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(**) E-mail: ricardo.gimeno@bde.es and manuel-m@bde.es.

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Abstract

In this paper we decompose nominal interest rates into real risk-free rates, inflation expectations and risk premia using an affine model that takes as factors the observed inflation rate and the parameters generated in the zero yield curve estimation. We apply this model to the Spanish economy during the 90s, which is an especially challenging exercise given the nominal convergence towards the European Monetary Union (EMU) then under way. The methodology seems to be suitable for other countries currently involved in convergence towards EMU. The evidence indicates that inflation expectations and risk premia account for most of the observed variation in nominal rates, while real risk-free interest rates show a reduction during this period lower than that suggested by other approaches.

JEL codes: G12, E43, E44 and C53.

Keywords: Real interest rates, Risk Premium, Inflation expectations, Affine Model.

1 Introduction

Nominal interest rates can be decomposed, from a theoretical perspective, into three components: real risk-free rates, inflation expectations and risk premia. Disentangling which component is the main driver of some of the changes seen in nominal interest rates is often crucial in several different realms such as bond pricing, the analysis of investment or other expenditure decisions made by firms or households or in the process of monetary policy decision-making. Unfortunately, however, the above-mentioned components are not directly observable and the literature only proposes partial solutions to obtain this decomposition.

The most common approach consists in subtracting the ex-post inflation rate from nominal interest rates to obtain an estimate of real risk-free rates. This implies assuming that there is no risk premium and also that agents are able to perfectly foresight the inflation rate. These assumptions are most likely to be only slightly restrictive in reasonably stable economies where the variability of prices is small. However, in other cases the risk premia or the inflation expectations errors could be significant. Moreover, if the economy exhibits some convergence process it seems natural to expect significant variations in risk premia and/or (rational) important mistakes when forecasting inflation. This is for instance the case of Spain and several other European countries involved in EMU creation where uncertainty over convergence finally achieved could have originated fluctuations on these variables. In that environment, ex-post real interest rates could provide a misleading approach to the actual behaviour of real risk-free rates.

An alternative to ex-post real rates is using the return on inflation-indexed bonds to approach real rates, whereas inflation expectations are estimated as the difference in respect of their nominal reference. However, these bonds are not traded in many countries or have been introduced only recently. In addition, although this approach provides an intuitive estimation of real rates, it does not considered the risk premia and thus provide a biased estimation of both real risk-free rates and inflation expectations [see Evans (1998)], a bias that in some circumstances —let us think of the Great Moderation process [see Summers (2005)], can hardly be negligible.

Against this background, this paper propose a methodology to decompose nominal interest rates into its three components and applies it to the Spanish nominal interest rates during the nominal convergence period that led the Spanish economy to be one of the eleven first members of the Economic and Monetary Union. This methodology is related to the macro-finance literature in which authors such as Diebold, *et ál.* (2004), Diebold, *et ál.* (2005), Carriero, *et ál.* (2006), and Ang, Bekaert and Wei (2007) (ABW) incorporate macro-determinants into a multi-factor yield curve model with non-arbitrage opportunities.

In particular, we consider a model where interest rates are affine relative to a vector of factors that includes inflation rates and exogenously determined factors based on the Nelson-Siegel exponential components of the yield curve [Nelson and Siegel (1987) in a similar vein to Carriero, Favero and Kaminska (2006) and Diebold and Li (2006)]. Moreover, in our case we include the condition of non-arbitrage opportunities along the yield curve as well as taking into account the risk-aversion. Taking these two conditions together allows us to decompose nominal interest rates as the sum of real risk-free interest rates, expected inflation and the risk premium. We therefore depart from Dai and Singleton (2000), Laubach and Williams (2003), and ABW (2007), who consider latent components, which they endogenously estimate. The latent component methodology depends heavily on the initial conditions and the arbitrary selection of some maturities that have to be observed without error as well as some *ad hoc* restrictions on the parameters [Kim and Orphanides (2005)]. Our proposal suppose a less restrictive approach and the results seems to be more robust.

The decomposition exercise shows a decline in Spanish real risk-free interest rates during the nineties of an order close to 3 pp, a figure significantly lower than the estimations with other methodologies. In fact, Blanco and Restoy (2007) highlights that traditional approaches, that produce much higher declines in real risk-free rate, are not compatible with the observed evolution of Spanish macroeconomic figures (such as GDP growth rate or employment rate) and the increase experienced by some asset prices, such as the case of stock or house prices. Moreover, the results show that, during some episodes, long-run expected inflation rates were systematically higher than ex-post actual inflation rates. This finding could very likely be reflecting the uncertainties surrounding the Spanish success in fulfilling the Maastrich criteria on time. In fact, risk premia upturns coincide with the shifts in inflation expectations. Thus changes in inflation expectations and inflation risk premia account for a substantial part of the decrease in nominal interest rates during the convergence process.

The rest of the paper is structured in four further sections. The second section offers a description of nominal interest rates and inflation in Spain during the nineties. In the third, we present the basic affine model used throughout the paper. In section four we derive the decomposition of nominal interest rates and analyse the main results for the Spanish economy. Finally, section five concludes.

2 Interest rate developments in Spain

In Spain, the evolution of interest rates during the nineties was closely related to the nominal convergence process associated with EMU entry, and by the structural and policy changes made during this period. In this sense, the economy moved from a scenario of high inflation rates and large public deficits to a new framework based on fiscal surpluses, moderate inflation and EMU membership. However, as Blanco and Restoy (2007) point out, this was plagued by uncertainty related to the EMU process itself and the ability by the Spanish economy to fulfil the Maastricht convergence criteria. Therefore, it is likely that these uncertainties impinged on the expectations on several macroeconomic variables such as the inflation rate. In particular, long run inflation expectations should have been a weighted average of the inflation rate under the convergence regime and the alternative inflation rate that could have prevailed under a non convergence scenario. The corresponding weights are likely to have change according to results of the changing assessments of the EMU process and of the performance of the Spanish economy. More importantly, the inflation rate prevailing under the non convergence scenario was not finally observed. Under those circumstances, even if we consider agents as rational, the observed inflation rate is not a good proxy for inflation expectations nor it is a simple way of estimating those expectations.

Agent expectations on inflation should be reflected in financial markets where asset valuation depends on this macro variable. In fact, as can be seen in Figure 1, Spanish nominal 5-years interest rate fell from 13-14% at the beginning of the decade to 3%-4% at the end of the nineties reflecting a similar process than the inflation rate that can be seen in Figure 2, where the Spanish inflation differential against the euro area narrowed from 3 pp at the beginning of the decade to a value close to 1pp at the end of the nineties. Two peaks can be observed in the course of the reduction of nominal interest rates indicating the aforementioned uncertainty episodes: first, the European Monetary System crisis at the end of 1992; and second, the widening of the ERM bands in 1995. Moreover, the Peseta exchange rate was also affected by the uncertainty over the process. As Figure 3 shows, the decade started with high variability of the Spanish Peseta exchange rate against the Deutsche Mark, which gave rise to four episodes of devaluation that coincide with the aforementioned peaks of 92-93 and 95[1].

Under this framework, ex-post real interest rates, which are normally used as a proxy for actual real interest rates, decreased significantly (see Figure 4). However, the magnitude of this reduction (more than 6 pp) seems to be excessive for it to be interpreted as a change in real risk-free interest rates. As mentioned by Blanco and Restoy (2007), there are good reasons to believe that nominal interest rates during this period incorporated a sizable compensation for the uncertainty related with EMU creation and with the Spanish accession. Thus, the observed decline in ex-post real interest rates could be incorporating some decrease in the risk premium as the economy approached EMU. Moreover, as we have suggested, during this period inflation expectations could systematically assign a non-zero probability to an alternative more inflationary scenario where Spain would have failed to fulfil the Maastricht convergence criteria. This would be incorporated into the nominal interest rates, and because that alternative scenario never materialized, also into the ex-post real rates, but it is not part of the real risk-free rate we are interested in.

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1. A detailed outline of developments in Spanish nominal interest rates and exchange rates can be found in Malo de Molina *et ál.* (1998).

3 Modelling Interest rates

3.1 The affine model

As stated by Piazzesi (2005), affine term structure models allow the risk premium to be separated from expectations about future interest rates. These models have been widely used in the financial literature to price fixed-income assets since the seminal works of Vasicek (1977) and Cox, Ingersoll and Ross (1985). Including inflation in the specification of the model, as in ABW (2007) and in Carriero, Favero and Kaminska (2006), will also make it possible to jointly estimate inflation expectations and real interest rates.

An affine model assumes that interest rates can be explained as a linear function of certain factors,

$$y_{t,t+k} = \frac{-1}{k}\left(A_k + B_k' X_t\right) + u_{t,t+k} \qquad u_t \sim N\left(0, \sigma^2 I\right) \tag{1}$$

where $y_{t,t+k}$ is the nominal interest rate in period t with term k, X_t is a vector of factors, A_k and B'_k are coefficients and $u_{t,t+k}$ represents the measurement error. Changes in interest rates across time will be the outcome of changes in the factors, whereas differences in the term structure will be driven by the coefficients A_k and B'_k applied.

There is extensive evidence on the predictability of interest rates [see Diebold and Li (2006)], and this feature is usually included in the affine model by assuming that X_t factors follow a VAR structure [in the same vein as Diebold, Rudebuch and Boragan (2004)],

$$X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t \qquad \varepsilon_t \sim N(0, I) \tag{2}$$

where μ is a vector of the constant drifts in the affine variables X_t, Σ is the variance-covariance matrix of the noise term and Φ is a matrix of the autoregressive coefficients. The VAR model accounts for the observed predictability in the interest rates but allows, at the same time, some degree of uncertainty in the future values of interest rates, represented by the noise vector ε_t that follows a standard i.i.d. Gaussian normal distribution. In order to avoid identification problems we will impose matrix Σ to be diagonal in equation 2, so relationships between factors X_t will be reflected by coefficients of matrix Φ rather than shocks[2].

In order to avoid arbitrage opportunities, the values of parameters A_k and B'_k of equation 1 should be restricted according to equation 3,

$$e^{A_{k+1} + B'_{k+1} X_t} = E_t\left[e^{A_1 + B'_1 X_t} e^{A_k + B'_k X_{t+1}}\right] \tag{3}$$

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2. A more general specification of Σ will imply a VARMA approach that would only affect the short-term forecasts but would create identification problems. Given that our focus is on the long-run forecast of the variables, we rely rather on VAR modelling.

The left hand-side of equation 3 represents the valuation of a zero-coupon bond with maturity in k+1 that under the non arbitrage condition should be equivalent to the expected value one period ahead of the same bond with maturity k discounted with the short-term interest rate. As can be seen in annex 1, solving forward equation 3 implies a recursive form for the A_k and B'_k coefficients.

