCEL FRATZSCHER AND CHRISTIAN THIMANN: The transmission of emerging market shocks to global equity markets.
0728 AGUSTÍN MARAVALL AND ANA DEL RÍO: Temporal aggregation, systematic sampling, and the Hodrick Prescott filter.
0729 LUIS J. ÁLVAREZ: What do micro price data tell us on the validity of the New Keynesian Phillips Curve?
0730 ALFREDO MARTÍN-OLIVER AND VICENTE SALAS-FUMÁS: How do intangible assets create economic value? An application to banks.
0731 REBECA JIMÉNEZ-RODRÍGUEZ: The industrial impact of oil price shocks: Evidence from the industries of six OECD countries.
0732 PILAR CUADRADO, AITOR LACUESTA, JOSÉ MARÍA MARTÍNEZ AND EDUARDO PÉREZ: El futuro de la tasa de actividad española: un enfoque generacional.
0733 PALOMA ACEVEDO, ENRIQUE ALBEROLA AND CARMEN BROTO: Local debt expansion… vulnerability reduction? An assessment for six crises-prone countries.
0734 PEDRO ALBARRÁN, RAQUEL CARRASCO AND MAITE MARTÍNEZ-GRANADO: Inequality for wage earners and self-employed: Evidence from panel data.0735 ANTÓN NÁKOV AND ANDREA PESCATORI: Oil and the Great Moderation.
0736 MICHIEL VAN LEUVENSTEIJN, JACOB A. BIKKER, ADRIAN VAN RIXTEL AND CHRISTOFFER KOK-SØRENSEN: A new approach to measuring competition in the loan markets of the euro area.
0737 MARIO GARCÍA-FERREIRA AND ERNESTO VILLANUEVA: Employment risk and household formation: Evidence from differences in firing costs.
0738 LAURA HOSPIDO: Modelling heterogeneity and dynamics in the volatility of individual wages.
0739 PALOMA LÓPEZ-GARCÍA, SERGIO PUENTE AND ÁNGEL LUIS GÓMEZ: Firm productivity dynamics in Spain.
0740 ALFREDO MARTÍN-OLIVER AND VICENTE SALAS-FUMÁS: The output and profit contribution of information technology and advertising investments in banks.
0741 ÓSCAR ARCE: Price determinacy under non-Ricardian fiscal strategies.
0801 ENRIQUE BENITO: Size, growth and bank dynamics.
0802 RICARDO GIMENO AND JOSÉ MANUEL MARQUÉS: Uncertainty and the price of risk in a nominal convergence process.
0803 ISABEL ARGIMÓN AND PABLO HERNÁNDEZ DE COS: Los determinantes de los saldos presupuestarios de las Comunidades Autónomas.
0804 OLYMPIA BOVER: Wealth inequality and household structure: US vs. Spain.
0805 JAVIER ANDRÉS, J. DAVID LÓPEZ-SALIDO AND EDWARD NELSON: Money and the natural rate of interest: structural estimates for the United States and the euro area.
0806 CARLOS THOMAS: Search frictions, real rigidities and inflation dynamics.
0807 MAXIMO CAMACHO AND GABRIEL PEREZ-QUIROS: Introducing the EURO-STING: Short Term INdicator of Euro Area Growth.
0808 RUBÉN SEGURA-CAYUELA AND JOSEP M. VILARRUBIA: The effect of foreign service on trade volumes and trade partners.
0809 AITOR ERCE: A structural model of sovereign debt issuance: assessing the role of financial factors.
0810 ALICIA GARCÍA-HERRERO AND JUAN M. RUIZ: Do trade and financial linkages foster business cycle synchronization in a small economy?
0811 RUBÉN SEGURA-CAYUELA AND JOSEP M. VILARRUBIA: Uncertainty and entry into export markets.
0812 CARMEN BROTO AND ESTHER RUIZ: Testing for conditional heteroscedasticity in the components of inflation.
0813 JUAN J. DOLADO, MARCEL JANSEN AND JUAN F. JIMENO: On the job search in a model with heterogeneous jobs and workers.
0814 SAMUEL BENTOLILA, JUAN J. DOLADO AND JUAN F. JIMENO: Does immigration affect the Phillips curve? Some evidence for Spain.
0815 ÓSCAR J. ARCE AND J. DAVID LÓPEZ-SALIDO: Housing bubbles.

