CARBON TAX SECTORAL (CATS) MODEL: A SECTORAL MODEL FOR ENERGY TRANSITION STRESS TEST SCENARIOS

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Pablo Aguilar

BANCO DE ESPAÑA

Beatriz González

BANCO DE ESPAÑA

Samuel Hurtado

BANCO DE ESPAÑA

(*) The authors belong to the Directorate General Economics, Statistics and Research of the Banco de España, and are grateful for the comments received from all reviewers. This article is the sole responsibility of the authors and does not necessarily reflect the opinion of the Banco de España or of the Eurosystem. Email for comments: samuel.hurtado.bde.es.

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Abstract

This paper presents a general equilibrium sectoral model designed to produce macroeconomic scenarios that incorporate transition risks associated with policies to curb climate change (but not physical risks associated with the long-term costs of climate change). The model is calibrated to the Spanish economy, and can simulate the impact of shocks to the price and coverage of greenhouse gas emission allowances, with particular attention to sectoral asymmetries arising from (i) the energy intensity of each industry, (ii) the source of that energy, and (iii) the interdependencies with other industries. We show that for an increase in the price of emission allowances similar to that observed in recent years (from approximately €25 per tonne of CO2 in 2019 to almost €100 per tonne in 2022) the model predicts a cumulative decline in Spanish GDP after three years of 0.37%. The loss in value added is very heterogeneous across industries, ranging from 4% in the most severely affected industries to virtually no impact in the least affected industries. In terms of the use of the model for stress testing, this heterogeneity points to potential risks for financial stability and the importance of the right diversification for banks to diminish their exposure to transition risks.

Keywords: climate change, stress test, input-output matrix.

JEL classification: Q48, H30.

Resumen

Este artículo presenta un modelo de equilibrio general diseñado para producir escenarios macroeconómicos que incorporen los riesgos de transición asociados a las políticas adoptadas para evitar el proceso de cambio climático (no los riesgos físicos asociados al coste del cambio climático a largo plazo). El modelo está calibrado para la economía española, y puede simular el impacto de *shocks* en el precio y la cobertura de derechos de emisión de gases efecto invernadero, con particular atención a las asimetrías sectoriales provenientes de i) la intensidad de uso de la energía; ii) la fuente de dicha energía, y iii) las interdependencias entre industrias. Mostramos que, tras un incremento en el precio de los derechos de emisión similar al observado en los últimos años (de aproximadamente 25 € por tonelada de CO_2 en 2019 a casi 100 € por tonelada en 2022), el modelo predice una caída acumulada del PIB después de tres años del 0,37 %. La pérdida de valor añadido es muy heterogénea entre distintas industrias, con valores que van de pérdidas del 4 % en las industrias más afectadas a un impacto prácticamente nulo en los sectores menos afectados. En relación con el uso del modelo para pruebas de estrés, esta heterogeneidad apunta a la existencia de riesgos potenciales para la estabilidad financiera, y a la importancia de una correcta diversificación bancaria para disminuir su exposición a los riesgos de transición.

Palabras clave: cambio climático, pruebas de estrés, matrices insumo producto.

Códigos JEL: Q48, H30.

Contents

1 Introduction

The challenges of climate change affect all aspects of the economy. Both physical risks (associated with the direct effect of climate change) and transition risks (associated with the tax and regulatory measures implemented to try to prevent climate change) may have large aggregate effects. These effect may also be asymmetric, revealing a special vulnerability in certain sectors or firms. This article presents a macroeconomic model with a strong sectoral component, capable of simulating the effects of transition risks on the Spanish economy (but not long-term physical risks). It is a static general equilibrium model with rich sectoral networks, based on Baqaee and Farhi (2019), extended to include two energy sectors and a carbon tax attached to the emissions of greenhouse gases.

The main use will be to generate macroeconomic scenarios for climate change stress testing. Under the most pessimistic scenarios, some financial institutions may find themselves in difficulty if they are poorly diversified in newly-relevant dimensions, e.g. if they are highly exposed to sectors that can be expected to display more negative effects in response to shocks related to climate change. Bank stress tests attempt to anticipate the possible emergence of this type of problem.[1] To carry out such tests, quantitative tools are required, to simulate the effects of shocks and their transmission throughout the economy and the financial system; the model presented in this article is one such tool.

This model does not attempt to include the effects of physical risks, which are those directly associated with the process of climate change. These include, inter alia, rising temperatures, ice melt and sea level rises, a higher frequency and intensity of adverse atmospheric phenomena, progressive degradation of environmental variables such as air and water quality, deforestation and biodiversity loss.[2] There is some evidence that these risks are beginning to materialize, causing significant damage (to capital goods and real estate, for example), reductions in productivity and ad hoc disruptions to production chains and, moreover, are expected to continue increasing in the next decades, whose effects will be concentrated in the long run. Transition risks, on the other hand, are those associated with initiatives to mitigate the climate change process: For instance, rising the cost of emission allowances, new taxes and subsidies to accelerate reductions in greenhouse gas emissions, new regulations requiring changes in agents' behavior to obtain these results, climate change legislation that affects financial insti-

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[1] Financial institutions include not only banks, but also other financial intermediaries, such as insurance companies and investment funds, which are closely linked to banks in Spain. In principle, the scenarios generated by this model may be used to analyse the effects of the shock on all of them.

[2] Various European and international bodies have published evidence on the long-term physical impact of climate change. See OECD (2015), G20 (2016),Giuzio et al. (2019) and Commission (2020).

tutions[3], technological changes that increase the rate at which capital is depreciated when replaced by less polluting options, or even consumer preference changes prompting a producer response, etc. Our model tries to assess the macroeconomic effects of some (but not all) of these transition risks.

In the case of transition risks, the scope of this paper, there is a greater probability of observing potentially sizable effects within more limited time periods, especially if a fast transition amplifies the short-term costs.[4] Following this reasoning, the model presented in this article is designed to produce macroeconomic scenarios relating to transition risks, to serve as the basis for stress tests to verify that every part of the financial system is prepared for possible adverse events of this type. The model doesn't include elements such as stranded assets or other big rigidities that could amplify the short-term effects of a disorderly transition, but by simulating big and fast shocks, it can generate sufficiently-costly scenarios.

The model is used in this article to simulate several shocks: a change in the international price of energy inputs (which is used to calibrate the main elasticities of substitution in the model), a steep rise in the price of CO_2 emission allowances, an expansion of the coverage of the system of emission allowances to all sectors in the economy, a combination of increased prices and increased coverage, and an additional simulation with revenue recycling through reduced labor taxes. The model simulates the impact of these shocks on the Spanish economy, paying particular attention to sectoral asymmetries arising from the intensity with which different types of energy are used in each industry, the interdependencies summarized in the input-output tables for the Spanish economy, and the general equilibrium effects in terms of relative price changes and sectoral reallocation.

The traditional approach to climate change in the literature is based on integrated assessment models (IAMs and DICE, in their dynamic version) that study how economic growth impacts climate conditions and what is the cost-benefit of policies that mitigate emissions seeking to reduce global warming in the long-run. The most notorious example is the neoclassical climate-change growth model of Nordhaus (2007) and Nordhaus (2017) that internalizes climate change damages, measured as greenhouse emissions, in the social welfare function when deciding the growth path that leads to

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[3] For example, the European Commission's "Action Plan: Financing Sustainable Growth" seeks to redirect capital flows towards sustainable investment, and the Taxonomy Regulation, also approved by the European Commission, defines the criteria for classifying economic activity environmentally. Legislative developments may also affect financial institutions' asset portfolios, including the EU Green Bond Standard, which will potentially have an impact on asset valuations, the inclusion of environmental aspects in the European Central Bank's (ECB) bank stress tests, and, more generally the ECB's strategy review. See Drudi et al. (2021).

[4] See Bank of England (2018), ESRB (2016) and Giuzio et al. (2019).

an optimal level of climate change mitigation. [5] However IAMs models are ofteen a too simplified representation of the economy (compared with standard macroeconomic models), lacking in transmission channels, such as commodity price changes during the transition, which reduce the level of detail in the macroeconomic impact of climate policy.

The development of Computable General Equilibrium (CGE) models, that introduce a detailed representation of the economy allows for tractable simulations of climate change policies, see for instance Branger and Quirion (2014) and Matsumoto and Fujimori (2019). Some institutions, such as the Bank of Canada, have incorporated this type of models for climate policy analysis. Other representations of the economy, based on macroeconometric relationships, are also popular for the analysis of climate policies. This is the case for the De Nederlandsche Bank (DNB) or the Banque de France that use the NiGEM model for policy scenarios.

More recently, the dynamic stochastical approach to general equilibrium (DSGE) model, now a fundamental tool in the quantitative analysis of the macroeconomy, has also been adapted to analyse environmental policies and simulate the effects of climate change. Environmental DSGE (E-DSGE) incorporate emissions in firms production function and internalize its cost, featuring environmental externalities, as in the models of Heutel (2012) and Golosov et al. (2014). Unlike these models, CATS works on a relatively short time horizon, as it will be primarily used to generate transition-risk climate stress scenarios that assess the different productive sectors' degree of exposure in the event of an increase in the price of emission allowances or an extension of EU-ETS coverage, hence abstracting from environmental externalities. Alternatively, other models introduce environmental externalities either in agents' utility (see Angelopoulos et al., 2013) or affecting indirectly their utility (see Chang et al., 2009). E-DSGE are also useful for the analysis of cross-border emissions and climate cooperation as in the multi-country model of Ernst et al. (2022), that shows how countries taxing emissions benefit from others cooperating, while there is no incentive for non-cooperating countries to join them.

Our model, based on a sectoral general equilibrium representation of Spain a la Baqaee and Farhi (2019), falls into the category of CGE models. The granularity of this sort of model, 51 non-energy industries and two energy industries (fuel and electricity) accounting for the cross-sectoral relationships contained in the input-output tables, has the advantge of allowing us to assess the implications of Spanish industries' asymmetrical exposure to the green transition process. In particular, we find that in the event of an increase in the price of emission allowances similar to that observed

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[5] Weitzman (2012), Wagner and Weitzman (2016), Dietz and Stern (2015) explore alternative damage functions and their implication to the optimal while Dietz et al. (2021) and Bernie and Lowe (2018) analyze alternative carbon cycle and atmosphere impact.

in recent years (from approximately €25 per tonne of CO2 in 2019 to almost €100 per tonne in early February 2022), the model predicts a cumulative decline after three years of 0.37% in Spanish GDP, with a very asymmetric impact across sectors. This stresseses how the transitional risks for banks may depend on their exposure to the most affected sectors, which may pose a threat to financial stability.

Section 2 discusses the main characteristics of the model and Section 3 details the calibration. Section 4 presents the results of the transition risk simulations and Section 5 concludes.