The consideration of risk-aversion in this framework implies some compensation for the uncertainty about longer maturities[3], in which the random shocks ε_t accumulate. In this respect, it is clear that the higher the variance of random shocks on VAR equation (2) (identified by matrix Σ), the greater the uncertainty about future values of interest rates. So, in order to compensate investors for lending money at longer terms, some risk premium related to Σ should be embedded in the nominal interest rates (see annex 1). Coefficients that translate matrix Σ into the risk premium are called prices of risk (λ_t) and, following the literature, these coefficients are affine to the same factors X_t,

$$\lambda_t = \lambda_0 + \lambda_1 X_t \tag{4}$$

where λ_0 is a vector and λ_1 a matrix of coefficients. If λ_1 is set to be equal to zero, then the risk premium will be constant, while if we leave it unrestricted, we will obtain a time-varying risk premium.

Taking together the no-arbitrage opportunities and the risk-aversion, it is possible, after some algebra (see annex 1), to transform equation 2 into a recursive system of equations represented by equations 5 and 6.

$$A_{k+1} = A_1 + A_k + B'_k \mu - B'_k \Sigma \lambda_0 + \frac{1}{2} B'_k \Sigma \Sigma' B'_k \tag{5}$$

$$B'_{k+1} = B'_1 + B'_k \Phi - B'_k \Sigma \lambda_1 \tag{6}$$

In equations 5 and 6 the coefficients determining the interest rates with maturity in k+1 (A_{k+1} and B'_{k+1}) are the result of the aggregation of the determinants of the short-term interest rate (A_1 and B'_1), the difference between actual short-term interest rate and its forecasted value (reflected by $A_k + B'_k \mu$ and $B'_k \Phi$ terms, respectively) a compensation for risk ($B'_k \Sigma \lambda_0$ and $B'_k \Sigma \lambda_1$ terms, respectively), and a quadratic term consequence of the Jensen Inequality ($\frac{1}{2} B'_k \Sigma \Sigma' B'_k$). As can be see, risk compensation depends on matrix Σ and the price of risk λ_t .

Therefore, the affine model to be estimated consists of equations 1 and 2, with the coefficients of equation 1 being subject to restrictions 5 and 6. Differences among several affine models will be the result of the chosen X_t factors.

3.2 Alternative specifications for factors

We should consider the variables that could determine the term structure of interest rates in order to select the factors in the model. In fact, there is ample evidence in the literature that the information content of the whole term structure could be shortened to a small number of factors. The choice of these factors depends on the purpose of the exercise. One alternative is to introduce an unobservable or latent component. This is the case, for example, in Duffie

3. Bekaert and Hodrick (2001) reviewed the evidence which suggests that expected returns on long bonds are, on average, higher than on short bonds, reflecting the existence of a risk premium and that this premium is time-varying.

and Kan (1996), Duffie and Singleton (1997), Dai and Singleton (2000), Duffie (2002), and Kim and Wright (2005). This specification allows a high degree of flexibility in the model but requires in the estimation to arbitrarily choose some interest rate to be observed without error, and the factors are harder to interpret.

Other approaches, based on certain macroeconomic factors [see Ang *et ál.* (2006), Dewachter and Lyrio (2006), or Dewachter *et ál.* (2006)], obtain a better interpretation of the course of interest rates. For example, in a context in which short-term interest rates are linked to central bank decisions, the whole term structure will be correlated with actual and forecast decisions on monetary policy. In this sense, it seems obvious to introduce inflation dynamics as one of the factors, since this variable is one of the principal elements in monetary policy decisions[4]. Furthermore, the incorporation of inflation rates as one of the components of vector X_t, helps us to obtain real rates decomposition at a later stage of our analysis, as shown by ABW (2006). Moreover, it is common to include other factors related to economic activity, such as GDP or the employment rate. Nevertheless, a model that uses only macroeconomic variables will give rise to poor fitness of the term structure of interest rates.

Another alternative, such as that suggested by Diebold and Li (2006) is to use some factors related to the estimation of the zero-coupon yield curve. This kind of model has been used for forecasting purposes given the good performance of the results. Moreover, as suggested by Carriero, Favero and Kaminska (2006), including macroeconomic variables in this framework could actually improve performance.

In this paper we will use two specifications for modelling nominal interest rates. The first one (that we will refer as endogenous factors model) was extensively used in the literature. We will follow closely the papers of Ang and Piazzesi (2003) and ABW (2007), where they proposed a model that include both macro and latent factors in order to obtain a decomposition of US interest rates similar to that we are looking for. In the latter, the basic structure of the financial affine model with two latent factors is extended into a macro-finance affine model by adding the CPI rate as an observable variable to the VAR model. This is precisely the model we will use, where equation 2 is re-specified as:

$$\begin{pmatrix} q_{t+1} \\ f_{t+1} \\ \pi_{t+1} \end{pmatrix} = \begin{pmatrix} \mu_q \\ \mu_f \\ \mu_\pi \end{pmatrix} + \begin{pmatrix} \Phi_{qq} & 0 & 0 \\ 0 & \Phi_{ff} & 0 \\ \Phi_{q\pi} & \Phi_{f\pi} & \Phi_{\pi\pi} \end{pmatrix} \begin{pmatrix} q_t \\ f_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} \sigma_q & 0 & 0 \\ 0 & \sigma_f & 0 \\ 0 & 0 & \sigma_\pi \end{pmatrix} \begin{pmatrix} \varepsilon_q \\ \varepsilon_f \\ \varepsilon_\pi \end{pmatrix} \tag{7}$$

In equation 7, q_t and f_t are the latent factors, while coefficient matrixes Φ and Σ include some restrictions in order to identify the estimation of the whole model[5]. Latent factors are endogenously estimated along with the rest of the parameters. Nevertheless, the estimation procedure generally imposes some extra problems, and, as was pointed out by Kim and Orphanides (2005), the estimation of this type of model is rather unstable (especially for small samples and transition phases). Moreover, as was quoted by this authors, the restrictions imposed on the parameters are highly arbitrary.

4. This relationship is particularly evident if the central bank has some independence in respect of the political cycle and pursues an inflation target (as was the case of the Spanish central bank from 1994).

5. Latent factors are set to be uncorrelated, whereas inflation depends on latent factors while latent factors are not affected by inflation shocks. Additionally, the price of inflation risk is fixed to zero.

Alternatively to this approach, we propose in this paper a model (we will refer as exogenous factors model) that simplifies the estimation procedure, and does not require any kind of *ad-hoc* restriction (like imposing an inflation price of risk equal to zero). Our approach consists in determining beforehand the factors that characterise the shape of the term structure and use them in the X matrix. In fact, in a model with latent factors, such as that proposed by ABW (2007), the factors obtained are usually identified with some of the characteristics of the yield curve, like the level of interest rates or the slope [Litterman and Scheinkman (1991), and Chen and Scott (1993)]. Along these lines, Diebold and Li (2006) propose, in order to improve the forecasts of the yield curve, an affine model that uses as factors the *level* (L_t), *slope* (S_t) and *curvature* (C_t) parameters from the Nelson and Siegel (1987) term structure specification. These factors can be found in most central bank estimations of the zero-coupon yield curve. This estimation imposes that nominal interest rates can be modelled[6] as in equation 8.

$$y_{t,t+k} = L_t + S_t \frac{1-e^{-k/\tau}}{k/\tau} + C_t\left(\frac{1-e^{-k/\tau}}{k/\tau} - e^{-k/\tau}\right) + u_{t,t+k} \tag{8}$$

In equation 8, τ, L_t, S_t and C_t are the parameters that give us the interest rate in time t with maturity in k periods. Diebold and Li (2006) propose fixing the value of τ in the mean value observed throughout the original sample[7], so interest rates can be considered affine to factors L_t, S_t and C_t. Therefore, values of these factors can be recovered as parameters in an OLS regression. Successive regressions in each period give us the time series of parameters L_t, S_t and C_t that can be considered as factors determining the term structure of interest rates. L_t, is the long-term interest rate (both forward and spot), S_t is the spread (difference between long-term and short-term interest), while C_t is a measure of the term structure curvature. Diebold and Li (2006) showed that all three parameters are needed in order to recover the whole structure of the yield curve[8]. Restricting the parameters to just the level and the spread will entail the loss of the information about short-term changes in interest rates, usually linked to movements in inflation expectations.

Hence, in a similar fashion to ABW (2007), we define a model with four factors, three of them related to the shape of the yield curve, as proposed by Diebold and Li (2006), while the fourth (inflation rate) is included in order to subsequently be able to decompose the nominal interest rate. Additionally, we also impose no-arbitrage conditions following annex 1, a suitable feature that was skipped by Diebold and Li (2006).

This solution goes in line with Kim and Orphanides (2005) proposal of including additional information in order to improve the estimations. Unfortunately, other kind of information apart of the parameters of the yield curve, such as survey forecast [Kim and Orphanides (2005)], or inflation-linked bond prices [D'Amico, *et ál.* (2007)] are not always available.

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6. The basic idea behind the Nelson and Siegel estimation was to obtain zero-coupon data from the observed yields of bonds with different coupons and maturities. There are two assumptions under this methodology: the smoothness of the yield curve and the convergence towards a long-run interest rate.

7. Trying to estimate τ jointly with L_t, S_t and C_t produced non-trivial problems of identification, as shown by Gimeno and Nave (2006).

8. In fact, Diebold and Li (2006) showed that it is possible to forecast properly these factors by VAR equations and obtain a good projection of the term structure.

Although including a fourth factor in the model may not be necessary in order to obtain a good fitting of the interest rate term structure if Nelson and Siegel model (equation 8) is considered[9], adding the inflation rates allows us to taking into account that the yield curve provides information that could be useful in order to forecast inflation, what would have a clear effect on equation 2.

$$X_t = \begin{bmatrix} L_t \\ S_t \\ C_t \\ \pi_t \end{bmatrix} \tag{9}$$

Since all factors in (9) are determined prior to the estimation of the affine model (equations 1-2), no restrictions, apart from (5) and (6) that ensure no-arbitrage and risk-aversion, are required on the model for the dynamics of X_t[10]. Furthermore, there is no need of fixing any interest rate as observed without error, while initial values for the maximum likelihood estimation are easily obtained via OLS regressions (see annex 2).

3.3 Endogenous versus exogenous factor model

In order to estimate the affine model proposed in the previous section, we use monthly spot nominal interest rates for the Spanish Government Yield Curve[11]. The interest rate time series considered in our analysis run from January 1991 to December 1998. The beginning of the sample is determined by the availability of data, while the end is given by entry into the European Monetary Union, and nominal interest rates began to be driven by European determinants. In the case of the endogenous model, we follow ABW (2007), and use a sample of 3-month, 1-year, 3-year and 5-year term interest rates. By contrast, exogenous model estimation requires the use of consecutive interest rates so, in this case, terms included in the sample span from 1-month interest rates to 5-year interest rates, which give us 60 interest rates for each of the months considered.

In the case of the endogenous model, the number of parameters that has to be estimated along with the latent factors that have to be recovered usually gives rise to some problems of identification. In order to solve them, it is usual to impose some restrictions on the set of parameters. For instance, factors q and f are set to be independent while past values of inflation do not affect either of the latent factors. The main drawback of this approach is the reduction in the flexibility of the model, which could cause some difficulties in the accuracy of the estimated term structure.

