DYNAMIC STABILITY IN REPEATED GAMES

2006

Sergio Puente

Documentos de Trabajo
N.º 0618

BANCO DE ESPAÑA

DYNAMIC STABILITY IN REPEATED GAMES

DYNAMIC STABILITY IN REPEATED GAMES (*) (**)

Sergio Puente (***)

BANCO DE ESPAÑA

(*) This work is part of the Ph.D. dissertation of the author, supervised by Manuel Santos, and developed at Universidad Carlos III.

(**) This paper has benefited from comments by Diego Moreno, Marco Celentani, M.ª Ángeles de Frutos and other seminar participants at Universidad Carlos III. I would especially like to thank Manuel Santos for his very helpful suggestions. The opinions and analyses herein are the responsibility of the author and, therefore, do not necessarily coincide with those of the Banco de España or the Eurosystem.

(***) Banco de España, Alcalá 48, 28014 Madrid, Spain. Tel: +34 91 338 5705. E-mail: sergio.puente@bde.es. The author acknowledges financial support from the Spanish MCYT.

Documentos de Trabajo. N.º 0618
2006

The Working Paper Series seeks to disseminate original research in economics and finance. All papers have been anonymously refereed. By publishing these papers, the Banco de España aims to contribute to economic analysis and, in particular, to knowledge of the Spanish economy and its international environment.

The opinions and analyses in the Working Paper Series are the responsibility of the authors and, therefore, do not necessarily coincide with those of the Banco de España or the Eurosystem.

The Banco de España disseminates its main reports and most of its publications via the INTERNET at the following website: http://www.bde.es.

ISSN: 0213-2710 (print)
ISSN: 1579-8666 (on line)
Depósito legal: M.35815-2006
Imprenta del Banco de España

Abstract

A concept of dynamic stability in infinitely repeated games with discounting is presented. For this purpose, one modification of the available theory is needed: we need to relax the assumption that the game starts in a given period. Under this new framework, we propose stable strategies such that a folk theorem with an additional stability requirement still holds. Under these strategies, convergence to the long run outcome is achieved in a finite number of periods, no matter what actions or deviations have been played in the past. Hence, we suggest a way in which a player can build up his reputation after a deviation.

Keywords: Repeated Games, Stability, Stable Strategies

JEL classification codes: C70, C72

1 Introduction

The dynamic features of repeated games have been analyzed in a number of studies. If the same players play the same game repeatedly, then they can choose their actions as a function of the history of what have been played. This implies that dynamic programming tools can be used in this context. This approach has been revealed as a very useful tool. One example of its applications is the work by Abreu, Pearce and Stachetti (1990).

A dynamic consequence of repeated games is that there can be equilibria that do not imply to play a static Nash equilibrium (NE from now on) each period. Players may choose in one period strategies which are different from the static best response in order to induce, through the history, some kind of play in the future. The natural question arises: What kind of actions (different from the static NE) can be supported in a repeated game? The 'folk' theorems provide an answer for subgame perfect equilibria (SPE).[1] Apart from some technical conditions, all action profiles with an individually rational static payoff can be supported as a SPE of an infinitely repeated game.

This result sometimes relies on the so-called trigger strategies. However, the use of this type of strategies brings additional problems. For example, if for whatever reason the history is slightly perturbed, the continuation path induced by trigger strategies changes dramatically, switching from a path of cooperation to a path of (usually) Nash reversion. Obviously, this cannot satisfy any concept of dynamic stability, as we will see later. This lack of dynamic stability is also present in other, more general, punishment schemes used in the literature. Optimal punishment schemes, like those described in Abreu (1998), are an example.

Against this background, we propose a concept of dynamic stability for repeated games. The first observation is that we need to modify the usual theory in the following way: We need a game with no beginning. In other words, it is often assumed that a repeated game has a first period in which the game has started. As a consequence, the history of the game is always of a finite and increasing dimension. It will be seen that this fact brings some problems for the question of dynamic stability. Hence, we propose a slightly different framework: the game has no beginning, i.e. history is always of the same (infinite) dimension.

This modification allows us to define stationary strategies, i.e. strategies that can be represented by only one function relating the past history with

[1] See Fudenberg and Tirole(1991), section 5.1.2.

the current action. Then, we will be able to define our concept of dynamic stability in repeated games.

Another step in the analysis is to see if the requirement of dynamic stability modifies in a substantial way the set of possible outcomes in a repeated game. Our approach here will be to show that the folk theorem also holds when we require dynamic stability. The proof of the theorem is constructive, since it yields the class of (stationary) strategies that can support a certain outcome. These strategies have the property that, after a deviation, the profile played each period converges again to the long run profile in a finite number of periods.

The following additional remark concerning the necessity of games with no beginning may be useful. As already pointed out, the strategies at any period are functions of the history, under a dynamic programming approach. One can interpret this history as the object that determines the reputation of both players, because it summarizes all cooperations and defections conducted by the players. But, if the game starts in a given period, nothing can be said about the initial reputation by looking at the history, since in the first period the history is the empty set. Indeed, it is implicitly assumed that play begins with full reputation, because strategies often recommend players to cooperate in the first period.[2] The available theory is silent about how this initial reputation is determined. Note that it is indiferent to make an implicit assumption, or to say that we assume explicitly (and exogenously) a certain degree of reputation at the beginning of the game: we are in both cases analyzing the convergence for only one initial reputation. By contrast, we make in this paper the complete exercise: we take all possible histories in a given period for a game that has no starting period, and we check if the strategies induce convergence for all these possible histories. In this way, we are studying convergence for all initial reputations, whereas the traditional analysis focuses only on one initial reputation.

Of course, this new approach is appropriate to study games with no beginning. However, it should be clear that our motivation is not (only) to study this class of games. In a game starting today, players need to fix some idea about the likelihood that their opponents will cooperate or not. Clearly, we might do this by assuming some prior distribution over a set of possible types of the opponent. The other possibility is to do what we're doing here. Take all possible histories of the game as if it would have no beginning. If the equilibrium converges to certain profile for all of them, then it is clear that this profile will be the long run outcome. This is precisely our definition of

[2] More precisely, the strategies that support a certain outcome usually start by choosing in the first period the profile to be supported.

dynamic stability. With this approach, we avoid the use of stochastic tools. Moreover, we obtain the additional result that following any deviation (or mistake), the equilibrium will converge to the long run outcome again, and in a finite number of periods. In this sense, our approach not only provides a more precise[3] answer to the question: "why are we cooperating?", but also answers the question: "if we are not in a cooperative situation (maybe because someone has deviated in the past), how can we reach it again?".