2 Model

The model is based on Baqaee and Farhi (2019), extended to include two energy sectors that have to pay a carbon tax for the emissions they generate. It is a static general equilibrium model with rich sectoral networks. The economy is closed, with the exception of basic energy inputs, which are imported. The structure of the model is summarized on Figure 1.

There is a representative household, that consumes and supplies her labor elastically, with constant relative risk aversion utility. Her labour is perfectly mobile across sectors. The model includes N_e energy producers, N_s non-energy producers, N_s non-energy intermediate product retailers, N_s energy intermediate product retailers, one energy consumption retailer, one non-energy consumption retailer, and one final consumption retailer.

Each one of the N_s non-energy production sectors use a production function with three inputs: it first combines homogeneous labor and a sector-specific bundle of intermediate non-energy inputs using a Cobb-Douglas technology, and then this is nested into a CES that combines it with a sector-specific bundle of energy inputs.

There are N_e energy production sectors. They use a naïve production function where imported basic energy goods are the only input, and output equals input, with no value added.[6]

There are N_s non-energy intermediate input retailers; each of them buys goods from all non-energy producers, combines them with Cobb-Douglas technology, and sells the resulting bundle to a specific non-energy production sector. And there are N_s energy intermediate input retailers, again one for each non-energy sector; they combine the energy goods coming from the N_e energy production sectors, using a CES aggregator to create unique energy good bundles to be sold to the non-energy sectors.

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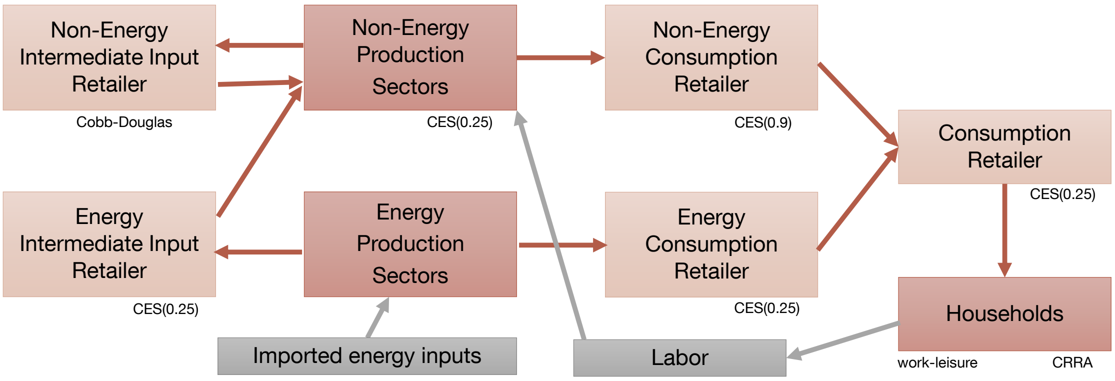
[6] Using a more complicated production function here, with other inputs such as elastic labor supply or non-energy goods is not necessarily an improvement, because even with realistic parametrization it could bring unrealistic results such as the following: a strong carbon tax that reduces the amount of energy used in the economy would free up those other inputs to be used by non-energy sectors, generating a posisive supply channel that reduces the negative effects for the rest of the economy.

There is another set of aggregators in the consumption side of the economy. One energy consumption retailer buys from the N_e energy producers and bundles those goods using a CES aggregator. One non-energy consumption retailer buys inputs from the N_s non-energy production sectors, and combines them using another CES to produce a single non-energy consumption good. Finally, there is a consumption retailer that buys the energy consumption bundle and the non-energy production bundle, and combines them through a new CES aggregator to produce the final good that households buy for consumption.

In practice, we use $N_s = 51$ non-energy sectors and $N_e = 2$ energy sectors. The two energy sectors represent electricity and fuel. Inside the model they only differ in terms of the amount of emissions that each unit of output requires, and what share of them are subject to the carbon tax. The way this simplified specification relates to real-world structures is not straightforward. In the case of fuels, their production does not generate a large amount of emissions, but their use does; it is the agents who use the fuels that have to acquire the associated emission allowances if subject to the ETS system, while the fuel producer receives a price that does not include the amount corresponding to such rights. Electricity, in contrast, generates emissions when it is produced[7], but not necessarily when it is used; thus, electricity users do not need to acquire emission allowances, but simply pay a price to electricity producers, who are responsible for obtaining the necessary emission allowances to be able to produce that

Figure 1
Schematic description of the model



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[7] A future version of the model could separate the electricity sector into renewables and non-renewables. The biggest challenge in this woul be the calibration, since input-output tables don't have that level of disagregation. In the meantime, in the current version electricity generation generates emissions at the average rate observed in each economy, which is a good description of the current situation but doesn't allow for substitution channels within the electricity sector as a response to higher carbon taxes; in time horizons of one to three years, for which the model is mostly used, such substitution would be expected to be very limited.

electricity. In the model, though, both sectors function in the same way: energy users pay a gross price that includes the electricity or fuel itself along with the emission allowances required to produce or consume it, and energy producers receive a net price from which the cost of these emission allowances has already been deducted. The fitting of the model to the data resolves this divergence between the real-world and model structures: the fuel price in the real world corresponds to its net price in the model, while the electricity price in the real world corresponds to its gross price in the model.

An additional issue related to this is that the structure in the model is a carbon tax, whereas the ETS system is not. Under certain assumptions, they could be equivalent, e.g. if emission rights are sold by the government in carefully-calibrated auctions so that it effectively sets the price instead of the quantity. More broadly, the structure in the model would correctly represent the current ETS system up to first order, but there is a second-order channel that would be missing: the endogenous fall in the price of ETS allowances when a shock makes agents in the model want to use less energy and reduce their greenhouse gas emissions. This could cause a second-order upwards bias in the simulations of the model (i.e. real-world effects could be slightly smaller than what the model predicts). A priori, this bias should be small. Further versions of the model may address this issue.

In the standard specification of the model, the revenue that the carbon tax generates for the public sector is given back to households in the form of lump sum transfers. Even though it is not out of line with the early stages of the ETS sytem, this is a relatively pessimistic assumption: it could also be used for reducing other distortionary taxes. For example, if it is used to reduce a proportional tax on labor income, it would induce a positive supply shock and can generate a positive aggregate effect on GDP and employment, as shown at the end of the section that presents the simulations. Even though these optimistic alternatives can still generate a negative impact on specific sectors, we only consider the lump sum option when generating scenarios for climate change stress tests, where there is an explicit aim of generating a sizeable but plausible fall in economic activity in the simulations.

Next, we summarize the decision problem of the different agents in this economy here, and Appendix A contains the full problem solved by each agent. For simplicity of exposition, we omit the time subscripts of the variables.

2.1 Household

The household chooses consumption of the final aggregate product, C, and the amount of labor to provide, L, to maximize its utility over time subject to a budget constrain.

$$max_{C_t} E_t \sum_{t=0}^{\infty} \beta^t \frac{C^{1-\sigma}}{1-\sigma} - \Upsilon \frac{L^{1+\vartheta}}{1+\vartheta}$$

$$s.t.\ PC = (1-\tau_l)WL + T,$$

2.2 Non-energy production sector

The production of the non-energy sector is determined by a CES function that uses energy, E_s, labor, L_s and non-energy intermediate input, H_s, that is sold to the non-energy consumption retailer, and the N_s non-energy intermediate-input retailers, $Z_s = C_s + \sum_{x=1}^{s} H_{x,s}$. The production function is the following:

$$Z_s = \left[\omega_{E,s}^{\frac{1}{\upsilon_E}} E_s^{\frac{\upsilon_E-1}{\upsilon_E}} + (1-\omega_{E,s})^{\frac{1}{\upsilon_E}} \left[L_s^{\alpha_s} H_s^{1-\alpha_s} \right]^{\frac{\upsilon_E-1}{\upsilon_E}} \right]^{\frac{\upsilon_E}{\upsilon_E-1}}$$

2.3 Energy production sector

There are N_e energy production sectors. In each of these sectors, denoted by the subscript e, production of energy is determined by the imported energy input from abroad, M_e

$$Z_e = M_e. \tag{1}$$

The energy good Z_e is sold to the consumption good consumption retailer, C_e , and to each of the energy intermediate-input retailer, $E_{e,s}$ (N_s):

$$Z_e = C_e + \sum_{s=1}^{N_s} E_{e,s} \tag{2}$$

2.4 Non-energy intermediate-input retailers

For each non-energy sector (N_s), there is a retailer that purchases the goods $H_{x,s}$ from each of the non-energy producers (N_s), and aggregates them into N_s non-energy intermediate-input bundle, H_s, using a Cobb-Douglas function with constant returns to scale:

$$H_s = \prod_{x=1}^{N_s} H_{x,s}^{\omega_{H,x,s}}, \ \sum_{x=1}^{N_s} \omega_{H,x,s} = 1,$$

where $H_{x,s}$ is the intermediate input produced from sector x and then used by sector s ($N_s x N_s$).

2.5 Energy intermediate input retailers

For each of the N_s non-energy sectors there is a retailer that purchases the energy intermediate goods e_e from the N_e energy sectors and aggregates them into an energy intermediate-input bundle for each non-energy sector, E_s, using a CES function as follows:

$$E_{s,t} = \left[\sum_{e=1}^{N_e} \omega_{e,s}^{\frac{1}{\upsilon_{H,E}}} e_{e,s}^{\frac{\upsilon_{H,E}-1}{\upsilon_{H,E}}} \right]^{\frac{\upsilon_{H,E}}{\upsilon_{H,E}-1}}, \quad \sum_{e=1}^{N_e} \omega_{e,s} = 1$$

where $\upsilon_{H,E}$ is the elasticity of substitution across different energy intermediate goods.