Moreover, the estimation procedure used by ABW (2007) requires assuming that a number of yields (equivalent to the number of latent variables) should be observed without error, in order to recover the unobserved latent factors. We have found that, at least in the Spanish nominal convergence process, the results are extremely sensitive to the interest rates

9. Nevertheless, some previous research [Núñez (1996)] found some evidence of the usefulness of an additional term in the yield curve in the form of the Svensson (1994) model that nest the Nelson and Siegel specification.

10. In fact, at least in the case of Spain, we have found a strong correlation between past values of inflation and actual values of C_t, the interpretation of which could be that unexpected movements in Inflation have an impact on the short-term expectations of the interest rates, determined by C_t.

11. These data have been computed from the Yield Curve estimations of the Statistical Department of the Banco de España that fits a Nelson and Siegel (1987) model from 1991 to 1996 and a Svensson (1994) model from 1996 [Núñez (1996)]. As shown by Gimeno and Nave (2006), the overlap of the two methodologies in the estimation for the yield curve did not cause any relevant shift.

selected as observed without error. We choose the 3-month – 5-year interest rates in order to take into account both extremes of the yield curve.

Finally, estimation results are extremely sensitive to the initial values of the parameters that have to be chosen arbitrarily. In order to avoid, to some extent, this problem, we have implemented a Genetic Algorithm [see Gimeno and Nave (2006)] excluding combinations of parameters that create meaningless estimations (such as consistently negative real interest rates or risk premia and extremely high values of inflation rates). This methodology, which is heavily demanding in computational terms, performs a better optimization by allowing the comparison of several sets of initial parameters.

By contrast, in the exogenous factor model the step-by-step estimation of equations 1 and 2 (see annex 2) is a good approach to the initial values in the joint maximum likelihood estimation for the whole model. Sensitivity to the initial values disappears, and Genetic Algorithms are not required either, reducing the computational time.

Table 1 shows the mean absolute error in both inflation rate forecasts and the interest rates fitting for both models (with endogenous and exogenous factors). Their goodness of fit differs depending on the equation considered. The VAR model (equation 2), irrespectively of the specification considered, is able to capture the information contained in the term structure about the future course of interest rates, since these models outperforms univariate ARIMA. Nevertheless, no clear preference could be established between them, although longer horizon forecasts seem to be slightly better in the case of the exogenous model (see Figure 5).

In the case of the term structure of the interest rates (equation 1), results clearly favour the model with exogenously determined factors. As can be seen in Table 1 and in Figure 6, the estimation procedure of the endogenously-determined factor model produces a perfect fitting in the terms considered without error (3 months and 5 years in the output presented), but entails a significant drawback, namely the lack of fitting in the other terms. This can be observed in Figure 6 for two specific periods, in which the model with exogenous factors captures all the term structure, while the unobserved component allows for significant deviations along the yield curve. Figure 7 also shows the lack of fitting of the endogenous factor model throughout the sample.

One of the reasons behind these fitting problems is that in this model, apart from the inflation rate we only have two factors to capture the term structure while, as was already highlighted by Litterman and Scheinkman (1991), three unrestricted factors are needed in order to accurately reproduce the yield curve. Diebold and Li (2006) suggested taking the coefficients estimated by Nelson and Siegel (1987) structure (Level, Slope and Curvature) as these three factors. In fact, we can see that the latent factors in the endogenous model follow a similar pattern to those of the Nelson and Siegel factors (see Figure 8). Latent factor q matches perfectly the evolution of the curvature parameter, while latent factor f seems to be an addition of the level and slope parameters. Moreover to estimate the endogenous model was necessary to impose some restrictions in the VAR model (equation 7) that would imply that curvature linked factor (q) does not affect long term interest rates (approximated by the evolution of factor f), while neither of them are influenced by inflation.

In addition to the lack of fitting in the endogenous factor model, the results with this framework will vary significantly depending on the terms considered as observed without error, prompting a significant robustness problem. Although this has minor consequences for inflation expectations, the disturbances in the yield curve estimation would be reflected in a worse determination of both real risk-free rates and risk premia.

Therefore, although the endogenous factor model has been previously used in the literature on the decomposition of nominal interest rates, we have found that, at least in a nominal convergence scenario, the results seems to be extremely sensitive to the initial parameters, the goodness of fit on the yield curve was not satisfactory and there seems to be a robustness problem with the terms considered as observed without error[12]. In this sense, we will focus on the decomposition from the exogenous factor model, since it overcomes most of the problems cited and produces a satisfactory fit for the whole structure of the yield curve.

12. The magnitude of this problem may be related to the structural break implicit in the Spanish economy. In this respect, we have obtained qualitatively the same results under a Markov switching regime model that allows for a different regime in the constant and variance of equation 2.

4 Nominal interest rate decomposition

4.1 Methodology

Once an affine model represented by equations 1, 2 and 4 has been estimated, it is possible to decompose *k*-period nominal interest rates ($y_{t,t+k}$) into real risk-free rates[13] ($Er_{t,t+k}$), inflation expectations ($E_t[\pi_{t,t+k}]$) and risk premia[14] (denoted by $\gamma_{t,t+k}$), according to equation 10.

$$y_{t,t+k} = Er_{t,t+k} + E_t[\pi_{t,t+k}] + \gamma_{t,t+k} \tag{10}$$

Therefore, real risk-free rates ($Er_{t,t+k}$) could be obtained by subtracting inflation expectations and risk premia from estimated nominal interest rates.

Firstly, inflation expectations are obtained from VAR equation (2). In fact, since vector X_t includes Inflation (π_t), expectations on this variable can be recovered from projections of the dynamics on the affine factors in the VAR equation (2).

$$E_t[X_{t+h}] = \left(1 + \Phi + \Phi^2 + \cdots + \Phi^{h-1}\right)\mu + \Phi^h X_t \tag{11}$$

Long-term nominal interest rate decomposition in equation 10 requires the average expected inflation for the aggregate period between *t* and *t+k* ($E_t[\pi_{t,t+k}]$), which could be recovered by integrating the forecasting values of inflation of equation 11 for consecutive periods between *t* and *t+k* ($E_t[\pi_{t+h,t+h+1}]$).

Secondly, risk premia are estimated as the difference between nominal interest rates and their risk-free counterpart. As stated in section 3.1, the risk premium appears as a consequence of investors' risk-aversion (see annex 1). This factor only reflects the existence of uncertainty in the future value of the affine factors driven by perturbations ε_t of the VAR equation. If investors were indifferent to risk, no risk premium would be needed to compensate them for the uncertainty of holding assets with longer maturities instead of shorter ones. In such a framework, the recursive formulas of annex 1.2 would be applied instead of those in annex 1.1. This is equivalent to assuming that the price of risk is zero ($\lambda_t = 0$) during the period considered [Ang and Piazzesi (2003)]. Consequently, we can now define risk-free rates $\tilde{y}_{t,t+k}$ as the interest rates obtained from affine equation 12.

$$\tilde{y}_{t,t+k} = \frac{-1}{k}\left(\tilde{A}_k + \tilde{B}_k' X_t\right) \tag{12}$$

where parameters $\tilde{A}_k$ and $\tilde{B}_k'$ are equivalent to those of equation 1, but assuming null prices of risk (annex 1.2). Differences between estimated nominal rates and estimated nominal

13. Real risk-free rates for longer terms may be seemed as the mean of the expected behavior of the short term real rate. In this sense, the real risk-free rate would be free of any term premia related to the uncertainty over its future evolution.

14. This risk premia is the result of the uncertainty on the future evolution of the affine factors. Therefore it includes both the inflation risk premia and the real term premia. Trying to separate both types of risk premia would imply an identification problem since inflation risk and monetary policy risk as closely related as recognize by ABW (2007).

risk-neutral rates will be the consequence of the introduction of risk-aversion and can be considered as risk compensation (risk premium).

$$\gamma_{t,t+k} = y_{t,t+k} - \tilde{y}_{t,t+k} \tag{13}$$

The risk premium ($\gamma_{t,t+k}$), defined by equation 13, will increase with the term considered as implied by the construction of the affine model (see annex 1), and will be time-varying (governed by the price of risk of equation 4).

4.2 Results

From equation 10 in the previous section, it is easy to see that once nominal interest rates have been stripped of both inflation expectations and risk premia, real risk-free rates will be the remaining value[15]. This decomposition is represented in Figure 9, where the course of the expected inflation in the next five years, and the risk premium associated with the uncertainty about term structure changes in this period are removed from five-year nominal interest rates.

As can be seen, most of the decline in nominal interest rates came from the reduction in inflation expectations and a further decline in the risk premia, while real interest rates fell by less than 3 pp during the sample period. The magnitude of this reduction in real risk-free interest rates is consistent with the findings of Blanco and Restoy (2007), in the sense that the evolution of macro and financial variables in the Spanish economy during this period did not support a reduction in the cost of capital similar to that suggested by ex-post real interest rates drop of 6.7 pp (Figure 10). Furthermore, results for real interest rates obtained with this model are in line with those obtained for other countries and methodologies [Laubach and Williams (2003), Manrique and Marqués (2004), Cuaresma, *et ál.* (2004) among others].

Most of the volatility in ex-post real interest rates is in fact captured by the estimated risk premium (see Figure 11 for the estimated five-year risk premium). As can be seen, this magnitude had ample volatility in the period considered and, in fact, was responsible for a sizable share of nominal interest rate movements. In fact, the major increases in the risk premium occurred in periods of weaknesses of the Spanish currency and could be linked to market uncertainty about future interest rates and inflation in the event of devaluation. From the mid-nineties this uncertainty vanished due to the fact that it became increasingly clear that Spain would enter the euro area[16] at the end of 1998.

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15. ABW (2006) propose an alternative approach to the decomposition of real interest rates, based on the restrictions imposed on their VAR model (equation 7), implying that inflation rates are affected by past values of the latent factors, but not the other way round. That allows them to assign all the uncertainty about latent factors to their own past and, therefore, they are not linked to inflation, letting real rates retain some part of the risk premium (term premium) not related to inflation expectations. Nevertheless, this decomposition is only possible when factors in the term structure are latent, so they can be arbitrarily transformed in order to comply with equation 7. In the case of exogenous factors, this is no longer possible, and the structure of the factor dynamics must be defined in a more general way (as in equation 2). It is thus possible for any factor to affect the future behaviour of the others, making it impossible to assign a specific random variable such as the uncertainty over inflation rates. In fact, in the Spanish case, the curvature parameter is closely related to inflation expectations, with both curvature and inflation responsible for most of the movements in the risk premium. Any restriction on the exogenous model that may allow to estimate ABW inflation risk premium maybe seen as arbitrary since we would need information on inflation-linked bond data in order to test the independence of both premiums. Unluckily this type of data does not exist, and the uncertainty over future monetary policy make extremely hard to disentangle the risk premium linked to inflation or monetary policy related sort term interest rates.

16. In this sense, risk-premium estimation can be considered as a market indicator of the credibility of a country entering a major monetary union.