Finally, note that the main contributions of this paper are the proposed definition of stability, the proposed stable strategies, the management of initial conditions in a repeated game, and how players can build up again their reputation after a deviation. The folk theorem is presented only as a complementary result. Of course, the proof is very similar to standard proofs of folk theorems in repeated games; it is just a matter of extending it to an environment without starting period and with reversion to the cooperative outcome.

The rest of the paper is organized as follows. Section 2 presents a review of the existing theory of repeated games, and proposes the main definitions in which the rest of the work is based, including the definition of a stable SPE. Section 3 presents a theorem useful to characterize outcomes supportable as a stable SPE. Section 4 analizes the memory requirements of the proposed stable strategies and relate them to the theory on bounded recall. Finally, concluding thoughts and possible extensions are presented in section 5.

1.1 Related Literature

Abreu (1988) argues that we can have a penalty greater than Nash reversion (the penalty usually assumed in trigger strategies). In this paper we also use a penalty greater than Nash reversion. In fact, we use a penalty worse than (or equal to) the minmax, but only for a finite number of periods. On the other hand, the penalty in Abreu (1988) is for an infinite number of periods (if there are no deviations in the punishment phase), and it is not worse than the minmax.

There are some recent studies concerned with the size of the penalty. Evans and Thomas (2001) use the concept of perturbed game to study repeated games and cooperation. They criticize the previous work by Aumann and Sorin (1989) and Anderlini and Sabourian (1995), showing that the result "perturbation implies efficiency" can be achieved only if there are 'draconian' penalties in the support of the perturbation, i.e. penalties designed to min-

[3] More precise in the sense that, under our framework, cooperation does not depend on initial conditions in the long term.

max a player almost all the time. We have the same concern. Even Nash reversion is a very big penalty to be fully credible. By contrast, our stability requirement ensures that penalty is at least limited in time.

The strategy presented below is related to at least two other strategies proposed in the previous literature. Green and Porter (1984) present a strategy that has a finite number of punishment periods. However, since their model is of imperfect information, the punishment phase is defined as a function of one observable variable (the price) that imperfectly reflects the actions of other players. Therefore, even when players do not deviate, the observable variable sometimes induces the punishment phase. This occurs when it falls below a trigger level, generating fluctuations in the behavior of players. Hence, this environment is not adequate to study stability issues; the motivation is totally different. A similar strategy can be found in Piccione (2002).

Fudenberg and Tirole (1991) use in their theorem 5.4 a strategy with three phases. One is the cooperative phase. If someone deviates, the game goes to a finite punishment phase. However, at the end of this second phase the play doesn't return to the cooperative phase. Instead, it goes to a third phase with payoffs between the other two. Therefore, the strategies do not induce a convergent equilibrium path. Moreover, it is imposed exogenously that play begins in the cooperative phase, which amounts to assume that players begin the game with reputation.

Finally, Kalai and Stanford (1988) and related papers study the possible implementation of strategies by finite automata. At first glance, one can interpret this approach as a concern about stationarity of strategies. However, this is only one requirement of our definition of stability (the other is convergence). Moreover, we think that their automata do not cover stationarity issues properly, because they assume that automata start the game in some initial "state of mind" and, as already discussed, this initial assumption is not innocuous. The same can be said about Kalai, Samet and Stanford (1988), with the addition that they show that reactive equilibria can exist only by chance, i.e. under a particular combination of parameters of measure zero.

2 Preliminaries

In this section we start with some basic concepts to be used. Then, we modify the concept of a repeated game with an initial period or node to define a repeated game without initial period. Finally, we define the concept of stable subgame perfect equilibrium.

2.1 Stage Game

Let's define the following symmetric game, which will be called the stage game. There are two players. Both of them must play an action simultaneously from the same set of possible actions. This set can be discrete or continuous, finite or infinite. Let's call this set S. The particular choices of the two players will be denoted by $s_1, s_2 \in S$. Let's define the instantaneous payoff function for player i, u_i as follows:

$$u_i : S \times S \to \mathbb{R}$$

Where the first argument in both payoff functions is the action of player 1 and the second the action of player 2. Each player is trying to maximize his own payoff.

Assumption A1: The functions u_i are bounded, and the (static) best response correspondences $BR_i(s_{-i}) = \arg\max_{s_i} u_i(s_1, s_2)$ are non-empty valued.

2.2 Repeated Game

We construct a new game by the infinite iteration of the stage game defined before, in which past actions are observable. Now the objective function is the sum of the instantaneous payoff functions, discounted by the parameter δ, which is the same for both players. It is usual in the literature to assume that the repeated game starts in a given period and continues up to infinity. Later we modify this concept using a repeated game that goes from minus infinity up to infinity. To facilitate comparison between the two concepts we present here some definitions about a repeated game with starting node.

Let's call period 0 the first stage game, period 1 the second stage game, and so on.

It is useful to define the object 'history' in period t as the actions played by both players in previous periods, and period t itself: $h^t = \{h_k^t\}_{k=0}^t ; h_k^t = \left\{h_k^{1,t}, h_k^{2,t}\right\}$. Denote the set of all possible histories at t by H^t.

Now we can define a strategy for player i in this repeated game as follows:

Definition 1 *A strategy σ_i for player i in the repeated game with starting node is a sequence of functions, one for each $t \in \{1, 2, \ldots\}$ of the form $s_i^t : H^{t-1} \to S$, and an action $s_i^0 \in S$ for period 0.*

Note that the set $H^{t-1} = (S \times S)^t$ varies with t.