---

1. Previously published Working Papers are listed in the Banco de España publications catalogue.

0816 GABRIEL JIMÉNEZ, VICENTE SALAS-FUMÁS AND JESÚS SAURINA: Organizational distance and use of collateral for business loans.
0817 CARMEN BROTO, JAVIER DÍAZ-CASSOU AND AITOR ERCE-DOMÍNGUEZ: Measuring and explaining the volatility of capital flows towards emerging countries.
0818 CARLOS THOMAS AND FRANCESCO ZANETTI: Labor market reform and price stability: an application to the Euro Area.
0819 DAVID G. MAYES, MARÍA J. NIETO AND LARRY WALL: Multiple safety net regulators and agency problems in the EU: Is Prompt Corrective Action partly the solution?
0820 CARMEN MARTÍNEZ-CARRASCAL AND ANNALISA FERRANDO: The impact of financial position on investment: an analysis for non-financial corporations in the euro area.
0821 GABRIEL JIMÉNEZ, JOSÉ A. LÓPEZ AND JESÚS SAURINA: Empirical analysis of corporate credit lines.
0822 RAMÓN MARÍA-DOLORES: Exchange rate pass through in New Member States and candidate countries of the EU.
0823 IGNACIO HERNANDO, MARÍA J. NIETO AND LARRY D. WALL: Determinants of domestic and cross-border bank acquisitions in the European Union.
0824 JAMES COSTAIN AND ANTÓN NÁKOV: Price adjustments in a general model of state-dependent pricing.
0825 ALFREDO MARTÍN-OLIVER, VICENTE SALAS-FUMÁS AND JESÚS SAURINA: Search cost and price dispersion in vertically related markets: the case of bank loans and deposits.
0826 CARMEN BROTO: Inflation targeting in Latin America: Empirical analysis using GARCH models.
0827 RAMÓN MARÍA-DOLORES AND JESÚS VAZQUEZ: Term structure and the estimated monetary policy rule in the eurozone.
0828 MICHIEL VAN LEUVENSTEIJN, CHRISTOFFER KOK SØRENSEN, JACOB A. BIKKER AND ADRIAN VAN RIXTEL: Impact of bank competition on the interest rate pass-through in the euro area.
0829 CRISTINA BARCELÓ: The impact of alternative imputation methods on the measurement of income and wealth: Evidence from the Spanish survey of household finances.
0830 JAVIER ANDRÉS AND ÓSCAR ARCE: Banking competition, housing prices and macroeconomic stability.
0831 JAMES COSTAIN AND ANTÓN NÁKOV: Dynamics of the price distribution in a general model of state-dependent pricing.
0832 JUAN A. ROJAS: Social Security reform with imperfect substitution between less and more experienced workers.
0833 GABRIEL JIMÉNEZ, STEVEN ONGENA, JOSÉ LUIS PEYDRÓ AND JESÚS SAURINA: Hazardous times for monetary policy: What do twenty-three million bank loans say about the effects of monetary policy on credit risk-taking?
0834 ENRIQUE ALBEROLA AND JOSÉ MARÍA SERENA: Sovereign external assets and the resilience of global imbalances.
0835 AITOR LACUESTA, SERGIO PUENTE AND PILAR CUADRADO: Omitted variables in the measure of a labour quality index: the case of Spain.
0836 CHIARA COLUZZI, ANNALISA FERRANDO AND CARMEN MARTÍNEZ-CARRASCAL: Financing obstacles and growth: An analysis for euro area non-financial corporations.
0837 ÓSCAR ARCE, JOSÉ MANUEL CAMPA AND ÁNGEL GAVILÁN: Asymmetric collateral requirements and output composition.
0838 ÁNGEL GAVILÁN AND JUAN A. ROJAS: Solving Portfolio Problems with the Smolyak-Parameterized Expectations Algorithm.

BANCO DE **ESPAÑA**
Eurosistema

Unidad de Publicaciones
Alcalá, 522; 28027 Madrid
Telephone +34 91 338 6363. Fax +34 91 338 6488
e-mail: publicaciones@bde.es
www.bde.es