2.6 Non-energy consumption retailer

The non-energy retailer purchases the goods C_s from the 51 non-energy produces and aggregates them into an unique non-energy consumption bundle, C_{NE},using the following CES function:

$$C_{NE} = \left[\sum_{s=1}^{N_s} \omega_{C,s}^{\frac{1}{\upsilon_{C,s}}} C_s^{\frac{\upsilon_{C,s}-1}{\upsilon_{C,s}}} \right]^{\frac{\upsilon_{C,s}}{\upsilon_{C,s}-1}} \quad \sum_{s=1}^{N_s} \omega_{C,s} = 1, \tag{3}$$

where $\upsilon_{C,s}$ is the elasticity of substitution across different non-energy final consumption goods. From the maximization problem, we obtain

$$P_s = \left(\omega_{C,s} \frac{C_{NE}}{C_s} \right)^{\frac{1}{\upsilon_{C,s}}} P_{NE}. \tag{4}$$

2.7 Energy consumption retailer

There is a retailer purchases the energy consumption goods, C_e, from the two kinds of energy producers (coke and refined petroleum and electrictiy and gas) and aggregates them into an energy consumption bundle, C_E,using the following CES function:

$$C_{E,t} = \left[\sum_{e=1}^{N_e} \omega_{C,E}^{\frac{1}{\upsilon_{C,E}}} C_e^{\frac{\upsilon_{C,E}-1}{\upsilon_{C,E}}} \right]^{\frac{\upsilon_{C,E}}{\upsilon_{C,E}-1}}, \quad \sum_{e=1}^{N_e} \omega_{C,E} = 1$$

where $\upsilon_{C,E}$ is the elasticity of substitution across different enegery consumption goods (i.e., energy consumption of different types are substitute goods, so that carbon tax can lead to a rise of "greener" energy sectors). From the profit maximization problem, and using the fact that there are $N_e = 2$ energy sectors in our calibration of the model, we obtain the relative prices between their products (see Appendix A.3 for details):

$$\frac{P_{e1}}{P_{e2}} = \left(\frac{\omega_{C,E1}}{\omega_{C,E2}} \right)^{\frac{1}{\upsilon_{C,E}}} \left(\frac{C_{e1}}{C_{e2}} \right)^{\frac{1}{\upsilon_{C,E}}} C_E, \tag{5}$$

and the price of the final consumption good becomes:

$$P_{E,t} = \left[\sum_{e=1}^{N_e} \omega_{C,E} P_{e,t}^{\frac{1}{\upsilon_{C,E}}} \right]. \tag{6}$$

2.8 Consumption retailer

The consumption retailer purchases the consumption energy good, C_E,from the energy consumption retailer and the non-energy consumption good, C_{NE}, from the non-energy consumption retailer and aggregate them into the final consumption good sold to the household under a CES production function:

$$C = \left[\omega_c^{\frac{1}{\upsilon_C}} C_{E,}^{\frac{\upsilon_C - 1}{\upsilon_C}} + (1 - \omega_c)^{\frac{1}{\upsilon_C}} C_{NE}^{\frac{\upsilon_C - 1}{\upsilon_C}} \right]^{\frac{\upsilon_C}{\upsilon_C - 1}},$$

where $\upsilon_C$is the elasticity of substitution between energy and non-energy consumption. From the profit maximization problem, we obtain the relative prices between energy and non-energy consumption goods (see Appendix A.2 for details on the derivations):

$$\frac{P_E}{P_{NE}} = \left(\frac{\omega_C}{1 - \omega_C} \right)^{\frac{1}{\upsilon_C}} \left(\frac{C_E}{C_{NE}} \right)^{\frac{1}{\upsilon_C}} C, \tag{7}$$

and the price of the final consumption good becomes:

$$P = \left[\omega_c P_E^{\frac{1}{\upsilon_C}} + (1 - \omega_c) P_{NE}^{\frac{1}{\upsilon_C}} \right]. \tag{8}$$

2.9 Closing the model

Taxes: government collects all the revenue from the carbon tax, and pays it back to the household either in as lump-sum transfers,

$$T = \sum_{s=1}^{N_e} \tau_e P_e Z_e. \tag{9}$$

Or, in an alternative specification, through a reduction in labor taxes,

$$-\tau_l W L = \sum_{s=1}^{N_e} \tau_e P_e Z_e. \tag{10}$$

Labor Market: The demand for labor equals the supply of labor,

$$L = \sum_{s=1}^{N_s} L_s. \tag{11}$$

Summary of agents and solution of the model: with $N_s = 51$ and $N_e = 2$, there are a total of 159 agents interacting in this economy: 1 representative household, 51 non-energy producers (who use employment, a basket of different energy intermediate products and a basket of different non-energy intermediate products), 2 energy producers (who use imported basic energy products), 51 energy intermediate product retailers (each of which combines 2 energy products, fuels and electricity), 51 non-energy intermediate product retailers (each of which combines 51 non-energy products), 1 energy consumption aggregator (who combines two products, fuels and electricity), 1 non-energy consumption aggregator (who combines 51 non-energy products), and 1 consumption aggregator (who combines two products, an energy bundle and a non-energy bundle). Computing the model equilibrium requires finding the 159 prices and the almost 3,000 quantities that simultaneously satisfy the optimality conditions of all these agents and the economy's aggregate constraints.

3 Calibration

One of the main features of the model is its detailed sectoral breakdown: given the fact that the risks associated with climate change have a very marked asymmetric component in this respect, it is essential for the model to be capable of capturing both the characteristics of each sector in terms of the use of energy, and also the interrelations between sectors. Table 1 sets out the sectoral breakdown currently used by the model, which corresponds to 2-digit NACE classification: 51 non-energy sectors and 2 energy production sectors ("fuels" and "electricity").

The model is calibrated with observed data for Spain in 2015[8]. Figure 3 shows how the model precisely replicates the share of each sector in final consumption[9] and replicates reasonably well (but not exactly, owing to the simplifications involved in the stylised form of the aggregator and production functions) the share of each non-energy sector in total energy used as input in production of other goods and services, and the relative size of the various industries in terms of value-added and production. Furthermore (not shown in the figure), the model also matches the full set of trade relationships for the Spanish economy summarized in the input-output table.

Apart from this economic data published by INE (the Spanish National Statistics Institute), the calibration of the model also uses a lot of information relating to the use

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[8] 2015 is the most recent year for which a suitable input-output table is available both for the Spanish economy and for the euro area, which allows for a homogenous alternative calibration that can highlight the effect of differences in the structure of the economy.

[9] Since the model is of a closed economy without capital, the concept of "household consumption" in the model is actually matched to the sectoral data for the sum of consumption plus gross fixed capital formation plus exports.

of energy, the taxation of emissions of greenhouse gases, etc. The main sources for this information are the industry CO_2 atmospheric emission accounts published by the INE (National Statistics Institute), and the EU Emissions Trading System (ETS) dashboard published by the European Environment Agency (EEA), which, apart from including their own information, also sumarizes data coming from the European Union Transaction Log (EUTL), the European Energy Exchange (EEX) and the Intercontinental Exchange (ICE).

That information is used for calibrating the structure of the carbon tax included in the model. For electricity, the tax rate is obtained from the relationship between the value of the emission allowances surrendered by the electricity production sector and

Table 1
Sectors in the model

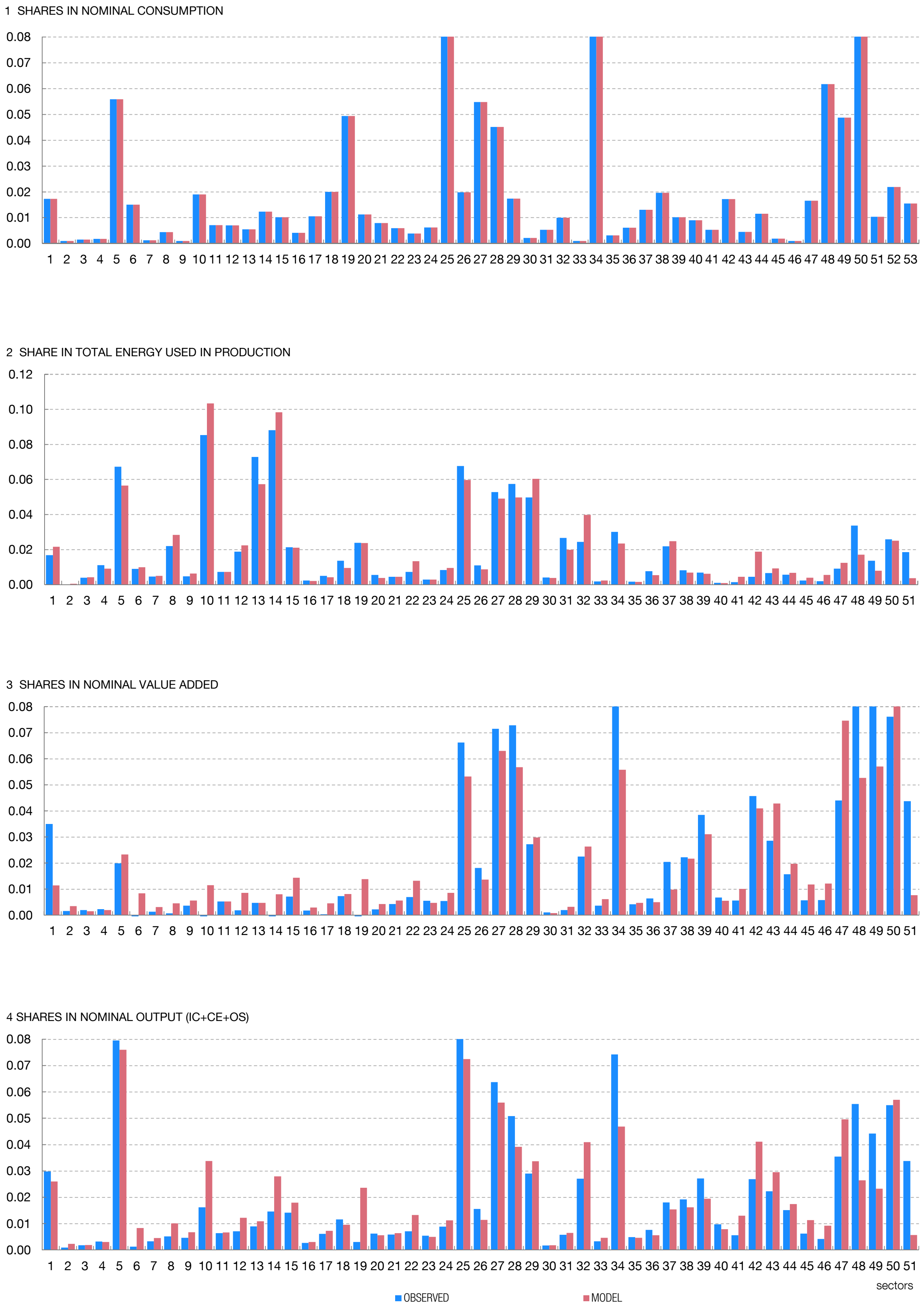
Non-energy sectors

1 Crop and animal production	27 Other wholesale trade
2 Forestry and logging	28 Other retail trade
3 Fishing and aquaculture	29 Land transport
4 Mining and quarrying	30 Water transport
5 Manufacture of food, beverages and tobacco products	31 Air transport
6 Manufacture of textiles, wearing apparel, leather	32 Warehousing & support activities for transportation
7 Manufacture of wood and wood products, except furniture	33 Postal and courier activities
8 Manufacture of paper and paper products	34 Accommodation and food service activities
9 Printing and reproduction	35 Publishing activities
10 Manufacture of chemicals and chemical products	36 Motion picture, video, television, music and radio
11 Manufacture of pharmaceutical products	37 Telecommunications
12 Manufacture of rubber and plastic products	38 Computer programming and information services
13 Manufacture of other non-metallic mineral products	39 Financial services, except insurance and pensions
14 Manufacture of basic metals	40 Insurance and pension funding
15 Manufacture of fabric. metal products, exc. mach. & equip.	41 Auxiliary activities to financial services
16 Manufacture of computer, electronic and optical products	42 Real estate activities
17 Manufacture of electrical equipment	43 Legal and accounting activities
18 Manufacture of machinery and equipment	44 Architectural and engineering activities
19 Manufacture of motor vehicles	45 Advertising
20 Manufacture of other transport equipment	46 Other professional services
21 Manufacture of furniture; other manufacturing	47 Administrative services
22 Repair and installation of machinery and equipment	48 Public administration and social security
23 Water collection, treatment and supply	49 Education
24 Sewerage & waste collection, treatment & disp. activities	50 Health
25 Construction	51 Other service activities
26 Wholesale and retail trade and repair of motor vehicles	

Energy sectors

52 Manufacture of coke and refined petroleum products	53 Electricity, gas, steam and air conditioning supply

Figure 2
Model calibration: fitting of the sectoral data.