In each subgame h^{t-1}, the strategies σ_1 and σ_2 determine a sequence of pairs of actions, or continuation path, composed of the actions that players would play if they followed the strategies σ_1 and σ_2 after history h^{t-1}. Let $P\left(h^{t-1},\sigma_1,\sigma_2\right)=\left\{P_k\left(h^{t-1},\sigma_1,\sigma_2\right)\right\}_{k=0}^{\infty}$ be the combination of a given history and its associated continuation path given σ_1 and σ_2. It can be defined in the following recursive manner:

$$P_k\left(h^{t-1},\sigma_1,\sigma_2\right)=\begin{cases} h_k^{t-1} & \text{if } k\leq t-1 \\ \left\{s_1^k(\{P_m\left(h^{t-1},\sigma_1,\sigma_2\right)\}_{m=0}^{k-1}),s_2^k(\{P_m\left(h^{t-1},\sigma_1,\sigma_2\right)\}_{m=0}^{k-1})\right\} & \text{if } k\geq t \end{cases}$$

A subgame perfect equilibrium (SPE) can be defined in the following way:

Definition 2 *A pair of strategies $\{\sigma_1,\sigma_2\}=\{s_1^t,s_2^t\}_{t=0}^{\infty}$ constitute a SPE of the discounted repeated game with starting node if, for all periods $t\in\{0,1,\ldots\}$, and for all histories $h^{t-1}\in H^{t-1}$ in each period, the following two conditions are satisfied:*

$$\begin{aligned}\sum_{k=t}^{\infty}\delta^{k-t}u_1\left(P_k\left(h^{t-1},\sigma_1,\sigma_2\right)\right) &\geq \sum_{k=t}^{\infty}\delta^{k-t}u_1\left(P_k\left(h^{t-1},\tilde{\sigma}_1,\sigma_2\right)\right),\forall\tilde{\sigma}_1\neq\sigma_1\\ \sum_{k=t}^{\infty}\delta^{k-t}u_2\left(P_k\left(h^{t-1},\sigma_1,\sigma_2\right)\right) &\geq \sum_{k=t}^{\infty}\delta^{k-t}u_2\left(P_k\left(h^{t-1},\sigma_1,\tilde{\sigma}_2\right)\right),\forall\tilde{\sigma}_2\neq\sigma_2\end{aligned}$$

Now we can focus on the repeated game without starting node. In this case history in period t is $h^t=\{h_k^t\}_{k=-\infty}^{t}$.[4] Note that here we need to include all periods up to minus infinity, since there is no starting node. Looking at the definition of h^t, it is clear that the set of all possible histories is now the same for all periods: $h^t\in H$, where $H=(S\times S)^{\infty}$.

The definition of a strategy and of an equilibrium are very similar to the previous ones:

Definition 3 *A strategy σ_i for player i in the repeated game without starting node is a sequence of functions, one for each $t\in\{\ldots,-1,0,1,\ldots\}$ of the form $s_i^t:H\rightarrow S$.*

Definition 4 *A pair of strategies $\{\sigma_1,\sigma_2\}=\{s_1^t,s_2^t\}_{t=-\infty}^{\infty}$ constitute a SPE of the discounted repeated game without starting node if, for all periods $t\in$*

[4] The number of periods in $P\left(h^{t-1},\sigma_1,\sigma_2\right)$ also changes, so now the continuation path is $P\left(h^{t-1},\sigma_1,\sigma_2\right)=\left\{P_k\left(h^{t-1},\sigma_1,\sigma_2\right)\right\}_{k=-\infty}^{\infty}$ with the same recursive definition exposed above.

$\{\ldots,-1,0,1,\ldots\}$, *and for all histories* $h^{t-1}\in H$ *in each period, the following two conditions are satisfied:*

$$\begin{aligned}\sum_{k=t}^{\infty}\delta^{k-t}u_1\left(P_k\left(h^{t-1},\sigma_1,\sigma_2\right)\right) &\geq \sum_{k=t}^{\infty}\delta^{k-t}u_1\left(P_k\left(h^{t-1},\tilde{\sigma}_1,\sigma_2\right)\right),\forall\tilde{\sigma}_1\neq\sigma_1\\ \sum_{k=t}^{\infty}\delta^{k-t}u_2\left(P_k\left(h^{t-1},\sigma_1,\sigma_2\right)\right) &\geq \sum_{k=t}^{\infty}\delta^{k-t}u_2\left(P_k\left(h^{t-1},\sigma_1,\tilde{\sigma}_2\right)\right),\forall\tilde{\sigma}_2\neq\sigma_2\end{aligned}$$

Note that in both cases we have an infinite countable set of conditions to be checked with an infinite countable set of functions that constitute the strategies.

In the traditional repeated games with initial node we have a very well defined utility function for the whole game (which is usually assumed to be additively separable over time). When we extend the model to cover a repeated game with no beginning, this is no longer true. But this is not a problem for definitions 3 and 4, as long as we keep the assumption of separability over time of utility functions. With this assumption, the payoff function of each player at each period is defined conditionally, and therefore the optimization problems are well defined, and deviations can be analysed exactly in the same way as in a game with initial period.

2.3 Stationary Strategies and Stable Equilibria

Once we have defined an equilibrium for both cases (with and without initial period), the next step is to define stationary strategies.

Definition 5 *A strategy* σ_i *for the repeated game is stationary if there is a function* s_i *such that* $s_i^t=s_i,\forall t$.

It is interesting to note that the concept of stationary strategies is not applicable to a repeated game with initial period,[5] because the functions s_i^t are defined over different sets (H^t) for different periods. Contrariwise, when there is no initial node, all functions s_i^t are defined over the same set (H), so it is possible to have stationary strategies. Here we have one advantage of our approach: it is possible to study stationary strategies, and this fact can simplify the analysis. Note that with stationary strategies, we do not need to check deviations in all periods in all histories. Instead, it is enough to fix one period t, and analyze deviations in all histories or subgames h^t,

[5]This is true unless the strategy is independent of history for all periods. If this is the case, we can only have as equilibrium one NE of the stage game every period.

but only for this period, because the only difference between two subgames is the previous history, not the period itself.

Now consider the continuation path $P(h^t, \sigma_1, \sigma_2) = \{P_k(h^t, \sigma_1, \sigma_2)\}_{k=-\infty}^{\infty}$ determined by the strategies σ_1 and σ_2 given the history h^t, defined above. The definition of a dynamically stable SPE is the following:

Definition 6 *A subgame perfect equilibrium for a repeated game with no starting node satisfies dynamic stability if the following two conditions are satisfied:*

(a) Stationarity: The strategies σ_1 and σ_2 are stationary.

(b) Convergence: $\lim_{k\to\infty} P_k(h^t, \sigma_1, \sigma_2) = \lim_{k\to\infty} P_k\left(\tilde{h}^t, \sigma_1, \sigma_2\right), \forall h^t, \tilde{h}^t \in H$

Condition (b) in the previous definition states that in a stable SPE, the continuation path converges to the same profile for all histories. Consequently, the long run payoff for player i, $u_i\left(\lim_{k\to\infty} P_k(h^t, \sigma_1, \sigma_2)\right)$, does not depend on the history.

Since the definitions of stationarity and stability cannot be applied to repeated games with initial period, the next section focuses only on repeated games with no starting period.