Indeed, if we compare (Figure 12) the estimated risk premium with the spread between the Spanish bond yield and the benchmark German bund, we can see that not only does the magnitude of the risk premium match the spread but also that the evolution is strongly similar. This result is in line with the intuition that in a nominal convergence process towards a monetary union the changes in the spread should be closely linked to the probabilities assigned to the EMU entry. In fact, during this period some practitioners used the spread to extract the probabilities of entry into EMU [see Bates (1999)]. It is easy to see that at the end of the decade both variables tend to diverge, signalling a switch in the link between the term structure and the Spanish CPI to the European index, once it was clear Spain was going to enter EMU[17].

Finally, in the case of the CPI, projections obtained for different periods with equation 11 are presented in Figure 13 (blue dotted lines) and compared against the observed CPI (unbroken red line). As can be seen, on average the paths of inflation projections are similar to those of observed inflation. Although the differences between projected and actual data increase with the prediction horizon, these differences are lower than those obtained via a univariate ARIMA (Table 1).

If we compare observed inflation and the CPI five-year forecast, following equation 11, as well as the evolution of the expectations derived from the Consensus Forecast, as shown in Figure 14, we can find periods with both inflation expectations were persistently higher than the final actual values (i.e. 1994-1995), producing a divergence between ex-ante and ex-post interest rates. These differences were in line with some concern about Spain's entry into EMU that would have affected inflation expectations as well as the associated risk premium, and would have been reflected in the evolution of the term structure. However, in the final year of the sample, where there was a general consensus about the entrance in the EMU, both types of inflation forecasts (survey and estimates) produced outcomes lower than the finally observed. The reason could be related with some expectations of convergence between the Spanish and the euro-area inflation rate that was not finally achieved after entering to the euro-zone. In fact, in the following years a persistent inflation gap was observed between Spain and the euro-zone. This evolution, which could be not anticipated by most of the analyst, could be due to the fact that the process of wage and price formation continued to be referred to national references instead to the euro area benchmark [see Alberola and Marques (2001)].

In fact, the information content in the fixed-income markets about the CPI via the term structure found with this model highlights the credibility given by the market to the inflation targets published by the Banco de España during that period in its inflation report. In Figure 15 the inflation targets set by the Banco de España (represented by the red square and the dotted red lines) are compared with the market projections implied by the term structure and obtained with equation 11, showing a close relationship between them. Same result is observed when inflation expectations are compared with the consensus forecast (Figure 16). In fact, this feature, reinforce the evidence of the term structure information contents on inflation expectations.

17. Further worthwhile research could be to apply the model presented here to the European CPI and interest rates in order to check the effect of short-term interest rates on inflation expectations and term structure forecasting potential on the CPI.

4.3 *Are results robust to regime switching?*

Some studies, such as ABW (2007), include a regime switch in their model affecting both the level of affine variables (the drift in equation 2) and the risk premium (via the Σ matrix, or the price of risk equation). In this sense, ABW (2007) consider two regimes in order to take into account differences in monetary policy strategy and different cyclical positions. The same approach could be of interest in the Spanish case as a possible means of capturing the nominal convergence process that ended upon EMU entry.

In order to implement a Markov regime switch in our exogenous model we consider two different states (s_t) which, in theory, could be linked to the convergence or not towards EMU. These two regimes would be associated with two states for the drifts $\mu(s_t)$ in equation 2 allowing for different inflation expectations depending on the state, as well as two different vectors of constants $\lambda_0(s_t)$ for the price of risk in equation 4 to account for different uncertainty valuation.

$$X_t = \mu(s_t) + \Phi X_{t-1} + \Sigma\varepsilon_t \tag{14}$$

$$\lambda_t = \lambda_0(s_t) + \lambda_1 X_t \tag{15}$$

Two regimes, one of them of low inflation, are therefore identified. As can be seen in Figure 17, the regime of high inflation appears to be most likely at the beginning of the nineties and around 1995. Both episodes were characterized by currency turbulence and serious market doubts about Spanish fulfilment of the Maastricht criteria and EMU entry.

The results of the (exogenous[18]) regime switching model are highlighted in Figure 18, which evidences the nominal interest rate decomposition, similar to that presented in the previous section for the model with just one regime. Slight differences between both models are reflected in a swap between inflation expectations and risk premia. The appearance of a second state of higher inflation increases both the overall expectations as well as the compensation given in nominal interest rates. But this increase implies that CPI expectations reflect the possibility of a higher inflation regime, reducing the upside risk and increasing the downside one, so this rise is balanced by a reduction of the same intensity in the risk premium.

In this respect, as shown by Figure 19, real risk-free rates seem to be not affected by the introduction of a regime switch, giving both estimations a very similar path for real risk-free rates. In fact, the intensity of the fall in interest rates is independent of the use or not of the Markov regime switch (table 2) and the transition in real risk-free rates produced by the convergence process of the nineties can be fully explained by our model without any need of adding regime switching.

18. The regime switching, when applied to the endogenous model, produce qualitatively similar results to the one regime model.

5 Conclusions

In this paper we discuss the decomposition of nominal interest rates for Spain under an affine model methodology that only imposes risk-aversion and no-arbitrage opportunities along the yield curve. We propose the use of exogenously determined factors based on the zero coupon yield curve estimation and compare the results with standard endogenously determined factors. Our results suggest that exogenously determined factors related to the estimation of the zero coupon curve exhibit the best properties in term of robustness, fitness and economic interpretation of the results.

Interest rate decomposition obtained for Spain points to a real risk-free interest rate reduction of less than 3 pp during the nineties, a figure substantially lower than the ex-post real interest rate reduction and the decline previously found in the literature with other methodologies. This magnitude seems to be close to that observed in other countries and reflects the fact that most of the reduction in nominal interest rates during that decade can be attributed to a decline in risk premia and the convergence of inflation expectations towards European values.

Therefore, this kind of exogenously determined factors, which has previously been used in the literature related to forecasting of the term structure, seems to be more appropriate for obtaining the decomposition of interest rates in a scenario of nominal convergence similar to Spain's entry into EMU. In this respect, this methodology could be of special interest for monitoring the current process experienced by other Monetary Union accession countries.

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ANNEX 1: RECURSIVE EXPRESSION OF TERM STRUCTURE PARAMETERS

1.1. Risk-aversion and no-arbitrage conditions

A no-arbitrage condition guarantees the existence of a risk-neutral measure (noted as *Q*) that allows interest rates to be expressed in terms of future term structure outcomes,

$$e^{A_{k+1}+B'_{k+1}X_t} = E_t^Q\left[e^{A_1+B'_1X_t}e^{A_k+B'_kX_{t+1}}\right] \tag{A.1}$$

Risk-neutral measures *(Q)* are usually converted into natural probabilities using the Radon-Nikodym derivative, as in Ang and Piazzesi (2003), denoted by ξ_t ,

$$e^{A_{k+1}+B'_{k+1}X_t} = E_t\left[e^{A_1+B'_1X_t}e^{A_k+B'_kX_{t+1}}\frac{\xi_{t+1}}{\xi_t}\right] \tag{A.2}$$

Usually, ξ_t in equation A.2 is assumed to follow a log-normal process,

$$\xi_{t+1} = \xi_t e^{\left(-\frac{1}{2}\lambda_t{}'\lambda_t - \lambda_t{}'\varepsilon_{t+1}\right)} \tag{A.3}$$

where λ_t is a time-varying vector that incorporates the concept of risk-aversion into the valuation framework. The first part of the exponent $(\lambda_t{}'\lambda_t)$ is the Jensen Convexity component that ensures that τ , while in the second, λ_t multiplies the perturbation vector ε_{t+1} , scaling the uncertainty in the random variables. This second term is responsible for the introduction of the risk premium in the valuation framework, whereby λ_t can be considered as a price of risk. Time-variant risk premia [Bekaert and Hodrick (2001)] will be the consequence of changes in this price of risk that is modelled assuming it to be also affine to the same factors X_t,

$$\lambda_t = \lambda_0 + \lambda_1 X_t \tag{A.4}$$

Finally, substituting A.3 in A.2, we arrive at a modified no-arbitrage condition that now takes into account investors' risk aversion.

$$e^{A_{k+1}+B'_{k+1}X_t} = E_t\left[e^{A_1+B'_1X_t}e^{A_k+B'_kX_{t+1}}e^{-\frac{1}{2}\lambda'_t\lambda_t-\lambda'_t\varepsilon_{t+1}}\right] \tag{A.5}$$

Only X_{t+1} and ε_{t+1} of expression (A.5) are not already known in period *t*, while the other terms in the exponents can be extracted from the expectations operator,

$$e^{A_{k+1}+B'_{k+1}X_t} = e^{A_1+A_k+B'_1X_t-\frac{1}{2}\lambda'_t\lambda_t}E_t\left[e^{B'_kX_{t+1}-\lambda'_t\varepsilon_{t+1}}\right] \tag{A.6}$$

Nevertheless, vector X_{t+1} can be forecast using VAR equation (2),

$$e^{A_{k+1}+B'_{k+1}X_t} = e^{A_1+A_k+B'_1X_t+B'_k\mu+B'_k\Phi X_t-\frac{1}{2}\lambda'_t\lambda_t} E_t\left[e^{(B'_k\Sigma-\lambda'_t)\varepsilon_{t+1}}\right] \qquad (A.7)$$

The exponent left in the expectations operator of expression (A.7) is solved taking into account the Jensen inequality.

$$e^{A_{k+1}+B'_{k+1}X_t} = e^{A_1+A_k+B'_k\mu+\frac{1}{2}B'_k\Sigma\Sigma'B'_k+(B'_1+B'_k\Phi)X_t+B'_k\Sigma\lambda_t} \qquad (A.8)$$

Finally, replacing the price of risk λ_t in A.8 by its definition (equation A.4), we arrive at expression (A.9),

$$e^{A_{k+1}+B'_{k+1}X_t} = e^{A_1+A_k+B'_k\mu+\frac{1}{2}B'_k\Sigma\Sigma'B'_k+(B'_1+B'_k\Phi)X_t+B'_k\Sigma(\lambda_0+\lambda_1X_t)} \qquad (A.9)$$

This last expression allows us to recover the recursive expression of coefficients A_{k+1} and B'_{k+1} in the affine representation as a function of the shorter terms,

$$A_{k+1} = A_1 + A_k + B'_k\mu - B'_k\Sigma\lambda_0 + \frac{1}{2}B'_k\Sigma\Sigma'B'_k \qquad (A.10)$$

$$B'_{k+1} = B'_1 + B'_k\Phi - B'_k\Sigma\lambda_1 \qquad (A.11)$$

1.2. Valuation without Risk compensation

The risk neutrality valuation framework used in (A.1) allowed us to incorporate the risk premium into the term structure. In order to recover risk-free rates we should consider a framework where agents are not concern about risk, so expectations derived from the non-arbitrage condition are evaluated under a natural measure,

$$e^{\tilde{A}_{k+1}+\tilde{B}'_{k+1}X_t} = E_t\left[e^{\tilde{A}_1+\tilde{B}'_1X_t}e^{\tilde{A}_k+\tilde{B}'_kX_{t+1}}\right] \qquad (B.1)$$

where $\tilde{A}_j$ and $\tilde{B}'_j$ are the coefficients of equation 1, that meet no-arbitrage conditions. Using the same reasoning of annex 1.1, replacing X_{t+1} by its forecast and applying Jensen inequality to solve the expectations operator we arrive at expression (B.2),

$$e^{\tilde{A}_{k+1}+\tilde{B}'_{k+1}X_t} = e^{\tilde{A}_1+\tilde{A}_k+\tilde{B}'_k\mu+\frac{1}{2}\tilde{B}'_k\Sigma\Sigma'\tilde{B}_k+(\tilde{B}'_1+\tilde{B}'_k\Phi)X_t} \qquad (B.2)$$