1 SHARES IN NOMINAL CONSUMPTION

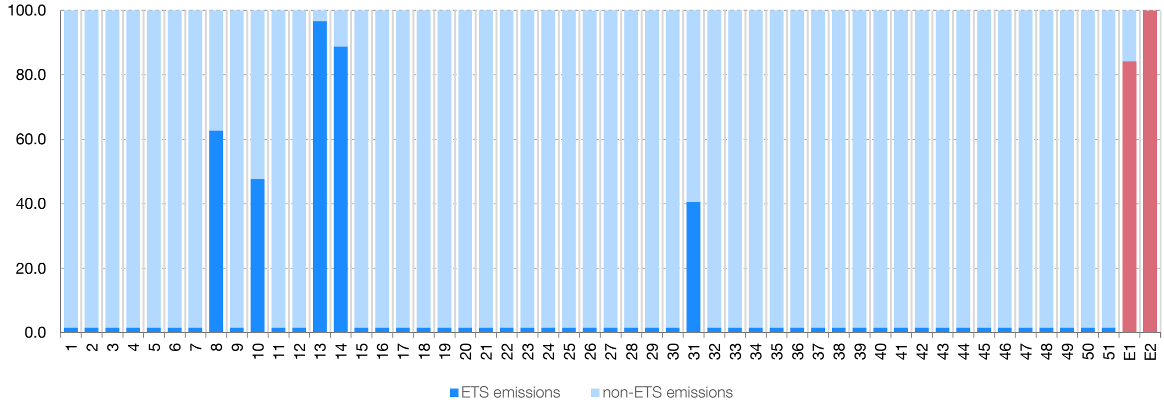
2 SHARE IN TOTAL ENERGY USED IN PRODUCTION

3 SHARES IN NOMINAL VALUE ADDED

4 SHARES IN NOMINAL OUTPUT (IC+CE+OS)

Figure 3

Share of emissions that are covered by the ETS system, by sector



the sectors' aggregate revenues net of these rights; it is therefore assumed that all non-energy sectors pay the same homogeneous carbon tax when they use electricity. In the case of fuels, the carbon tax rate is sector-specific: is estimated using the relationship between the value of the emission allowances surrendered by each non-energy sector, and the sector's expenditure in fuels as intermediate inputs, net of the allowances it uses. This implies that that carbon tax structure in the model is the multiplication of three components:

- A technological component that captures the amount of emissions per unit of energy consumed.

- A regulatory component that captures the ratio of surrendered emission rights to total emissions.

- A pricing component that is just the price of surrendered emission rights, in euros per equivalent tonne of CO_2.

Figure 3 shows the regulatory component of each sector. As can be seen, as of 2022, only the firms from a few select sectors have to surrender ETS rights when they emmit greenhouse gases, meaning that most regulatory coefficients are zero, and only a few sectors have high regulatory coefficients, mainly a few industrial sectors with big factories generating big amounts of emissions, and airlines.

In model, electricity is a homogeneous good, meaning that the technological and regulatory components related to electricity are identical across sectors. This is not the case for fuels, where each sector has a different technological and regulatory component. The pricing component is common to all agents and energy types. All of these components are calibrated using observed data for 2019.

The most difficult parameters to calibrate in this model are the ones that do not relate to the static structure of the economy, but to the speed and degree of adjustment in response to shocks or policy changes. This is the case of the numerous parameters in aggregator and production functions that control the degree of substitution between different goods. These parameters are calibrated at values bigger than zero but usually smaller than one, indicating that some – albeit limited – substitution between goods is to be expected in response to a shock. Their value must depend on the simulation horizon: under the current calibration, that assumes a common elasticity of sustitution for all sectors, a rise in the price of emission allowances would not be expected to lead to significant substitution between fuels and electricity in any given sector (e.g. in the road transport sector) within a 3-year period, but a strong substitution could be expected if the relevant time horizon is 10 or 20 years.

Different approaches have been used to select the value for each of these elasticity-of-substitution parameters. In the case of non-energy inputs used in the production of non-energy goods, since the functional form used for the aggregator is a Cobb-Douglas, substitution is one-to-one, meaning that the quantities react proportionately to the relative-price changes, so that the nominal weight of the different sectors in the basket of intermediate products acquired by each non-energy producer remains constant. This is imposed in order to simplify the biggest block in the model, and make computation simpler; this assumption could be revised in the future in order to reduce the degree of substitution in this part of the model.

For the elasticity of substitution between non-energy goods in consumption ($\upsilon_{C,s}$), a slightly lower value of 0.9 is selected, following the literature: this is the value used by Atalay (2017), Baqaee and Farhi (2019), and Allen et al. (2020).

The elasticities of substitution involving energy are calibrated through an empirical exercise. This includes the elasticity of substitution between energy and non-energy inputs in production (υ_E), between different energy inputs in production ($\upsilon_{H,E}$), between energy and non-energy goods in consumption (υ_C), and between different energy goods in consumption ($\upsilon_{C,E}$); given the limitations of the exercise, these four parameters are assumed to share a single value. This data-matching exercise ended up selecting values that are lower than what is often used in the literature; this is somehow to-be-expected, as this model is being used for shorter time horizons than is common among its peers, and substitution is more difficult in the short term. With all other elasticities fixed at the values detailed above[10], the model is used to simulate a 32.5% fall in the price of oil in euros. This corresponds to the observed fall between 2014 and 2016 (which was 46.3%), adjusted by a factor of 0.7 to compensate for the fact that oil makes 100% of the input of the fuel sector in the model, but only 70% in the real world. We simulate the shock with the model using different values for the elasticities of substitution involving energy, and the results for each parameter value are compared with the observed data

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[10] This empirical methodology could not be used for the non-energy elasticity of substitution, as the signal-to-noise ratio in the response to an oil price shock is too low in these sectors: a 46% fall in oil prices can be expected to be the biggest shock for the fuels and electricity sectors over a period of three years, but may be just another factor affecting the telecommunications or advertising sector.

in terms of the cumulative change in nominal production of the energy sectors between 2014 and 2017[11]. A value of 0.25 for all the elasticities of substitution involving energy is the one that most closely matches the evolution observed in the data, in the sense that it minimizes the unweighted distance from the simulated responses of nominal production of the two energy sectors to the observed ones, and that is the value we select for simulations with a time horizon of approximately three years (see Table 3). Bigger values for these parameters (which are common in the literature) would imply a much bigger substitution in real terms, and a fall in nominal terms that would be much smaller than what was observed in Spain in this episode; this was to be expected, because these higher values used in the literature usually refer to longer time horizons than just three years.

The rest of the parameters in the model take standard values from the literature. This is the case of the relative risk aversion (with a value of one) and the Frisch elasticity of labor (also one).

Table 2
Simulation exercise used for calibrating the elasticity of substitution parameters involving energy

	Fuels sector	**Electricity sector**	**distance**
Observed (2014-2017)	**-19.3**	**-18.8**	
Elasticity:			
0.10	-28.3	-8.1	9.91
0.15	-26.8	-7.6	9.57
0.20	-25.3	-7.1	9.32
0.25	**-23.9**	**-6.7**	**9.21**
0.30	-28.2	-8.1	9.89
0.35	-21.1	-5.7	9.33
0.40	-19.5	-5.3	9.60
0.45	-18.0	-4.8	9.98
0.50	-16.4	-4.3	10.48
0.55	-14.8	-3.8	11.09
0.60	-13.3	-3.3	11.77
0.65	-11.6	-2.8	12.59
0.70	-10.0	-2.3	13.44

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[11] It is therefore a two-years-shock, evaluated at a three-years-horizon; longer time frames can't be used regarding this episode because the oil price grew back in the following years. Since the model can't pin down prices, the observed growth of the GDP deflator is added to the simulation results before comparing with the data for nominal production. Using sector-specific value added deflators, or comparing value added instead of production, or real variables instead of nominal variables, didn't seem adequate, as these more complex measures, which involve more granular data or indirectly-constructed data, are very volatile in the National Accounts data: nominal energy production fell approximately 21-22% in both energy sectors between 2014 and 2017, which seems reasonable; but, at the same time, nominal value added grew almost 300% in the fuels sector and fell slightly in the electricity sector; real value added grew 640% in the fuels sector, and just 3.8% in the electricity sector. It wouldn't be a good idea to make the model relicate these extremely noisy dynamics, so the more moderate measure for nominal production is used.

4 Simulation exercises

4.1 An increase of 10% in the international price of oil and natural gas

Here we present a simulation that is similar to the one used in the previous section for calibrating the elaticities of substitution involving energy, but now with more details about the results, and using a shock with a standardized size: instead of replicating the change observed in a real-world episode, now it is a 10% increase in the international prices of oil (which is the input used by the fuels sector in the model, and the primary one in the real world too) and natural gas (which is the input used by the electricity sector in the model, and the primary one in the technology that often sets the marginal price of electricity in the real world). Given the naïve production function that the energy sectors have in the model, this is equivalent to an exogenous increase in the price of fuels and electricity, but with higher rents flowing towards the rest of the world instead of an increase in national profits. Table 3 summarizes the results of the simulations in terms of the main macroeconomic aggregates, and Figure 4 details the response of the real value added of the 53 sectors included in the model.

Table 3
Simulation of a 10% increase in the price of oil and natural gas

	% change
Real GDP	-0.31
Real consumption	-0.47
Employment	-0.44
Use of fuels	-2.4
Use of electricity	-2.5
Emissions	-2.6

After this increase in the international price of all energy inputs, the model predicts a fall in real GDP of 0.31% in three years, which is in line with the results from other macroeconomic models for the Spanish economy after three years[12]. As firms reduce their energy intensity, the use of both fuels and electricity falls significantly more than

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[12] For example, in simulations with MTBE, the main macro model used in the projections for the Spanish economy elaborated by Banco de España, a 10% increase in the price of oil generates a fall in GDP of 0.25% after three years (see Arencibia et al., 2017), and adding the effect of an increase in the price of natural gas would increase this figure.

other inputs, and energy sectors display the highest reductions in real output[13]. Among non-energy sectors, the biggest impact is experienced by some sectors that have high energy intensity (e.g. land, water and air transport) and also by those that receive particularly big indirect effects (e.g. warehousing and support for transportation, which has a very low energy intensity but has close commercial ties with the transport sectors).