Now, we can investigate the existence of a stable SPE. For this purpose we cannot rely on the traditional arguments of existence of NE in each subgame, because we are focusing on pure strategy equilibria. Existence, however, can be obtained if there is at least one NE in pure strategies of the stage game. This is true because the strategies "play the static NE in each period, regardless of history" are always a stable SPE. In order to obtain a more general existence result, we should have a definition of dynamic stability also for mixed strategies, which is beyond the scope of this paper.[6]

3 Outcomes supportable as a stable subgame perfect equilibrium

In this section we present a folk theorem with an additional stability requirement. We start by defining the minmax value in pure strategies for player 1, $v = \min_{s_2}\max_{s_1} u_1(s_1, s_2)$. Since, for simplicity, we are focusing on symmetric games, this is also the minmax value in pure strategies for player 2. Let $m = \{m_1, m_2\}$ be one strategy profile in which this minmax value is attained for player 1. Again by symmetry, minmax for player 2 is attained

[6] Some comments about mixed strategies can be found in the concluding section.

at $\{m_2, m_1\}$. Suppose we want to support a certain outcome $\{c_1, c_2\}$, and for simplicity assume $c_1 = c_2 = c$. We will make two more simplifying assumptions. Later on we will suggest how they can be relaxed. The first one is:

Assumption A2: $\exists p$ such that $m_2 \in \arg\max_s u_1(s, p)$.

Take p according to the previous assumption, and take $q = m_2$. Normalize $u_1(p, q) = u_2(q, p) = 0$. Then it should be clear that $v \geq 0$. Furthermore, under assumption A2, q is the (static) best response to p, so $u_1(q, p) = u_2(p, q) \geq v \geq 0$. Another simplifying assumption is:

Assumption A3: $p \neq c$, $q \neq c$.

The three following subsections are as follows. The first presents the type of strategies that will be used in the proof of the folk theorem, which is presented in the second subsection. Finally, we study how long can be the punishment interval T that follows a deviation for a given value of δ.

3.1 Strategies

Our strategy of proof is to support a certain outcome $\{c_1, c_2\}$, by imposing a penalty of T periods in the case of a deviation, in which the deviating player receives a payoff smaller than or equal to v. As stated above, it is assumed for simplicity that this outcome is such that $c_1 = c_2 = c$.

The intuition of the strategy that is going to be used to support the outcome is very simple. In fact, the strategy can be defined very easily in the case of an initial period in the following way. Start in the cooperative phase, with the two players playing c. If player 1 deviates, switch to phase 1, and if player 2 deviates, switch to phase 2. In phase 1, the prescribed profile is $\{p, q\}$. The game remains in this phase until $\{p, q\}$ is observed for T consecutive periods, in which case the game switches again to the cooperative phase, or until a deviation occurs, in which case a penalty to the deviating player starts again. The same for phase 2, but with a prescribed profile $\{q, p\}$.

The problem, however, arises when there is no initial node, since we cannot impose a given phase at the start of the game. The alternative is to define the phase as a function of the previous history. Also, we need strategies to be independent of calendar time, by stationarity.

We will present now the strategy in a formal way. It is very similar to the one just described for games with initial period. First, we try to find a suitable starting point in history (τ), and then we use θ_j to record the phase in which the game was in period $j > \tau$. The expression $I(A)$ denotes a function that takes the value 1 if the condition A is true, and 0 otherwise.

The strategy for player 1 with T periods of punishment is as follows:[7]

Step 1: Analyze previous history h^{-1}. If $\sum_{j=-\infty}^{-1} I\left(h_j^{-1} = \{c,c\}\right) \geq 1$, then take $\tau = \max\left\{k : h_k^{-1} = \{c,c\}\right\}$, initialize $\theta_{\tau+1} = 0$, and go to step 2. If $\sum_{j=-\infty}^{-1} I\left(h_k^{-1} = \{c,c\}\right) = 0$ and $\sum_{j=-\infty}^{-1}\left(I\left(h_j^{1,-1} = c\right) + I\left(h_j^{2,-1} = c\right)\right) \geq 2$, then take $\tau = \max\left\{k < 0 : \sum_{j=k}^{-1}\left(I\left(h_j^{1,-1} = c\right) + I\left(h_j^{2,-1} = c\right)\right) = 2\right\}$, and initialize $\theta_{\tau+1} = 1$ if $h_\tau^{2,-1} = c$, and $\theta_{\tau+1} = 2$ if $h_\tau^{1,-1} = c$, and go to step 2. If $\sum_{j=-\infty}^{-1} I\left(h_j^{-1} = \{c,c\}\right) = 0$ and $\sum_{j=-\infty}^{-1}\left(I\left(h_j^{1,-1} = c\right) + I\left(h_j^{2,-1} = c\right)\right) = 1$, then take $\tau = \left\{k : I\left(h_k^{1,-1} = c\right) + I\left(h_k^{2,-1} = c\right) = 1\right\}$, and initialize $\theta_{\tau+1} = 1$ if $h_\tau^{2,-1} = c$, and $\theta_{\tau+1} = 2$ if $h_\tau^{1,-1} = c$, and go to step 2. Finally, if $\sum_{j=-\infty}^{-1}\left(I\left(h_j^{1,-1} = c\right) + I\left(h_j^{2,-1} = c\right)\right) = 0$, then initialize $\theta_0 = 0$ and go to step 3.

Step 2: If $\tau + 1 = 0$, then go to step 3. If not, compute $\theta_{\tau+2}$ as follows. If $\theta_{\tau+1} = 0$ and $h_{\tau+1}^{-1} = \{c,c\}$ or both arguments are different from c, then $\theta_{\tau+2} = 0$. If $\theta_{\tau+1} = 0$ and $h_{\tau+1}^{i,-1} \neq c, h_{\tau+1}^{-i,-1} = c$, then $\theta_{\tau+2} = i$. If $\theta_{\tau+1} = 1$, then if $h_k^{-1} = \{p,q\}, \forall k \in \{\tau+2-T, \ldots, \tau+1\}$, $\theta_{\tau+2} = 0$, else $\theta_{\tau+2} = i$, where i is 1 unless $h_{\tau+1}^{1,-1} = p$ and $h_{\tau+1}^{2,-1} \neq q$, in which case $i = 2$. If $\theta_{\tau+1} = 2$, then if $h_k^{-1} = \{q,p\}, \forall k \in \{\tau+2-T, \ldots, \tau+1\}$, $\theta_{\tau+2} = 0$, else $\theta_{\tau+2} = i$, where i is 2 unless $h_{\tau+1}^{2,-1} = p$ and $h_{\tau+1}^{1,-1} \neq q$ in which case $i = 1$. Iterate until θ_0 is computed.