As can be seen, expression (B.2) is equivalent to (A.8), the only difference being that once you avoid risk-aversion the term $B'_k\Sigma\lambda_t$ is no longer needed. In fact, this was the term that added a risk premium for each extra period of investment. A risk-neutral individual would have a null price of risk, with both expressions becoming equivalent. Under this assumption, the term structure recursive expression would be now,

$$\widetilde{A}_{k+1} = \widetilde{A}_1 + \widetilde{A}_k + \widetilde{B}'_k \mu + \frac{1}{2} \widetilde{B}'_k \Sigma \Sigma' \widetilde{B}_k \tag{B.3}$$

$$\widetilde{B}'_{k+1} = \widetilde{B}'_1 + \widetilde{B}'_k \Phi \tag{B.4}$$

ANNEX 2: EXOGENOUS MODEL ESTIMATION

Prior to the estimation of the affine model we have to determine the factors related to the term structure. Following Diebold and Li (2006), we use the Nelson and Siegel (1987) formula of the term structure.

$$y_{t,t+k} = L_t + S_t \frac{1-e^{-k/\tau}}{k/\tau} + C_t \left(\frac{1-e^{-k/\tau}}{k/\tau} - e^{-k/\tau} \right) \qquad \text{(C.1)}$$

Diebold and Li (2006) fixed the value of τ to be the mean throughout the sample. Once τ is constant, equation C.1 can be estimated by OLS for each period, regressing interest rates for different terms *(k)* against matrix Z_t.

$$Z_k = \begin{bmatrix} 1 & \frac{1-e^{-k/\tau}}{k/\tau} & \frac{1-e^{-k/\tau}}{k/\tau} - e^{-k/\tau} \end{bmatrix} \qquad \text{(C.2)}$$

Once L_t, S_t and C_t are estimated as the parameters of these regressions for each period, they can be used as factors for the affine model. As vector X_t is completely determined, we no longer need to fix any interest rate as observed without error. In fact, it is now quite easy to recover initial values of the parameters via OLS estimations in three steps.

Since vector X_t is exogenously determined, we can estimate the VAR equation via OLS, which allows initial values to be obtained of μ, Φ and Σ

$$X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t \qquad \varepsilon_t \sim N(0, I) \qquad \text{(C.3)}$$

We can also use vector X_t to regress it against nominal interest rates for different terms using the term structure equation, in order to estimate consecutive values of A_k and B'_k,

$$-k \cdot y_{t,t+k} = A_k + B'_k X_t + u_{t,t+k} \qquad u_t \sim N(0, \sigma^2 I) \qquad \text{(C.4)}$$

Finally, in order to incorporating non-arbitrage condition and risk aversion we go further than Diebold and Li (2006) and use $\hat{A}_k$ and $\hat{B}'_k$ estimations to regress them against shorter terms values, rearranging equations 5 and 6. Once we have tentative values from C.3 and C.4, then equations 5 and 6 become,

$$\left(\hat{A}_{k+1} - \hat{A}_k\right) - \hat{A}_1 - \hat{B}'_k \mu - \frac{1}{2} \hat{B}'_k \Sigma \Sigma' \hat{B}_k = -\hat{B}'_k \Sigma \lambda_0 \qquad \text{(C.5)}$$

$$\left(\hat{B}'_{k+1} - \hat{B}'_k \Phi\right) - \hat{B}'_1 = -\hat{B}'_k \Sigma \lambda_1 \qquad \text{(C.6)}$$

Equations C.5 and C.6 are linear with respect to λ_0 and λ_1, and therefore, these parameters can also be estimated by OLS.

Once we have estimated separately C.3, C.4, C.5 and C.6 equations, we have tentative initial values of the affine model that allow for a fast computation of the joint maximum likelihood estimation of the affine model given by,

$$y_{t,t+k} = \frac{-1}{k}\left(A_k + B_k' X_t\right) + u_{t,t+k} \qquad u_t \sim N\left(0, \sigma^2 I\right)$$

$$X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t \qquad \varepsilon_t \sim N(0, I)$$

subject to (C.7)

$$A_{k+1} = A_1 + A_k + B_k' \mu - B_k' \Sigma \lambda_0 + \frac{1}{2} B_k' \Sigma \Sigma' B_k'$$

$$B_{k+1}' = B_1' + B_k' \Phi - B_k' \Sigma \lambda_1$$

Figure 1: Nominal interest rates in the nineties

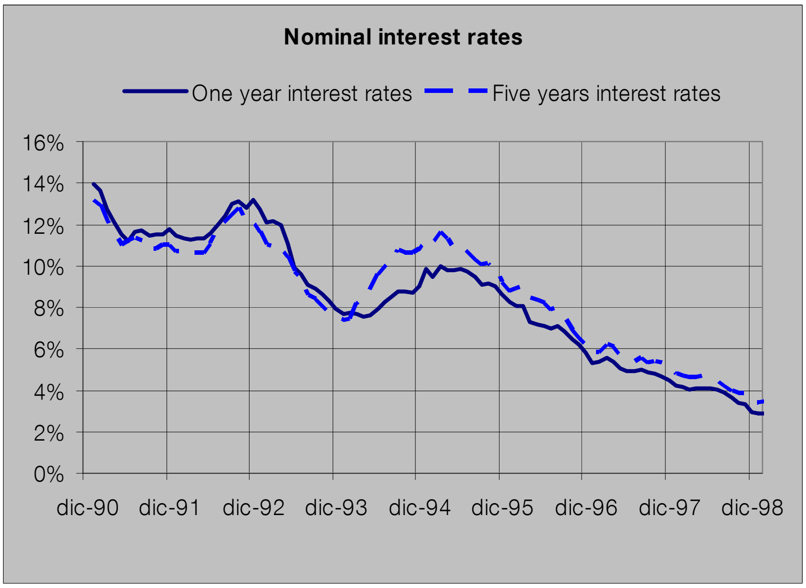


Figure 2: Spanish and European Consumer Price Indices

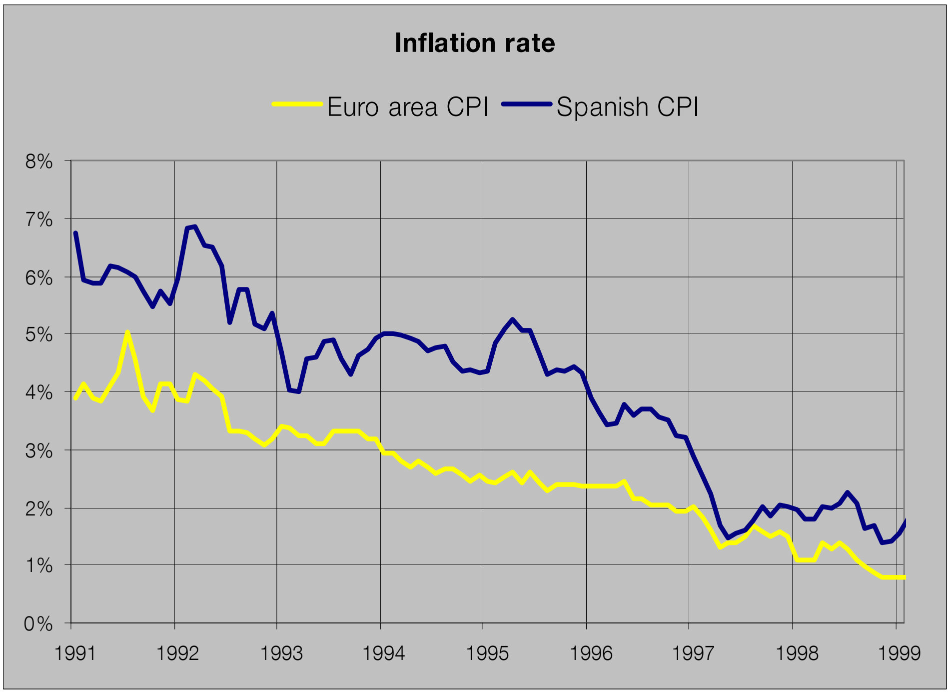
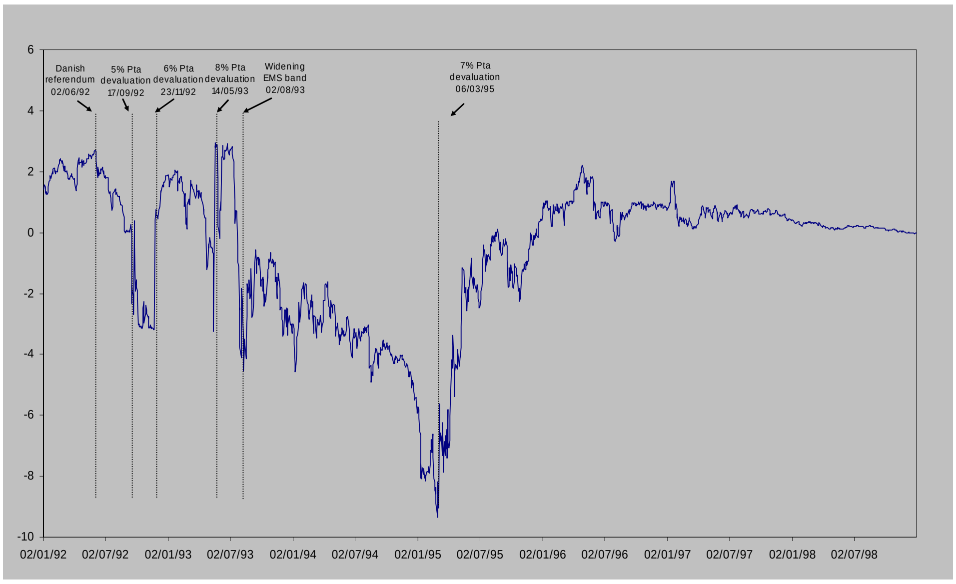


Figure 3: Spanish currency against the Deutsche mark



Source: Banco de España

Note: Appreciation (+), depreciation (-) of nominal exchange rate of the Peseta against Deutsche mark central parity