4.2 A increase in the price of greenhouse gas emissions

The main use of the model is to evaluate the efects on the Spanish economy of changes in the EU Emissions Trading System (EU ETS), which is the scheme that the carbon tax in the model tries to replicate, as detailed in the calibration section. Table 4 and Figure 5 show the results of a simulation in which the price of emission allowances

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[13] Since in the model these sectors have a naïve production function Y=E, there is no value added for them, either before or after the shock. Thus for energy sectors, the figure shows the percentage fall in real production, which under alternative simplistic assumptions would equal the percentage fall in real value added. The aim is just to showcase that there is a bigger fall in these sectors than in non-energy sectors, as would be seen in an additional figure representing the effect on sectoral production, which we don't show to reduce clout.

Figure 4
Simulation of a 10% increase in the price of oil and natural gas

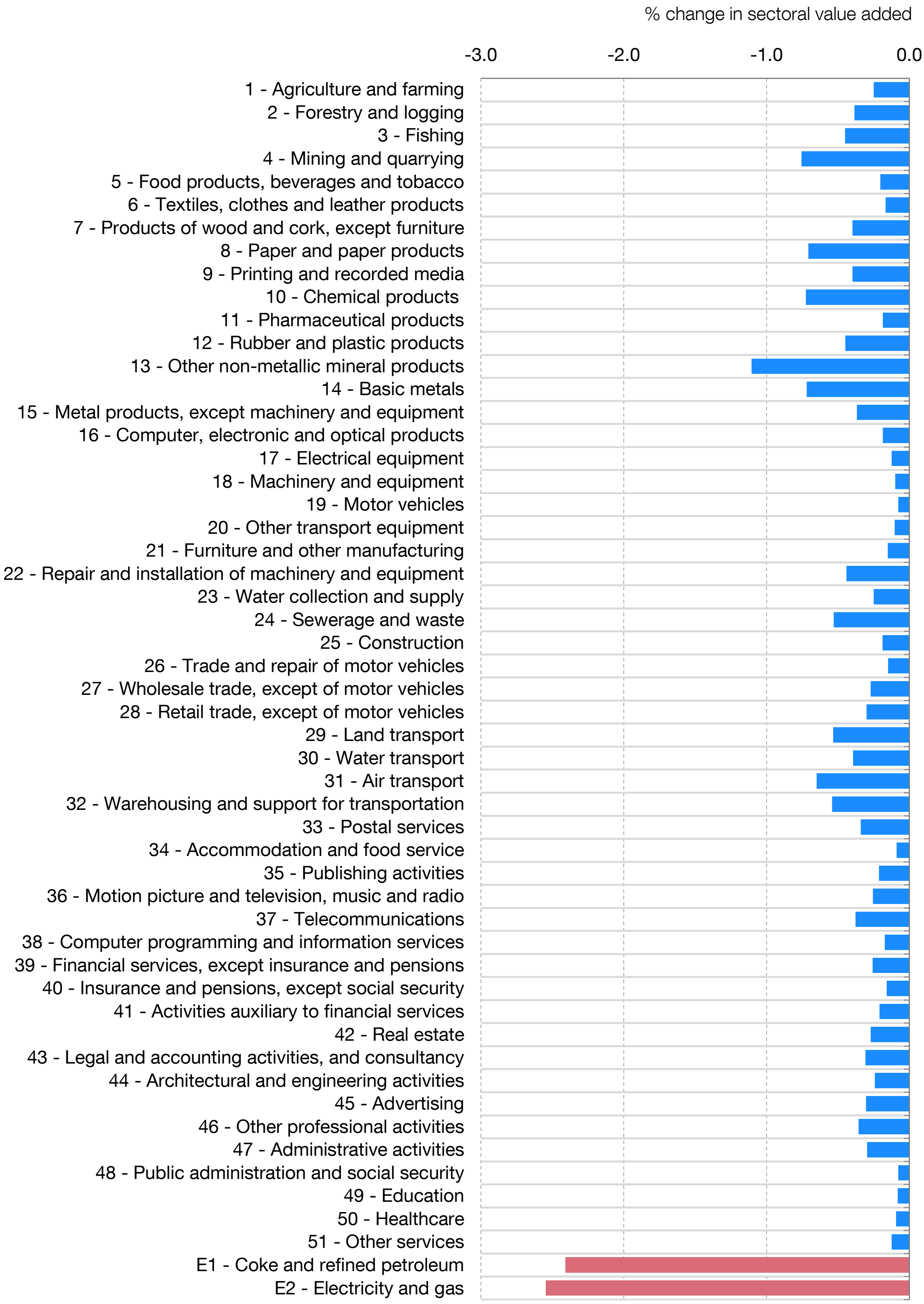

Table 4
Simulation of an increase in the price of CO_2 emissions, from 25 to 100 euros per tonne

	% change
Real GDP	-0.37
Real consumption	-0.63
Employment	-0.58
Use of fuels	-5.8
Use of electricity	-3.1
Emissions	-9.7

changes from 25 euros per ton of CO2 (which was approximately the average price in 2020) to 100 euros in 2023 (it has already been hovering above 80 for most of 2022). In the simulation, the average rate of the carbon tax (the total ETS costs paid by firms, including for electricity generation, divided by the total net cost of their energy intermediate inputs) goes from 3.6% to 14.2%[14].

As emissions become more expensive, firms reduce their energy intensity, with a 9.7% fall in emisssions, a 0.37% fall of GDP, and a reduction of employment of 0.58%. The increase in the cost of emissions affects both energy sectors, but the resulting reduction in the use of fuels is slightly larger than for electricity. And there are heterogeneous sectoral effects: Figure 6 plots the relationship between the impact on sectoral real value added and both initial carbon taxes paid per unit of output, and initial greenhouse gas emissions per unit of output (measured as the potential ETS that the sector would have to pay if all of its emissions were taxed at the initial rate, as a percentage over total production costs). In this simulation, the non-energy sectors that suffer the biggest fall in real value added are not necessarily those generating the most emissions, but those that were covered by the ETS system in the initial situation. As in most simulations with CATS, the model also identifies some sectors that suffer a big impact even if they are not directly affected by the shock; this is the case, for example, of printing and recorded media, that buys a lot of inputs from paper manufacturers, and of repair and

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[14] The focus on relatively big shocks is in line with the purpose of the model of serving as a tool to generate climate change stress test scenarios that help Banco de España evaluate the exposure of the financial system to transition risks in time horizons of approximately three years.

Figure 5
Simulation of an increase in the price of CO_2 emissions, from 25 to 100 euros per tonne

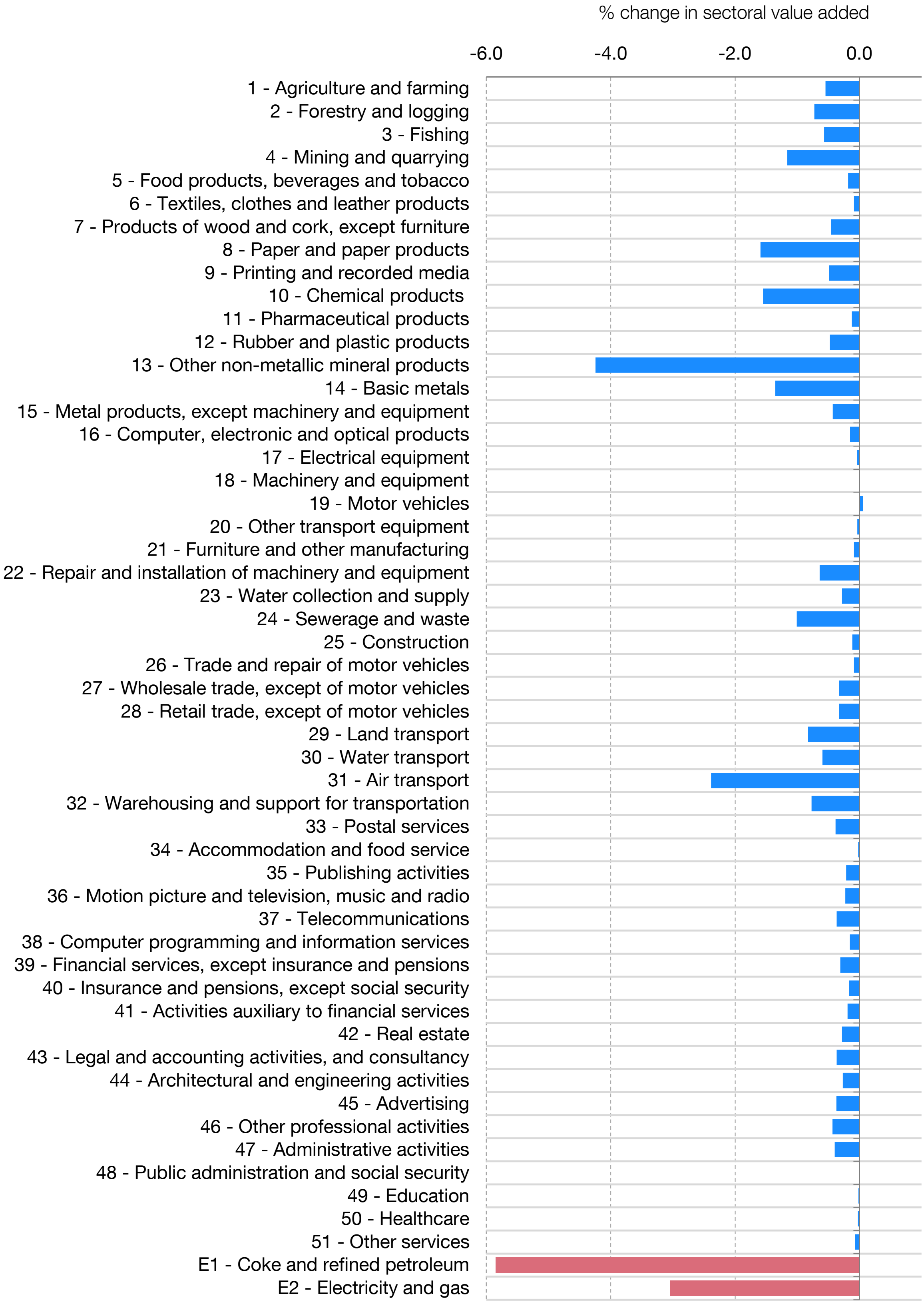
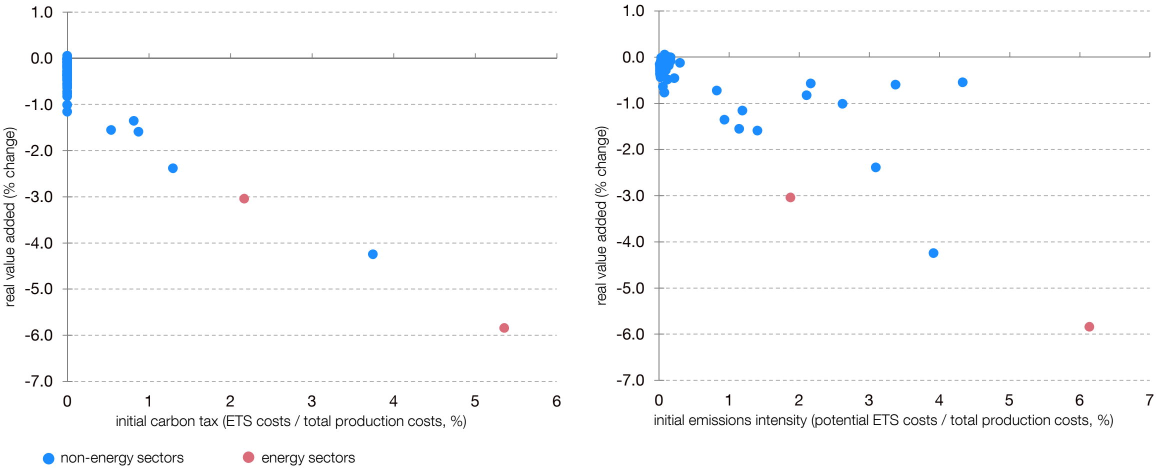