Step 3: If $\theta_0 = 0$, then play c. If $\theta_0 = 1$, then play p. If $\theta_0 = 2$, then play q.

Note that the previous maxima always exist because, at any given point, the history finishes in the previous period.

As anticipated above, the variable θ can be interpreted as the phase in which the game is. Note that it is only used to determine the action to be played, with no other future effects. The strategy is presented in this way to avoid confusion between imposing a particular phase in a particular period, and determining the phase in one period endogenously. This is done in step 1. In step 2 we analyze deviations from cooperation, or from a punishment phase, that have been occurred in the past, to be sure that at each time we know who is the player that has deviated most recently. In step 3 we simply require the players to play according to the current phase, which has been determined in the two previous steps. It is worth noting that we need to do the three steps at each period. Formally, it is not possible to make them once

[7] We assume that period is equal to 0 without loss of generality, by stationarity of the strategy. Also, only strategy for player 1 is presented, by symmetry.

and then follow the argument at the beginning of this subsection for games with initial period (because we require stationarity), although the intuition is similar.

3.2 The folk theorem

The proof of the following folk theorem for stable equilibria is quite standard, and hence we will only focus on the steps needed to fulfill the new stability requirement (for further details see Fudenberg and Tirole (1991)).

Theorem 7 *Suppose we have a stage game in normal form satisfying assumption A1, and consider the associated repeated game with no initial period. Then (a)* $\exists\bar{\delta} \in (0,1)$ *such that, for all* $\delta \in \left[\bar{\delta}, 1\right)$*, every pure strategy profile with payoffs greater than the minmax values can be the limit of the equilibrium path of a stable SPE, for all histories; and (b) for all* $\delta \in \left[\bar{\delta}, 1\right)$*, convergence to this limit can be achieved in a finite number of periods, for all histories.*

Proof. Part (b) is done in detail in the following subsection. For simplicity, the proof will make use of assumptions (A2) and (A3).[8]

Take the strategies defined in the previous section, with a penalty length T arbitrarily chosen, and $\{c, c\}$ as the profile to be supported.[9] If $\{c, c\}$ is a Nash equilibrium of the stage game, the proof is trivial, so assume that it is not (note that this implies that $\max_s u_1(s, c) - u_1(c, c)$ is strictly greater

[8] Assumptions A1 and A2 are made for simplicity. They are not necessary to derive the result, and in this note we are going to suggest how the result could be proved if they don't hold.

Suppose players are in phase 2 for certain history. Under A1, player 1 has no incentives to deviate because he is playing his static best response. On the other hand, if A1 doesn't hold, then player 1 could deviate for a short run profit. After this deviation, player 1 will be punished for T consecutive periods with the profile (p, q). But, the profit is for only one period, so it is bounded, and the punishment is more intense the larger are δ or T. Hence, for δ and T sufficiently large the incentive disappears.

Concerning assumption A2, please note that the second part of the assumption is not restrictive, because $q = m_2$ and hence if $q = c$ then $u_i(c, c) \leq v$.

Finally, if we need the punishment profile to be such that $p = c$, it would be difficult to assess whether $\{q, c\}$ is a deviation of player 1 or the punishment phase that follows a deviation of player 2. In the second case, if player 1 deviates and plays c, the strategy in section 3.1 would identify $\{c, c\}$ as a cooperation, instead of a deviation of player 1. But note that q is the static best response to c, so this deviation is not profitable.

[9] The same proof can be replicated for asymmetric games or symmetric games with $c_1 \neq c_2$, although with a little bit more notation. Simply we would have two different $\bar{\delta}$, one for each player, and it suffices to take the bigger of the two.

than 0). We need to show three facts. First, the strategies are stationary. Second, they induce a convergent continuation path, for all histories. And third, there are no profitable deviations. Since the game is symmetric, only deviations of player 1 will be considered.

The first and second parts are straightforward, given the definition of the strategy.

Concerning deviations, the following conditions[10] are enough to avoid them:

$$u_1(q,p)\left(1+\delta+\cdots+\delta^{t-1}\right)+\tfrac{\delta^t}{1-\delta}\cdot u_1(c,c)\geq$$

$$\geq \max_{s\in S-\{q\}} u_1(s,p)+\tfrac{\delta^{T+1}}{1-\delta}\cdot u_1(c,c)\,;1\leq t\leq T \tag{1}$$

$$\frac{\delta^t}{1-\delta}\cdot u_1(c,c)\geq \max_{s\in S-\{p\}} u_1(s,q)+\frac{\delta^{T+1}\cdot u_1(c,c)}{1-\delta};1\leq t\leq T \tag{2}$$

$$\frac{u_1(c,c)}{1-\delta}\geq \max_s u_1(s,c)+\frac{\delta^{T+1}}{1-\delta}\cdot u_1(c,c) \tag{3}$$

Inequalities (1) and (2) rule out deviations in histories following a deviation of player 2 and 1, respectively, with t remaining punishment periods. Finally, inequality (3) rules out deviations when players are supposed to cooperate and play c.

It is easy to see that, under our assumptions, condition (1) is always satisfied.

Condition (2) is equivalent to

$$\delta^T\geq \frac{\max\limits_{s\in S-\{p\}} u_1(s,q)}{u_1(c,c)} \tag{4}$$

Note that $\max\limits_{s\in S-\{p\}} u_1(s,q)\leq \max\limits_s u_1(s,q)=v<u_1(c,c)$. Hence, for a finite T there exists a $\delta_1\in(0,1)$ such that (4) is satisfied for all $\delta\in[\delta_1,1)$.

Finally, condition (3) can be expressed as:

$$u_1(c,c)\cdot\delta^{T+1}-\left(\max_s u_1(s,c)\right)\delta+\left(\max_s u_1(s,c)-u_1(c,c)\right)\leq 0 \tag{5}$$

[10] We are using the principle that, if the strategies constitute an equilibrium, then it is enough to check deviations as the following: deviate in one period and then follow again the strategy. See Fudenberg and Tirole (1991), section 4.2.