Figure 4: 5-year nominal and ex-post real interest rates

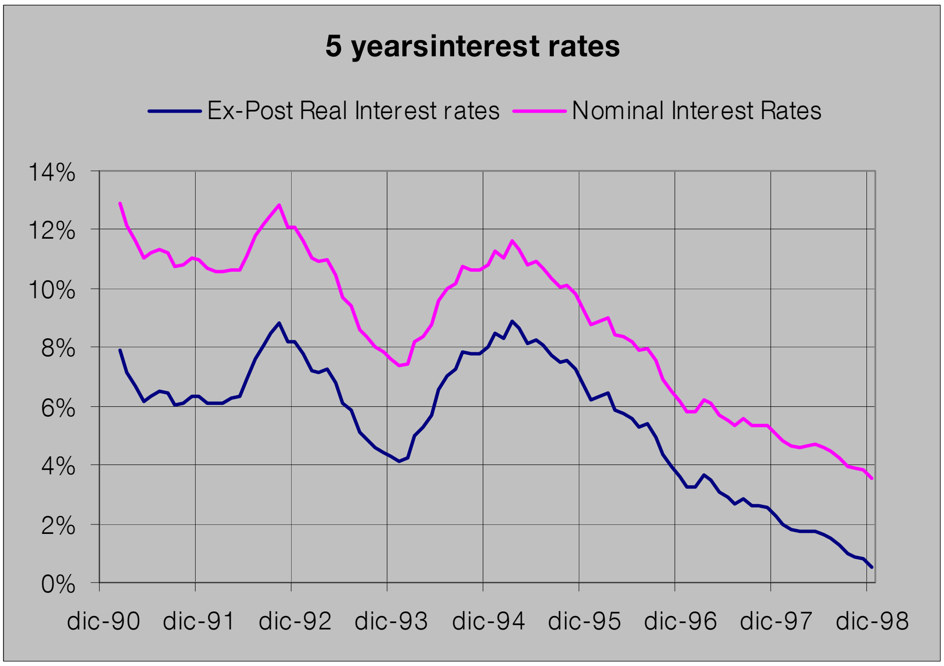


Table 1: Mean absolute error in the estimated models

inflation rate (mean absolute error)				
	1 month	**3 months**	**1 year**	**5 years**
Endogenous model	0.016%	0.158%	0.378%	0.978%
Exogenous model	0.050%	0.161%	0.371%	0.832%
Univariate ARIMA	0.075%	0.277%	0.462%	1.131%

Nominal interest rates (mean absolut error)				
	3 months	**1 year**	**3 years**	**5 years**
Endogenous model	0.000%	0.169%	0.132%	0.000%
Exogenous model	0.019%	0.023%	0.014%	0.021%

Figure 5: Inflation rate forecasts

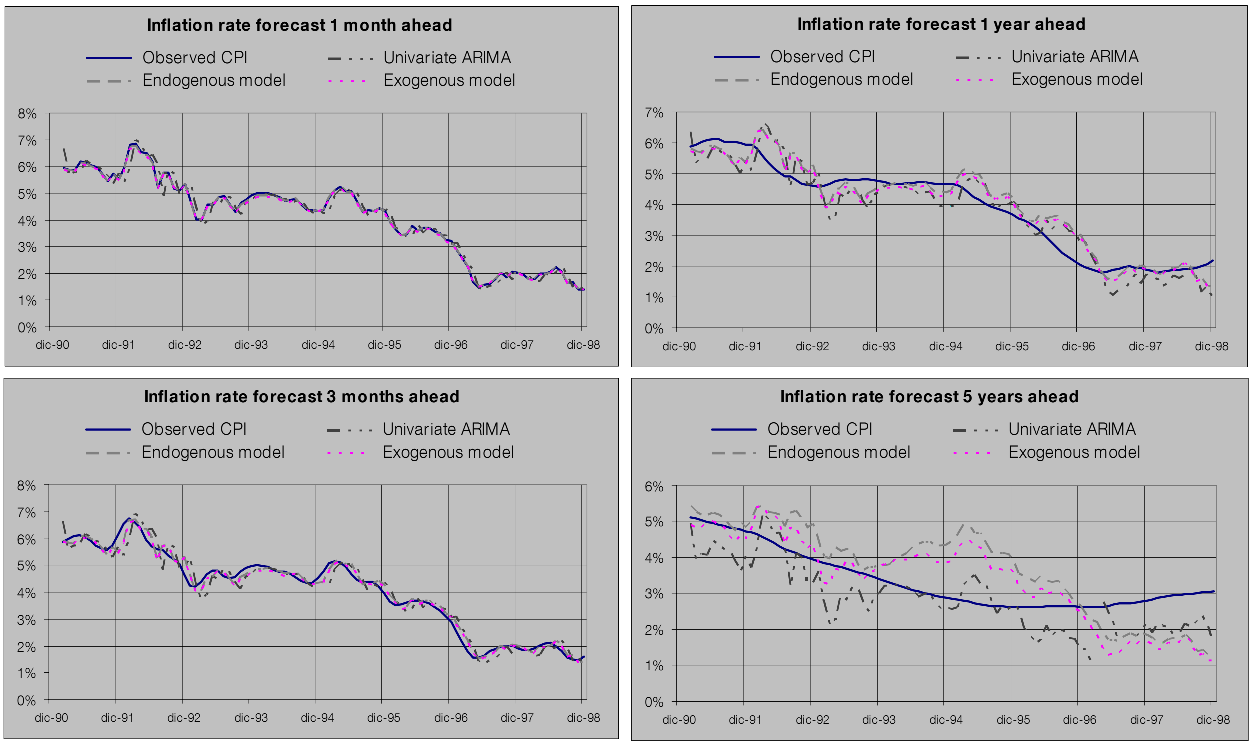


Figure 6: Nominal term structure

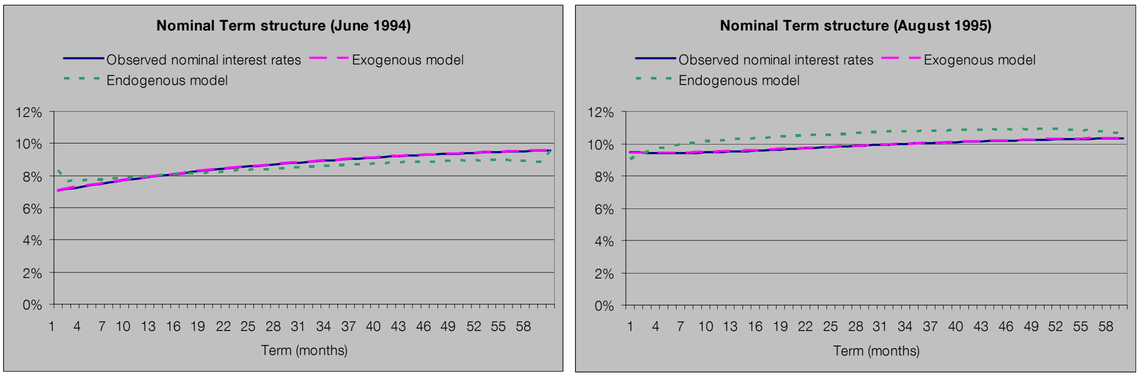


Figure 7: Estimation of goodness of fit (endogenous and exogenous models).

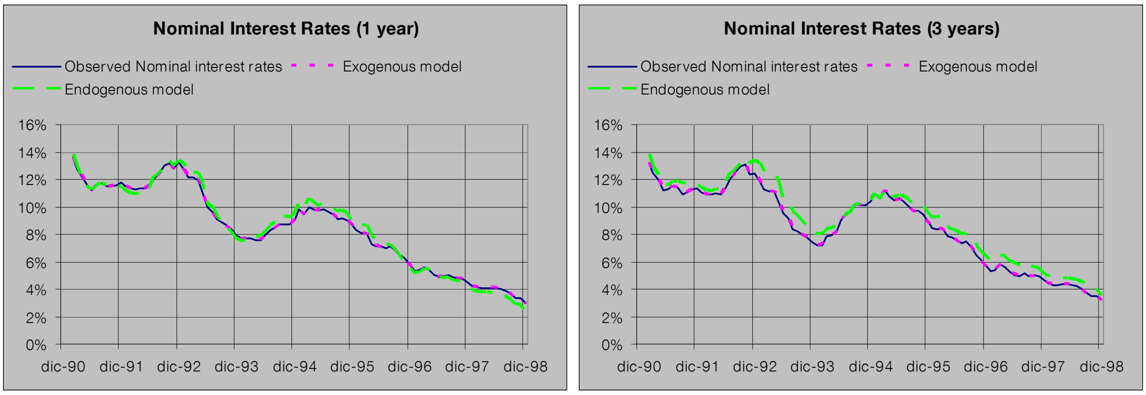


Figure 8: Dynamics of the affine factors

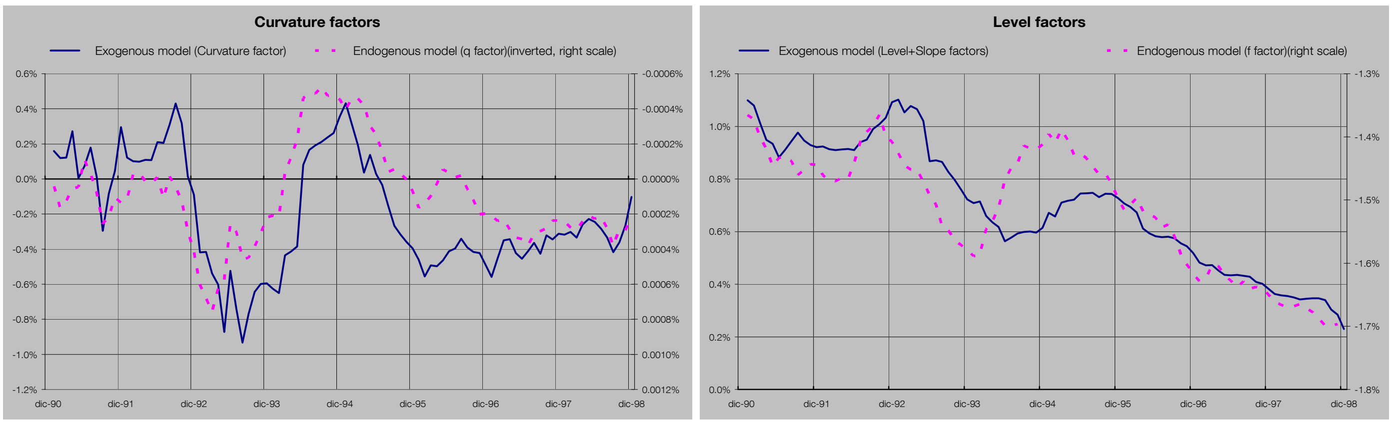


Figure 9: decomposition of 5-year interest rates.