Figure 6
Simulation of an increase in the price of CO_2 emissions, from 25 to 100 euros per tonne



installation of machinery and equipment, that sells a lot of their products and services to various chemical and metal manufacturing sectors. On the other hand, sectors such as agriculture and fishing, that have relatively high emissions intensity, are not particularly affected by this shock, since their regulatory ETS coverage is low.

4.3 An expansion of the coverage of the ETS system to all sectors

A second way of increasing the carbon tax in the model is to extend the coverage of the ETS system, so that all emissions by all sectors are taxed. As of 2022, only the firms from a few select sectors have to surrender ETS rights when they emmit greenhouse gases. In the section about the calibration of the model this was reflected as a low or zero regulatory coefficient for most sectors, and a non-zero one for some, mainly a few industrial sectors with big factories generating big amounts of emissions, and airlines. The simulation in this section keeps the price of greenhouse gas emissions at 25 euros per ton of CO2, but increases coverage so that all emissions from all sectors are taxed at this price (all regulatory coefficients are set to one). The results of this simulation are summarized by Table 5 and Figures 7 and 8.

Compared with the previous simulation (the increase in price), this one represents a smaller shock if we look at metrics such as the effect on GDP (-0.12%), but it generates a bigger fall in emissions (-14.5%), because its impact is focused on some secors with

Table 5
Simulation of an expansion of the coverage of the ETS system, to fully cover all emissions from all firms, of all sectors

	% change
Real GDP	-0.12
Real consumption	-0.24
Employment	-0.19
Use of fuels	-4.4
Use of electricity	-0.5
Emissions	-14.5

relatively high emissions that are currently exempt from the carbon tax, and therefore only responded indirectly to the shock in the previous subsection. One important channel through which this simulation achieves bigger reductions in emissions with lower cost in terms of GDP or employment, is electrification: whereas the increase in the price of emissions generated a relatively similar reduction in the use of fuels and electricity, this expansion of the coverage of the ETS system affects the cost of using fuels to a bigger extent, and induces a substitution towards electricity. The rate at which this is feasible is controlled by the elasticities of substitution that were discussed in the section about the calibration of the model, and most notably $\upsilon_{H,E}$, which controls the rate at which one energy input is substituted for another when their relative prices change.

In terms of the non-energy sectors most affected by this shock, they tend to be those with higher emission intensity (agriculture, fishing, transport, sewerage and waste), minus the ones that are already highly covered by the ETS system (chemicals, basic metals and non-metallic products, that display a sizeable effect but not as much as their emissions intensity would suggest), plus some that are indirectly hit through their commercial relations with other highly-affected sectors (warehousing and support for transportation, repair and installation of machinery and equipment, water collection and supply, etc).

Figure 7
Simulation of an expansion of the coverage of the ETS system, to fully cover all emissions from all firms, of all sectors

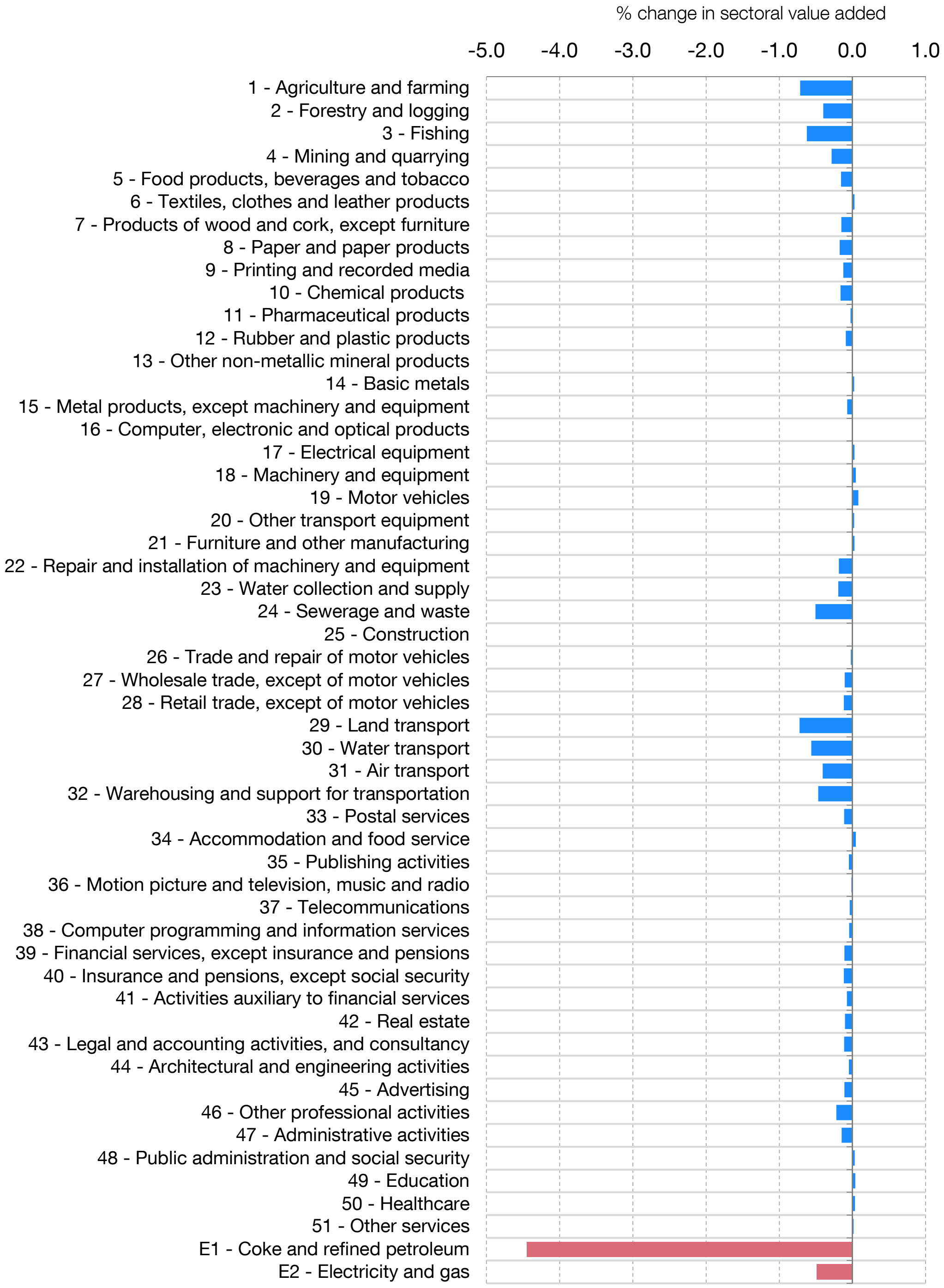


Figure 8
Simulation of an expansion of the coverage of the ETS system, to fully cover all emissions from all firms, of all sectors

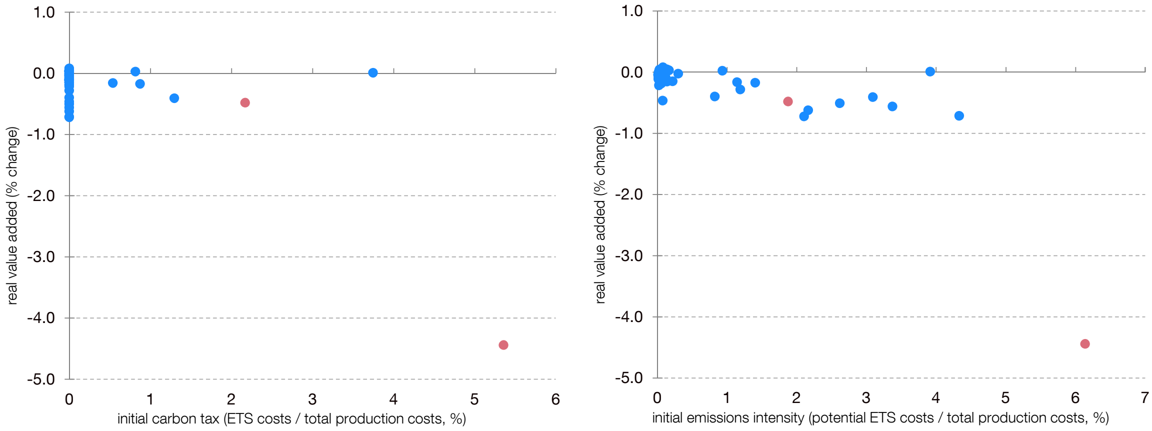


Table 6
Simulation of an increase in the price of CO_2 emissions, to 100 €/ton, plus an expansion of the coverage, to all firms in all sectors

	% change
Real GDP	-0.90
Real consumption	-1.52
Employment	-1.27
Use of fuels	-15.9
Use of electricity	-4.8
Emissions	-31.1

4.4 Increase in the price plus expansion of the coverage

When both shocks are implemented at the same time, the total effect is much bigger than the sum of both individual simulations presented above: it is equivalent to first increasing the price, and then, at this higher price, expanding the coverage. Table 6 and Figures 9 and 10 summarize the results from this simulation.