Note that the left hand side of (5) is a polynomial (in δ) of degree $T+1$. Let's call it $Y(\delta)$, so condition (5) is equivalent to $Y(\delta) \leq 0$. It's easy to see that $Y(0) = \max_s u_1(s,c) - u_1(c,c) > 0$, $Y(1) = 0$, $Y'(\delta) = ((T+1) \cdot u_1(c,c)) \cdot \delta^T - \left(\max_s u_1(s,c)\right)$, and that $Y(\delta)$ is (strictly) convex in $(0,\infty)$, since $u_1(c,c) > 0$.

Now, two cases are possible:

Case 1: $\frac{\max_s u_1(s,c)}{u_1(c,c)\cdot(T+1)} \geq 1$: In this case the polynomial $Y(\delta)$ is (strictly) decreasing in $(0,1)$. Since $Y(0) = \max_s u_1(s,c) - u_1(c,c) > 0$ and $Y(1) = 0$, then $Y(\delta) > 0$ for $\delta \in (0,1)$. Therefore, there is no equilibrium in this case.

Case 2: $\frac{\max_s u_1(s,c)}{u_1(c,c)\cdot(T+1)} < 1$: Here the function $Y(\delta)$ is decreasing in $\left(0,\hat{\delta}\right)$, and increasing in $\left(\hat{\delta},1\right)$, with $\hat{\delta} = \left(\frac{\max_s u_1(s,c)}{u_1(c,c)\cdot(T+1)}\right)^{\frac{1}{T}} \in (0,1)$. Again, since $Y(0) = \max_s u_1(s,c) - u_1(c,c) > 0$ and $Y(1) = 0$, then $\exists \delta_2 \in (0,1)$ such that $Y(\delta) \leq 0$ for all $\delta \in [\delta_2, 1)$.

Note that the value of T determines whether we are in case 1 or 2, and case 1 can be avoided by taking $T > \frac{\max_s u_1(s,c)}{u_1(c,c)} - 1$.

The proof is complete by considering $\bar{\delta} = \max\{\delta_1, \delta_2\}$. ■

3.3 Bounds for the punishment interval

In this subsection we make a different type of analysis. Once we have shown that a stable perfect equilibrium exists for sufficiently patient players, we can ask ourselves the following: for a given (sufficiently high) δ, what values of T constitute a stable SPE?

We have seen that a necessary condition is $\frac{\max_s u_1(s,c)}{u_1(c,c)\cdot(T+1)} < 1$, so the first constraint on T is:

$$T > \frac{\max_s u_1(s,c)}{u_1(c,c)} - 1 \tag{6}$$

Other constraints on T are conditions (4) and (5), but now for a fixed δ. With some algebra we can obtain the following from (4):

$$T \leq \frac{\log\left(\frac{\max_{s \in S-\{p\}} u_1(s,q)}{u_1(c,c)}\right)}{\log(\delta)} \tag{7}$$

The numerator and the denominator are negative, and δ is big enough so that the denominator is small enough (in absolute value) to produce a sensible bound.

Now, from (5) we get:

$$T \geq \frac{\log\left(-\frac{\max\limits_{s} u_1(s,c)}{u_1(c,c)} \cdot (1-\delta)+1\right)}{\log \delta} - 1 \tag{8}$$

Therefore the admissible values for T are the positive integers such that (6), (7) and (8) are satisfied. The lower bound is[11]

$$\max\left\{\frac{\log\left(-\frac{\max\limits_{s} u_1(s,c)}{u_1(c,c)} \cdot (1-\delta)+1\right)}{\log \delta} - 1, \frac{\max\limits_{s} u_1(s,c)}{u_1(c,c)} - 1, 1\right\}$$

and the upper bound is

$$\frac{\log\left(\frac{\max\limits_{s \in S-\{p\}} u_1(s,q)}{u_1(c,c)}\right)}{\log(\delta)}$$

Note that the numerator is finite under assumption A1, because $\max\limits_{s \in S-\{p\}} u_1(s,q) < u_1(c,c)$. The denominator is also finite and different from 0 for $\delta \in (0,1)$. Therefore, the upper bound is strictly finite, so convergence must be achieved in a finite number of periods.

The intuition for the upper bound is very simple. Consider the incentives for a player that is being punished. Clearly, deviation is more profitable if the punishment period is longer. Therefore, the punishment period cannot be very long, because it would induce deviations.

4 Memory Requirements

In this section we study the memory requirements of the strategies presented in section 3.1, in an attempt to compare our approach with the theory on bounded recall. First we will show that our strategies do not work under bounded recall. This could cast doubt on the complexity of our strategy and hence on its applicability in the real world, as Aumann (1997) suggests. For this reason we define a concept of memory requirements weaker than bounded

[11] Remember that condition (6) is a strict inequality, and the other two conditions are weak inequalities.

recall, and we provide upper bounds for memory requirements under this weaker definition.

We start by defining absolute memory of a strategy:

Definition 8 *A strategy σ_i has absolute memory $\Phi(\sigma_i) = \lambda$ if $\lambda \in \mathbb{N}$ is the minimum number such that $s_i^t(h^{t-1}) = s_i^t(\tilde{h}^{t-1})$, for any period $t \in \mathbb{Z}$, and for every pair of histories $h^{t-1} \in H$ and $\tilde{h}^{t-1} \in H$ such that $h_k^t = \tilde{h}_k^t, \forall k \in \{t-\lambda, \ldots, t-1\}$.*

We can say that a strategy σ_i satisfy bounded recall if and only if $\Phi(\sigma_i) < \infty$. Now, it is easy to see that the strategy defined in section 3.1 does not satisfy bounded recall, as the following proposition shows:

Proposition 9 *The strategy defined in 3.1 has infinite absolute memory.*

Proof. Consider a history $h^{t-1} \in H$ such that $h_k^{t-1} = \{q, q\}, \forall k \leq t-1$. Now, consider another history $\tilde{h}^{t-1} \in H$ such that $\tilde{h}_k^{t-1} = \{q, q\}, \forall k \leq t-1, k \neq t-\lambda$, and $\tilde{h}_{t-\lambda}^{t-1} = \{c, q\}$, for some $\lambda \geq 1$. According to the strategy defined in 3.1, player 1 should play c after history h^{t-1}, whereas he should play q after history $\tilde{h}^{t-1}$. The proof is complete by taking the limit $\lambda \to \infty$. ■

The previous proposition shows that our stable strategies cannot be defined under a bounded recall framework. But this does not mean that our strategies are infinitely complicated, because we can provide bounds for their memory requirements using a definition weaker than absolute memory. We call this weaker concept conditional memory, and it is defined as follows:

Definition 10 *A strategy σ_i has conditional memory $\phi(\sigma_i, h^{t-1}) = \lambda$ in subgame h^{t-1} if $\lambda \in \mathbb{N}$ is the minimum number such that $s_i^t(h^{t-1}) = s_i^t(\tilde{h}^{t-1})$, for every history $\tilde{h}^{t-1} \in H$ such that $h_k^t = \tilde{h}_k^t, \forall k \in \{t-\lambda, \ldots, t-1\}$.*

The definition of conditional memory calculates memory requirements, conditional on being in a particular subgame. On the other hand, absolute memory is defined without any conditioning, so it can be interpreted as the maximum conditional memory over all possible histories.