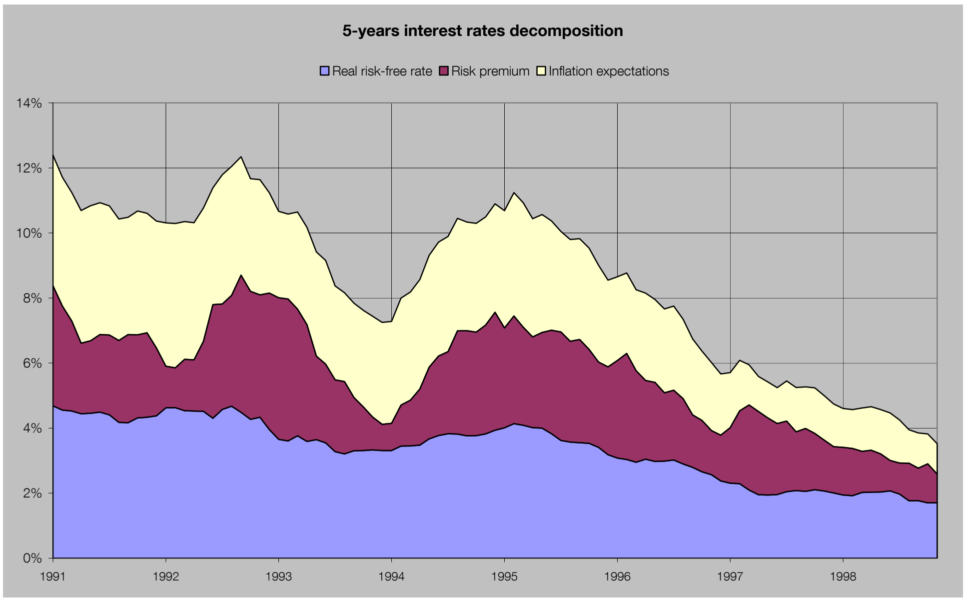


Figure 10: ex-post real interest rates vs. ex-ante real risk-free rates

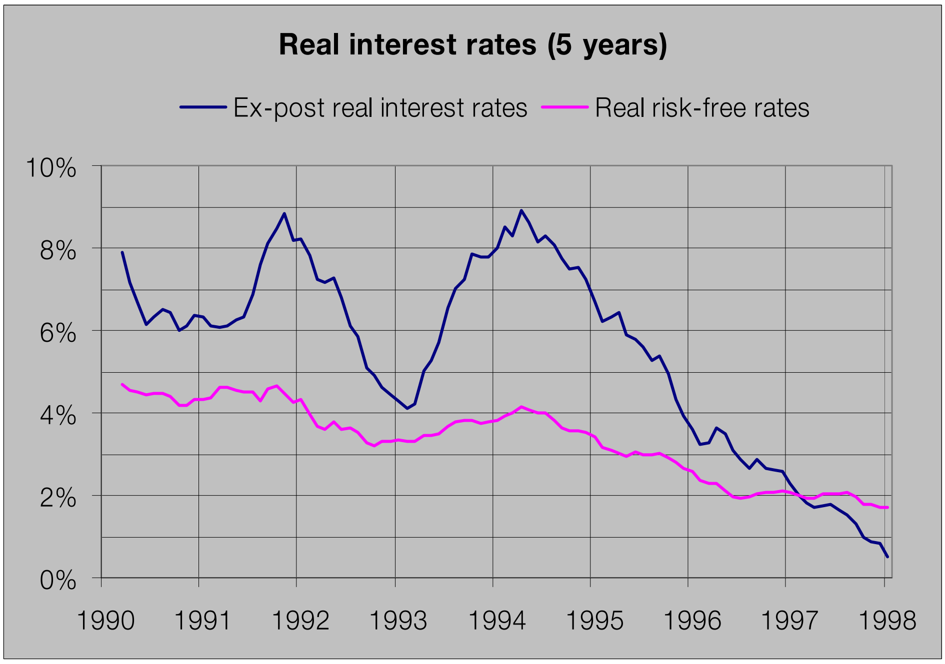


Figure 11: Risk premium for 5-year term

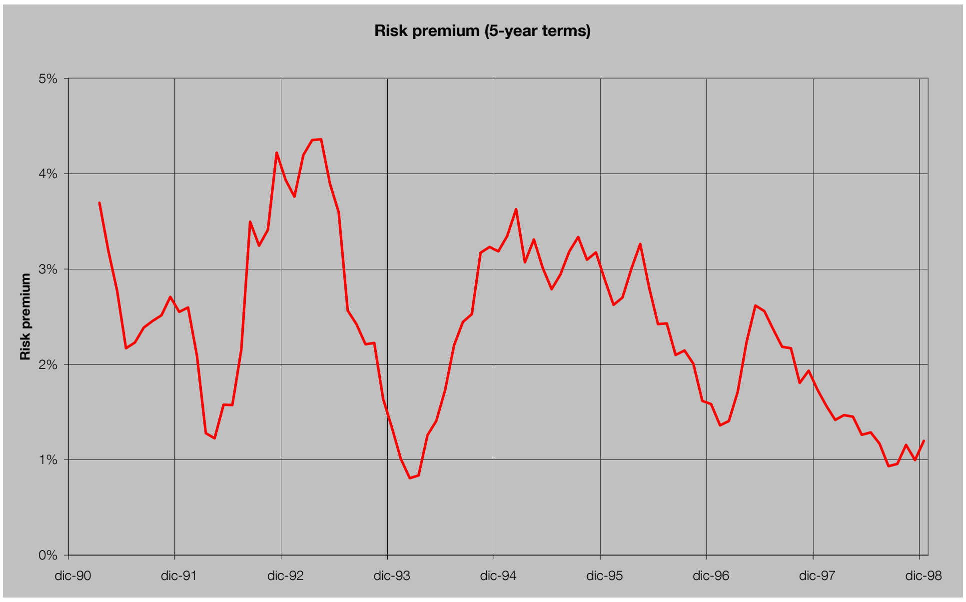


Figure 12: Comparison of five-year risk premium and German-Spanish spread.