Figure 9

Simulation of an increase in the price of CO_2 emissions, to 100 €/ton, plus an expansion of the coverage, to all firms in all sectors

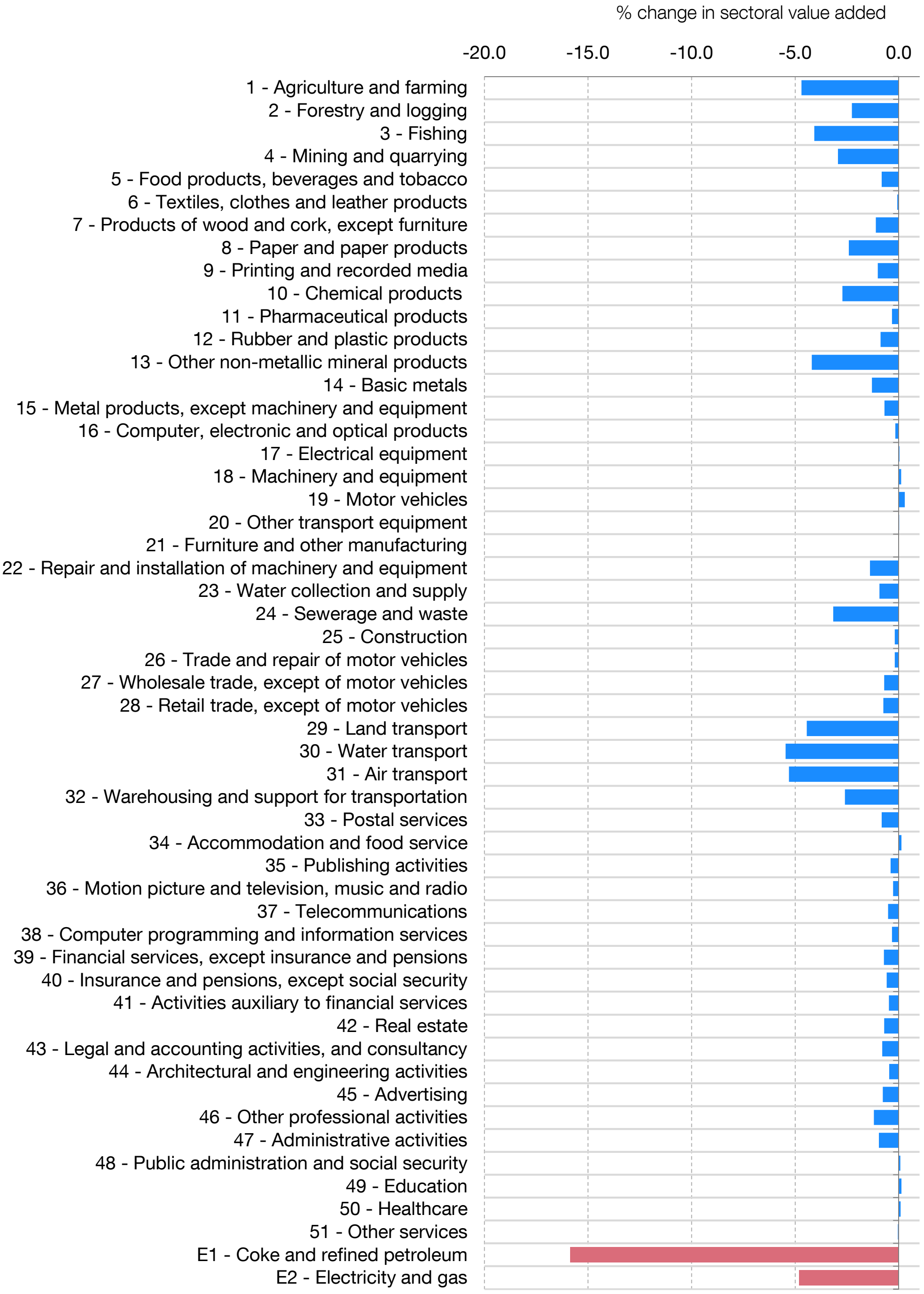
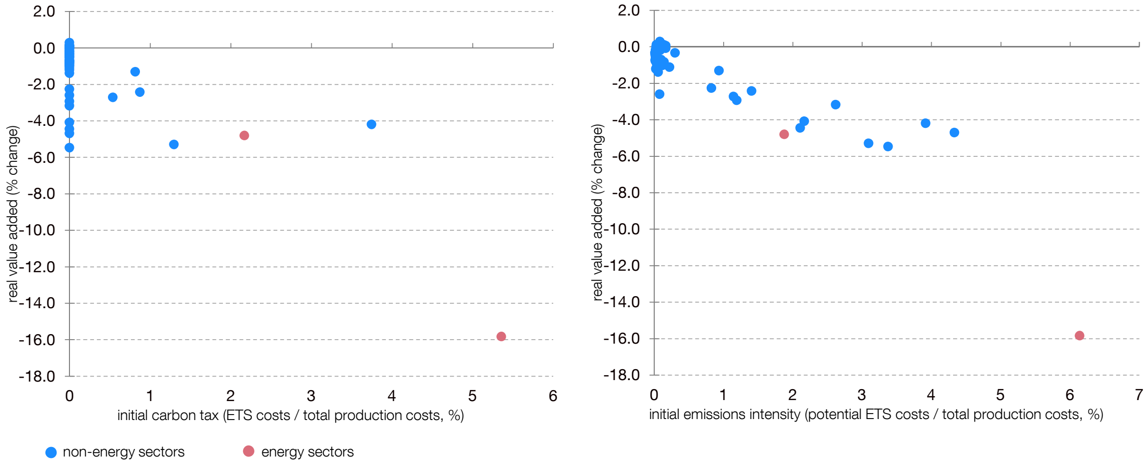


Figure 10
Simulation of an increase in the price of CO_2 emissions, to 100 €/ton, plus an expansion of the coverage, to all firms in all sectors



In terms of the GDP cost of reducing emissions by a given amount, this simulation falls in an intermediate point between the previous two, with 0.03 pp of GDP lost for every percentage point of emissions reduction, against 0.04 pp for the price increase and 0.01 pp for the expansion of coverge. The electrification channel is still present, with a reduction in electricity use that is 3.3 times smaller than the reduction in the use of fuels. In terms of the impact across non-energy sectors, those that have high emissions intensity show an almost-linear relationship, as they are now all directly affected by the shock (both the ones covered and not covered currently by the ETS system), whereas for those with low emissions intensity the size of the effect is determined by their network of commercial relations with other sectors, as defined by the input-output tables that the model captures.

4.5 Increase in the price plus expansion of the coverage, with compensation through lower labor taxes

All the simulations presented above are based on the standard configuration of the model, in which revenue from the carbon tax is used to reduce the lump-sum taxes paid by households. This is a relatively negative assumption: more productive ways of recycling the carbon tax revenue could generate smaller output losses, or even gains, as is common in the literature. For example, see Beiser-McGrath and Bernauer (2019) and Douenne (2020).

Table 7
Simulation of an increase in the price of CO_2 emissions, to 100 €/ton, plus an expansion of the coverage, to all firms in all sectors, with the revenue from those measures used to finance a reduction in labor taxes

	% change
Real GDP	2.47
Real consumption	1.84
Employment	2.09
Use of fuels	-13.0
Use of electricity	-1.6
Emissions	-28.8

In the case of CATS, the standard configuration with revenue-recycling through lump-sum taxes makes sense because the main use of the model is to generate stress test scenarios that try to identify the productive sectors that would be particularly hit in the event of a big negative shock. But the model is also able to generate positive results if the revenue from the carbon tax is used to reduce a proportional labor tax. The differnce with the lump-sum taxes is that in this case there's an additional incentive to expand labor supply[15], and this greater compensation of the direct negative effects of the carbon tax allows a net positive result.

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[15] The size of this effect is controlled mainly by the Frisch elasticity of labor. As explained in the section about the calibration of the model, this parameter is taken from the literature and has a value of one. Sensitivity analysis has been carried out, with results as expected: when households expand their labor supply to a greater extent in response to the reduction in labor taxes, effects become more positive. Even if the size may change, the positive sign of the effects remains for a wide range of reasonable values of this parameter.

Figure 11

Simulation of an increase in the price of CO_2 emissions, to 100 €/ton, plus an expansion of the coverage, to all firms in all sectors, with the revenue from those measures used to finance a reduction in labor taxes

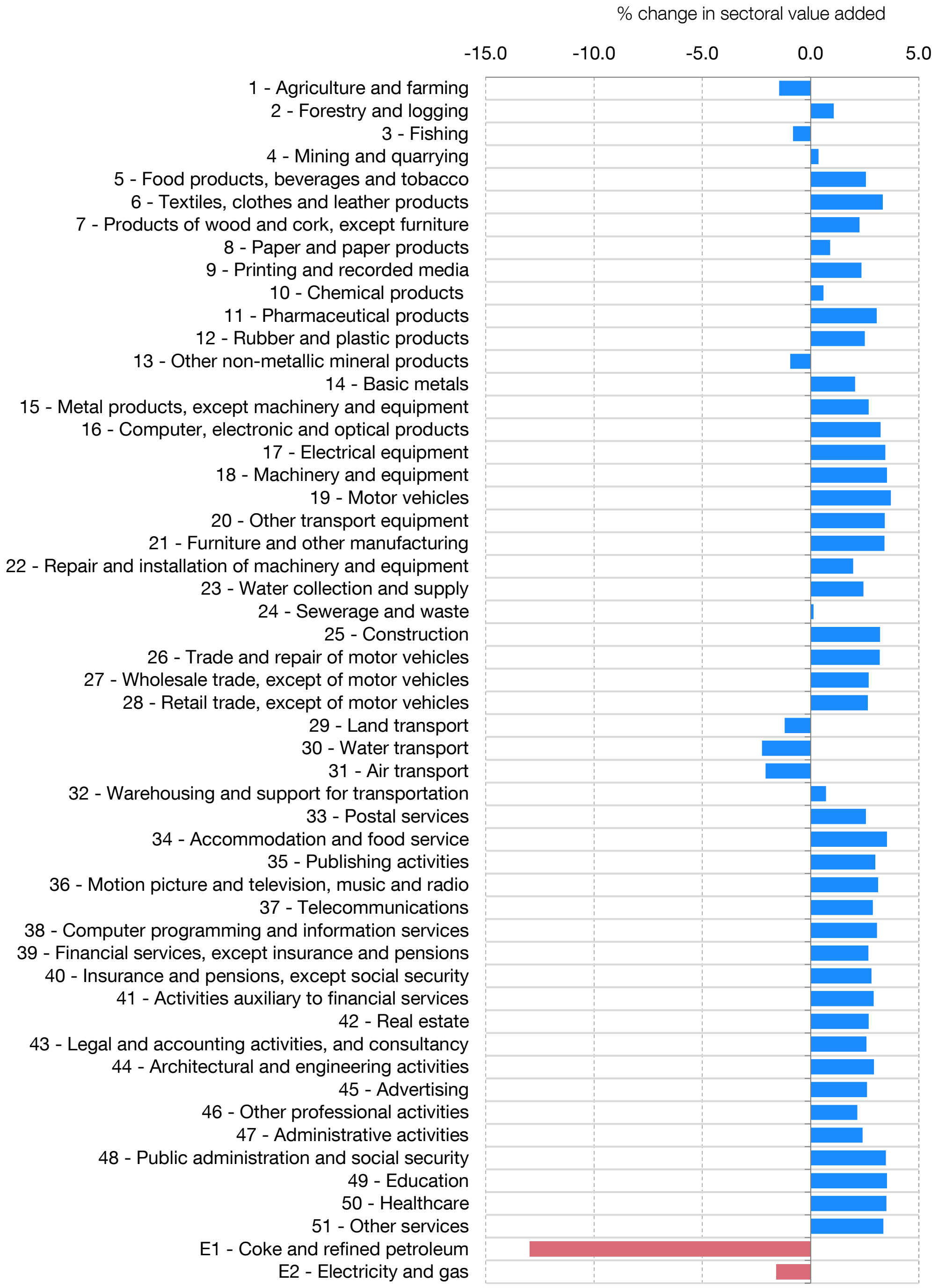
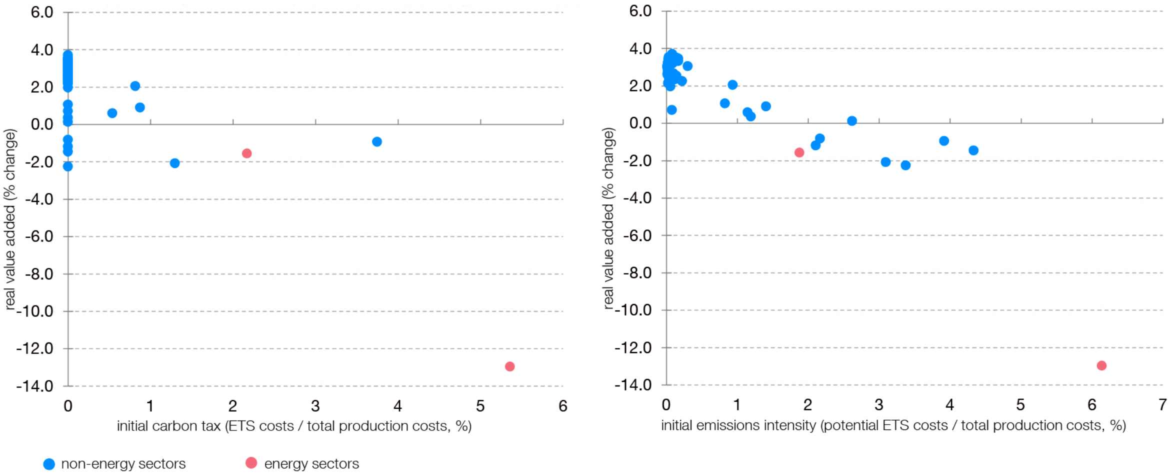