Now, we are in a position to provide bounds for conditional memory requirements of the stable strategy defined in 3.1:

Theorem 11 *Let σ_1 and σ_2 be the stable strategies defined in 3.1 for player 1 and 2, respectively. Then, $\phi(\sigma_i, P_k(h^{t-1}, \sigma_1, \sigma_2)) = 1$ for any $t \in \mathbb{Z}, h^{t-1} \in H, i \in \{1, 2\}, k \in \mathbb{Z}$ such that $k \geq t + T$.*

Proof. If players play according to the stable strategies defined in 3.1, then $P_k(h^{t-1}, \sigma_1, \sigma_2) \in \{\{c, c\}, \{p, q\}, \{q, p\}\}, \forall k \geq t$, depending on the value of θ_k computed in steps 1 and 2 in the definition of the strategy. Moreover, the profiles $\{p, q\}$ and $\{q, p\}$ can only last for at most T periods, so $P_k(h^{t-1}, \sigma_1, \sigma_2) = \{c, c\}, \forall k \geq t + T$. The proof is complete by noting that step 1 in the definition of the strategy ignores the part of the history observed before the most recent $\{c, c\}$. ■

The previous theorem states that, if players play according to the stable strategies defined in 3.1, then conditional memory requirements eventually collapse to 1, because eventually both players will be playing c. This means that conditional memory requirements are finite and small, except from histories that are very far from the equilibrium path implied by our stable strategies.

5 Concluding Comments

In this paper we have proposed a concept of dynamic stability in repeated games with discounting for which a modification of the traditional theory is needed. In particular, we need to introduce a game with no beginning, i.e. a game with an infinite history at all periods. One should keep in mind issues like reputation or robustness to initial conditions when interpreting the proposed concept of dynamic stability.

A characterization[12] of payoffs supported as a stable equilibrium is also presented, with the additional result that convergence to the long-run strategy profile can be achieved in a finite number of periods, for all previous histories. The proof is constructive, giving the strategies that support the long run profile. We have also shown that memory requirements of these strategies are bounded, but under a concept of memory requirements weaker than bounded recall.

One can look at the contributions of this work in at least three different ways. The most direct is the study of games with an infinite history, i.e. with no beginning. A more interesting interpretation is that the theory developed here provides a formal justification for the folk results often used in the literature, because all equilibrium paths converge to the cooperative outcome. In other words, we are checking the robustness of the folk results to the assumption that strategies begin with certain profile. Finally, the analysis of the convergence process is interesting by itself. If for whatever reason players are not cooperating (maybe because someone has deviated in

[12] This characterization is incomplete in the sense that we have focused on pure strategies, and on games with no uncertainty.

the past), the strategies presented here provide a way in which players can build up again their reputations.

The analysis developed here has nevertheless several limitations, and further research will be useful. The main limitation is that the theorem requires a supported outcome in pure strategies. Although the rest of the assumptions are innocuous, and are made for simplicity, the requirement of pure strategies cannot be generalized in a straightforward manner. If we want to support convex combinations of the pure strategy payoffs, we cannot apply the usual argument of a public randomizing device and a correlated distribution. The reason is that we require the equilibrium path to be convergent, not to switch at random over a set of outcomes. But one can still have a notion of dynamic stability in these cases. Perhaps the simplest way is to substitute condition (b) in the definition of a stable SPE by convergence in expected payoffs, not in the profile played. Maybe there are other possibilities, like convergence in distribution over outcomes, even though it may be difficult to develop stationary strategies with the property that future probability distribution over outcomes is invariant, irrespective of the realization of the present (stochastic) outcome.

Second, we have focused on games with no uncertainty, so a concept of stability for games with some source of uncertainty may be useful. Probably, the definition will be close to Ely and Välimäki (2002) and Green and Porter (1984), but the extension is not straightforward. One possibility would be to make the definition analogous to those used in stochastic processes, assuming that from now on all stochastic variables take a realization equal to their mean value (including variables related to imperfect information). Then, the definition of convergence could be modified accordingly.

Third, it is interesting to note that the folk result presented here is a limit result. The theorem in section 3.2 states that we can support stable equilibria for a sufficiently high discount factor. It doesn't say anything about optimal punishment schemes (satisfying stability), in the sense that there may exist other strategies that require weaker conditions for $\bar{\delta}$. This is obviously another possible extension.

References

Abreu, D. (1988). "On the Theory of Infinitely Repeated Games with Discounting," *Econometrica,* 56, pp. 383-396.

Abreu, D., D. Pearce and E. Stachetti (1990). "Toward a theory of discounted repeated games with imperfect monitoring," *Econometrica,* 58, pp. 1041-1064.

Anderlini, L., and H. Sabourian (1995). "Cooperation and Effective Computability," *Econometrica,* 63, pp. 1337-1369.

Aumann, R. (1997). "Rationality and Bounded Rationality," *Games and Economic Behavior,* 21, pp. 2-14.

Aumann, R., and S. Sorin (1989). "Cooperation and Bounded Recall," *Games and Economic Behavior,* 1, pp. 5-39.

Ely, J. C., and J. Välimäki (2002). "A Robust Folk Theorem for the Prisoner's Dilemma," *Journal of Economic Theory,* 102, pp. 84-105.

Evans, R., and J. P. Thomas (2001). "Cooperation and Punishment," *Econometrica,* 69 (4), pp. 1061-1075.

Fudenberg, D., and J. Tirole (1991). "Game Theory," *MIT Press.*

Green, E. J., and R. H. Porter (1984). "Noncooperative Collusion under Imperfect Price Information," *Econometrica,* 52 (1), pp. 87-100.

Kalai, E., D. Samet and W. Stanford (1988). "A Note on Reactive Equilibria in the Discounted Prisoners's Dilemma and Associated Games," *International Journal of Game Theory,* 17 (3), pp. 177-186.