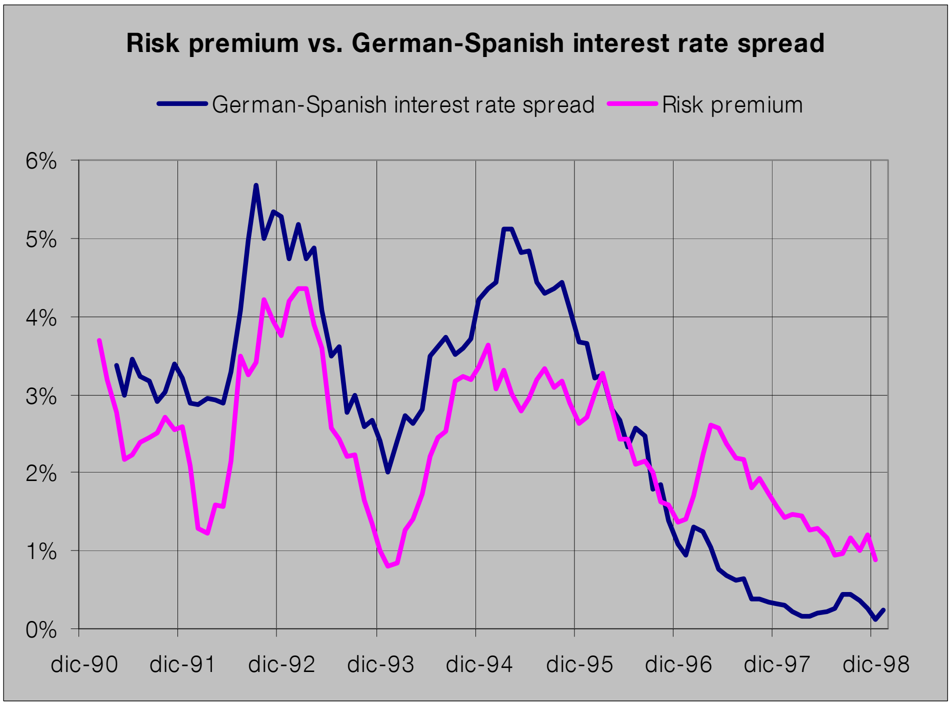


Figure 13: Inflation Projections

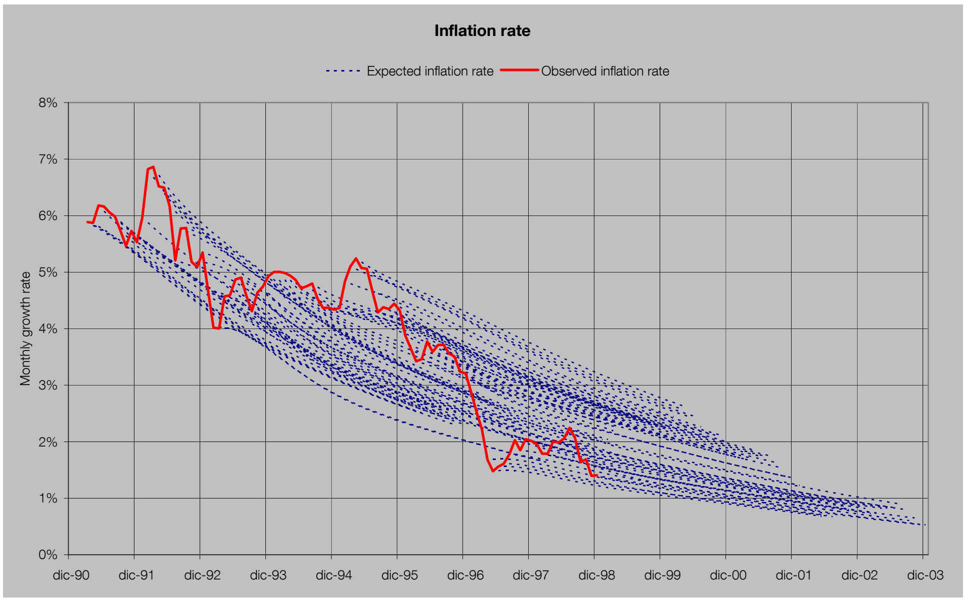


Figure 14: Inflation expectations vs. observed average inflation

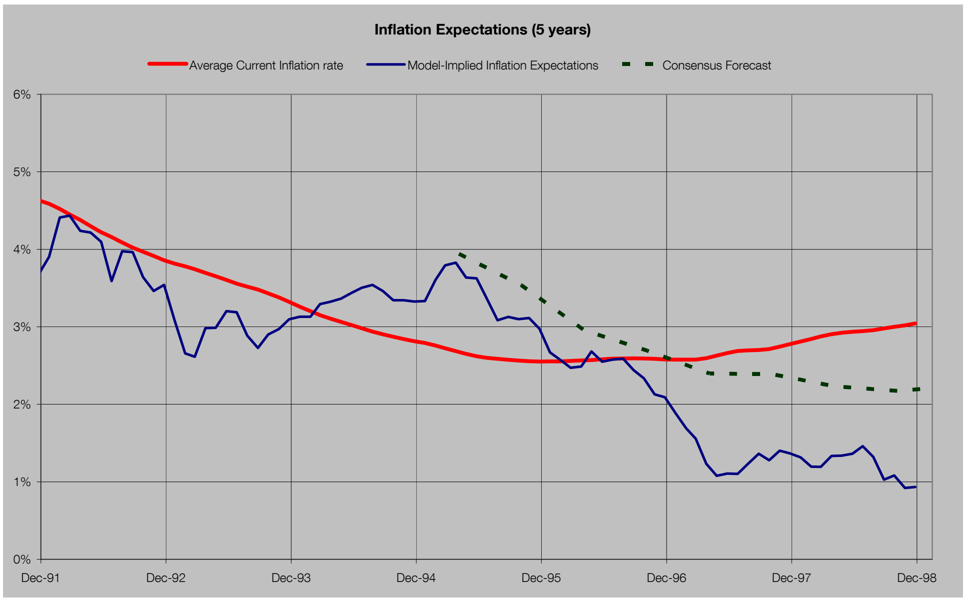
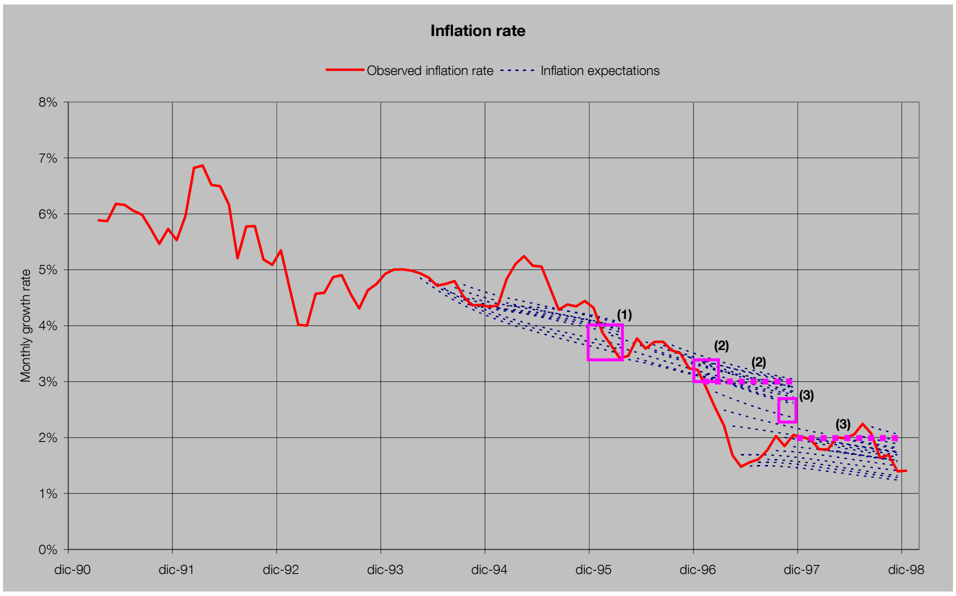


Figure 15: Banco de España inflation targets and model projections



(1) CPI target: between 3.5% and 4% at the beginning of 1996 (p. 13, Economic Bulletin, December 94, Banco de España)

(2) CPI target: close to 3% at the beginning of 1997 and below it during the year (p. 12, Economic Bulletin, December 95, Banco de España)

(3) CPI target: around 2.5% at the end of 1997 and close to 2% in 1998 (p. 12, Economic Bulletin, December 96, Banco de España)

Figure 16: Consensus Forecast on inflation rates and model projections

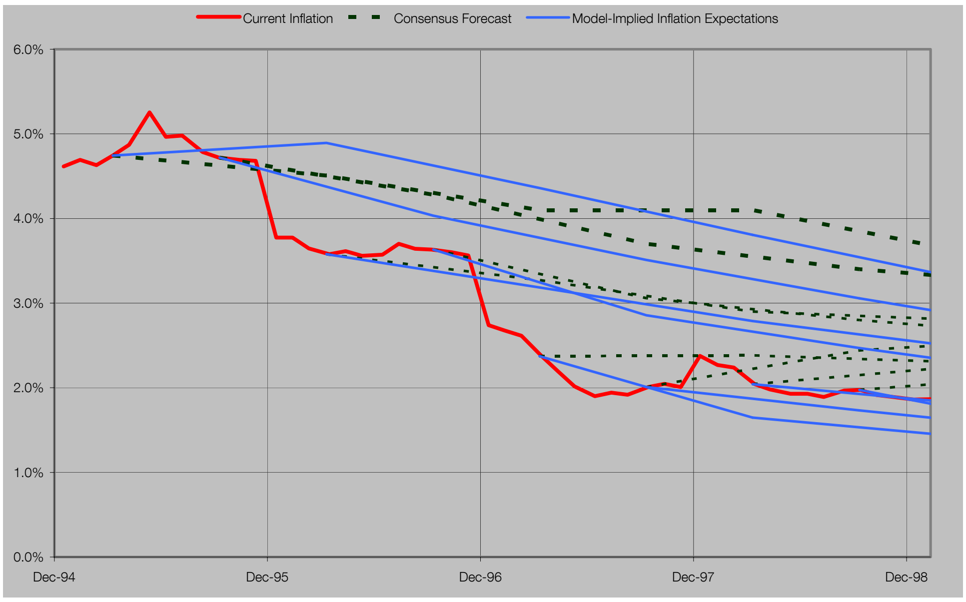


Figure 17: Probability of high-inflation regime.

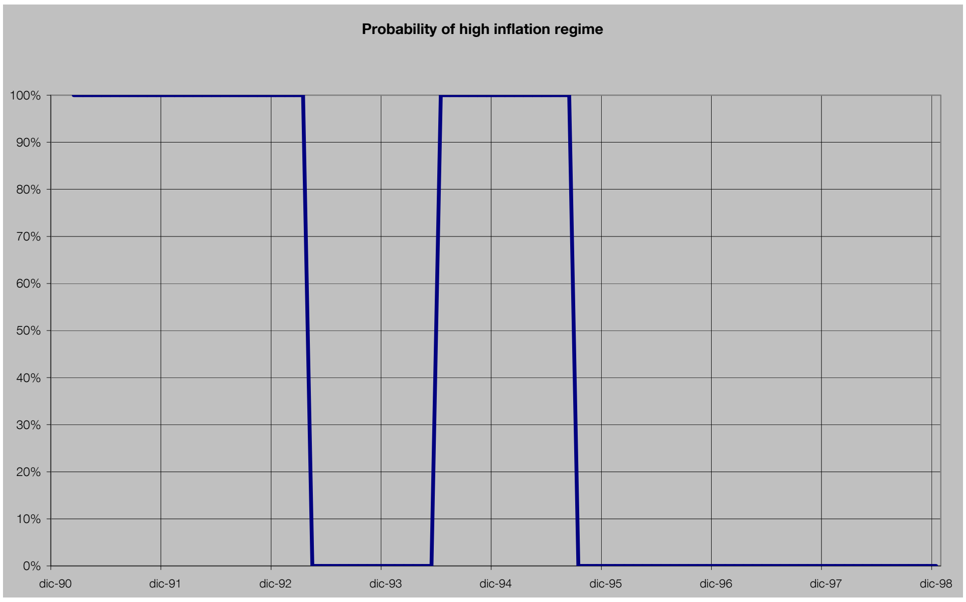


Figure 18: decomposition of 5-year interest rates. Regime switching model

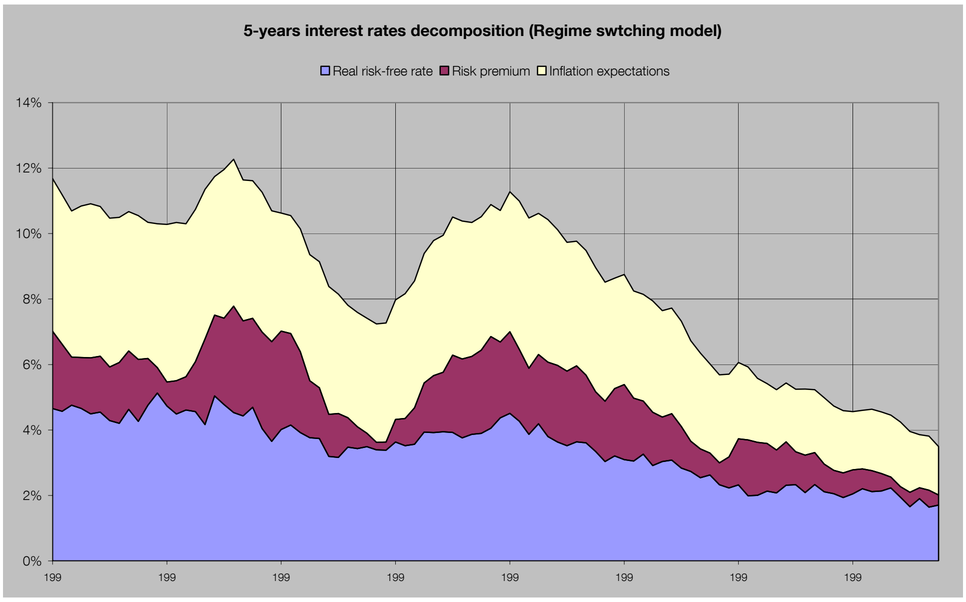


Figure 19: Real risk-free interest rates (5-year term). 1 regime vs. 2 regimes

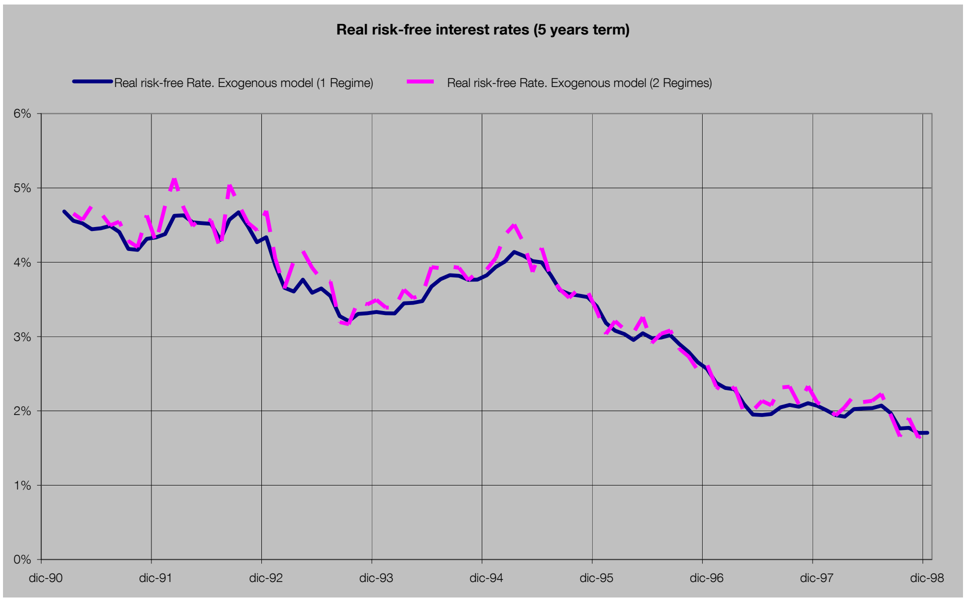


Table 2: Comparison of model with and without Markov regime switching

	Mean interest rate	Std. Deviation	Difference (Mar 91 - Dec 98)
Real risk-free rate	3.32%	0.92%	2.85%
Real risk-ree rate (regime swtching	3.42%	0.97%	2.89%

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