Kalai, E., and W. Stanford (1988). "Finite Rationality and Interpersonal Complexity in Repeated Games," *Econometrica,* 56 (2), pp. 397-410.

Piccione, M. (2002). "The Repeated Prisoner's Dilemma with Imperfect Private Monitoring," *Journal of Economic Theory,* 102, pp. 70-83.

BANCO DE ESPAÑA PUBLICATIONS

WORKING PAPERS[1]

0527 LUIS J. ÁLVAREZ, PABLO BURRIEL AND IGNACIO HERNANDO: Price setting behaviour in Spain: evidence from micro PPI data.

0528 EMMANUEL DHYNE, LUIS J. ÁLVAREZ, HERVÉ LE BIHAN, GIOVANNI VERONESE, DANIEL DIAS, JOHANNES HOFFMANN, NICOLE JONKER, PATRICK LÜNNEMANN, FABIO RUMLER AND JOUKO VILMUNEN: Price setting in the euro area: some stylized facts from individual consumer price data.

0529 TERESA SASTRE AND JOSÉ LUIS FERNÁNDEZ-SÁNCHEZ: Un modelo empírico de las decisiones de gasto de las familias españolas.

0530 ALFREDO MARTÍN-OLIVER, VICENTE SALAS-FUMÁS AND JESÚS SAURINA: A test of the law of one price in retail banking.

0531 GABRIEL JIMÉNEZ AND JESÚS SAURINA: Credit cycles, credit risk, and prudential regulation.

0532 BEATRIZ DE-BLAS-PÉREZ: Exchange rate dynamics in economies with portfolio rigidities.

0533 ÓSCAR J. ARCE: Reflections on fiscalist divergent price-paths.

0534 M.ª DE LOS LLANOS MATEA AND MIGUEL PÉREZ: Differences in changes in fresh food prices by type of establishment. (The Spanish original of this publication has the same number.)

0535 JOSÉ MANUEL MARQUÉS, FERNANDO NIETO AND ANA DEL RÍO: Una aproximación a los determinantes de la financiación de las sociedades no financieras en España.

0536 S. FABIANI, M. DRUANT, I. HERNANDO, C. KWAPIL, B. LANDAU, C. LOUPIAS, F. MARTINS, T. MATHÄ, R. SABBATINI, H. STAHL AND A. STOKMAN: The pricing behaviour of firms in the euro area: new survey evidence.

0537 LUIS J. ÁLVAREZ AND I. HERNANDO: The price setting behaviour of Spanish firms: evidence from survey data.

0538 JOSÉ MANUEL CAMPA, LINDA S. GOLDBERG AND JOSÉ M. GONZÁLEZ-MÍNGUEZ: Exchange-rate pass-through to import prices in the euro area.

0539 RAQUEL LAGO-GONZÁLEZ AND VICENTE SALAS-FUMÁS: Market power and bank interest rate adjustments.

0540 FERNANDO RESTOY AND ROSA RODRÍGUEZ: Can fundamentals explain cross-country correlations of asset returns?

0541 FRANCISCO ALONSO AND ROBERTO BLANCO: Is the volatility of the EONIA transmitted to longer-term euro money market interest rates?

0542 LUIS J. ÁLVAREZ, EMMANUEL DHYNE, MARCO M. HOEBERICHTS, CLAUDIA KWAPIL, HERVÉ LE BIHAN, PATRICK LÜNNEMANN, FERNANDO MARTINS, ROBERTO SABBATINI, HARALD STAHL, PHILIP VERMEULEN AND JOUKO VILMUNEN: Sticky prices in the euro area: a summary of new micro evidence.

0601 ARTURO GALINDO, ALEJANDRO IZQUIERDO AND JOSÉ MANUEL MONTERO: Real exchange rates, dollarization and industrial employment in Latin America.

0602 JUAN A. ROJAS AND CARLOS URRUTIA: Social security reform with uninsurable income risk and endogenous borrowing constraints.

0603 CRISTINA BARCELÓ: Housing tenure and labour mobility: a comparison across European countries.

0604 FRANCISCO DE CASTRO AND PABLO HERNÁNDEZ DE COS: The economic effects of exogenous fiscal shocks in Spain: a SVAR approach.

0605 RICARDO GIMENO AND CARMEN MARTÍNEZ-CARRASCAL: The interaction between house prices and loans for house purchase. The Spanish case.

0606 JAVIER DELGADO, VICENTE SALAS AND JESÚS SAURINA: The joint size and ownership specialization in banks' lending.

0607 ÓSCAR J. ARCE: Speculative hyperinflations: When can we rule them out?

0608 PALOMA LÓPEZ-GARCÍA AND SERGIO PUENTE: Business demography in Spain: determinants of firm survival.

0609 JUAN AYUSO AND FERNANDO RESTOY: House prices and rents in Spain: Does the discount factor matter?

0610 ÓSCAR J. ARCE AND J. DAVID LÓPEZ-SALIDO: House prices, rents, and interest rates under collateral constraints.

0611 ENRIQUE ALBEROLA AND JOSÉ MANUEL MONTERO: Debt sustainability and procyclical fiscal policies in Latin America.

---

1. Previously published Working Papers are listed in the Banco de España publications calalogue.

0612 GABRIEL JIMÉNEZ, VICENTE SALAS AND JESÚS SAURINA: Credit market competition, collateral and firms' finance.
0613 ÁNGEL GAVILÁN: Wage inequality, segregation by skill and the price of capital in an assignment model.
0614 DANIEL PÉREZ, VICENTE SALAS AND JESÚS SAURINA: Earnings and capital management in alternative loan loss provision regulatory regimes.
0615 MARIO IZQUIERDO AND AITOR LACUESTA: Wage inequality in Spain: Recent developments.
0616 K. C. FUNG, ALICIA GARCÍA-HERRERO, HITOMI IIZAKA AND ALAN SUI: Hard or soft? Institutional reforms and infraestructure spending as determinants of foreign direct investment in China.
0617 JAVIER DÍAZ-CASSOU, ALICIA GARCÍA-HERRERO AND LUIS MOLINA: What kind of capital flows does the IMF catalyze and when?
0618 SERGIO PUENTE: Dynamic stability in repeated games.

BANCO DE ESPAÑA	Unidad de Publicaciones Alcalá, 522; 28027 Madrid Telephone +34 91 338 6363. Fax +34 91 338 6488 *e-mail:* Publicaciones@bde.es www.bde.es