Figure 12
Simulation of an increase in the price of CO_2 emissions, to 100 €/ton, plus an expansion of the coverage, to all firms in all sectors, with the revenue from those measures used to finance a reduction in labor taxes



As seen in Table 7, in aggregate terms the effect on output is now strongly positive. This in turn affects energy use and greenhouse gas emissions: the reduction achieved is smaller than in the case with revenue recycling through lump-sum taxes. But because there's no negative output cost in this case, the fact that the simulation still shows a strong reduction in emissions showcases that, if revenue from a carbon tax is used in ways that reduce inefficiencies and improve productivity or incentives, it can become a revenue-neutral measure that incentivizes growth while reducing grenhouse gas emissions.

Still, figures 11 and 12, where the sectoral effects of this simulation are presented, show that, while the effect can be positive on aggregate and for most sectors, there can still be some sectors with a strongly negative effect. In this case, as the positive effect of increased output supply has a broad positive effect on all the economy, the ones showing negative effects are the same ones that showed a strongly negative effect in the simulation where carbon tax revenues were recycled through lump-sum taxes.

5 Conclusions

Both climate change and the policies implemented to counter it may have negative effects on the economy, which would be transmitted to financial institutions through their exposure to the firms and sectors most affected. We present a model for the Spanish economy, designed as a tool for generating macronomic scenarios that can be used as an initial ingredient in climate-change stress test exercises. The model closely aproximates the productive structure of the Spanish economy and allows reasonably realistic simulations to be formulated.

The model is primarily used to generate medium-term climate stress scenarios, as we focus on capturing transition risks associated with regulatory measues, to assess the different productive sectors' degree of exposure in the event of an increase in the price of emission allowances or an extension of EU-ETS coverage. We show that in the event of an increase in the price of emission allowances similar to that observed in recent years (from approximately €25 per tonne of CO2 in 2019 to almost €100 per tonne in early February 2022), the model predicts a cumulative decline after three years of 0.37% in Spanish GDP. The losses in value added among industries are very heterogeneous, ranging from losses of 4% to virtually no losses. This stresseses how the transitional risks for banks depend on their exposure to the most affected sectors, being a latent threat to financial stability.

Further ahead, the model could be expanded into an open economy model, with imports and exports (including the export of the home produced basic energy good), and to include capital in the production function, enhancing the realism with which the model fits the data and allowing effects on assets used by firms as loan collateral to be incorporated into the simulations. The electricity-generation sector could be divided into renewables and non-renewables, with asymmetric investment allowing a gradual increase in the weight of renewables. There is a lot of work still to be done in terms of enhancing the modelization of climate change issues for the Spanish economy, and the current version of CATS is but a first attempt at Banco de España, to be followed by other, bigger, projects.

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6 Appendix

Appendix

A Further details on the model

The model described in the paper is formed using the first order conditions as detailed in the appendix from households, retailers, the energy and non-energy sectors.

A.1 Household

The household chooses consumption of the final aggregate product, C, and the amount of labor to provide, L, to maximize its utility over time subject to a budget constrain.

$$max_{C_t} E_t \sum_{t=0}^{\infty} \beta^t \frac{C^{1-\sigma}}{1-\sigma} - \Upsilon \frac{L^{1+\vartheta}}{1+\vartheta}$$

$$s.t. \; PC = WL + T,$$

The first order condition associated to the choice of consumption implies the following shadow price $\lambda = {}^{1}\!/_{PC^{\sigma}}$.

A.2 Consumption retailers

The consumption retailer purchases the consumption energy good, C_E,from the energy consumption retailer and the non-energy consumption good, C_{NE}, from the non-energy consumption retailer and aggregate them into the final consumption good sold to the household under a CES production function:

$$C = \left[\omega_c^{\frac{1}{\upsilon_C}} C_E^{\frac{\upsilon_C - 1}{\upsilon_C}} + (1-\omega_c)^{\frac{1}{\upsilon_C}} C_{NE}^{\frac{\upsilon_C - 1}{\upsilon_C}} \right]^{\frac{\upsilon_C}{\upsilon_C - 1}},$$

where $\upsilon_C < 1$is the elasticity of substitution between energy and non-energy consumption (in this case, there is a strong complementarity between energy and non-energy consumption in the production function of the consumption retailer, meaning that the ability of households final good, C_t, to move away from energy consumption is limited.

The demand for the energy and non-energy consumption goods is determinated by the profit maximization:

$$max_{C_E, C_{NE}} P_t C_t - P_E C_E - P_{NE} C_{NE}$$

$$s.t.\, C_t = \left[\omega_c^{\frac{1}{\upsilon_C}} C_E^{\frac{\upsilon_C - 1}{\upsilon_C}} + (1-\omega_c)^{\frac{1}{\upsilon_C}} C_{NE}^{\frac{\upsilon_C - 1}{\upsilon_C}} \right]^{\frac{\upsilon_C}{\upsilon_C - 1}},$$

The respective first order conditions for C_E and C_{NE} are the following:

$$Foc_{C_E} : \; P \left[\omega_c^{\frac{1}{\upsilon_C}} C_E^{\frac{\upsilon_C - 1}{\upsilon_C}} + (1-\omega_c)^{\frac{1}{\upsilon_C}} C_{NE}^{\frac{\upsilon_C - 1}{\upsilon_C}} \right]^{\frac{-1}{\upsilon_C - 1}} \omega_c^{\frac{1}{\upsilon_C}} C_E^{\frac{-1}{\upsilon_C}} = P_E,$$

If we evaluate this expresion to the power of υ_C, it becomes:

$$P^{\upsilon_C} C_E \omega_C C_E^{-1} = P_E^{\upsilon_C},$$

$$C_E = \omega_C \left(\frac{P}{P_E} \right)^{\upsilon_C} C \tag{12}$$

$$Foc_{C_{NE}} : \; P \left[\omega_c^{\frac{1}{\upsilon_C}} C_E^{\frac{\upsilon_C - 1}{\upsilon_C}} + (1-\omega_c)^{\frac{1}{\upsilon_C}} C_{NE}^{\frac{\upsilon_C - 1}{\upsilon_C}} \right]^{\frac{-1}{\upsilon_C - 1}} (1-\omega_c)^{\frac{1}{\upsilon_C}} C_{NE}^{\frac{-1}{\upsilon_C}} = P_{NE},$$

$$C_{NE} = (1-\omega_C) \left(\frac{P}{P_{NE}} \right)^{\upsilon_C} C \tag{13}$$

Using these two equations we can obtain the relative prices between energy and non-energy consumption goods:

$$\frac{P_E}{P_{NE}} = \left(\frac{\omega_C}{1-\omega_C} \right)^{\frac{1}{\upsilon_C}} \left(\frac{C_E}{C_{NE}} \right)^{\frac{1}{\upsilon_C}} \tag{14}$$

and the price of the final consumption good becomes:

$$P = \left[\omega_c P_E^{\frac{1}{\upsilon_C}} + (1-\omega_c) P_{NE}^{\frac{1}{\upsilon_C}} \right] \tag{15}$$

Aditionally we can define:

A.3 Energy consumption retailers

This retailer purchases the energy consumption goods, C_e, from the energy producers. In this model, there are two energy sectors, $N_e = 2$ (coke and refined petroleum, and electrictiy and gas), so we already express the following formulas taking this into account. The retailer aggregates the energy consumption goods into an energy consumption bundle, C_E,using the following CES function:

$$C_E = \left[\omega_{C,e}^{\frac{1}{\upsilon_{C,E}}} C_{e1}^{\frac{\upsilon_{C,E} - 1}{\upsilon_{C,E}}} + (1-\omega_{C,e})^{\frac{1}{\upsilon_{C,E}}} C_{e2}^{\frac{\upsilon_{C,E} - 1}{\upsilon_{C,E}}} \right]^{\frac{\upsilon_{C,E}}{\upsilon_{C,E} - 1}},$$

where $\upsilon_{C,E} > 1$ is the elasticity of substitution across different enegery consumption goods (i.e., energy consumption of different types are substitute goods, so that carbon tax can lead to a rise of “greener” energy sectors).

The demand for the different energy types is determinated from the following profit maximization:

$$max_{C_{e1},C_{e2}} P_E C_E - P_{e1} C_{e1} - P_{e2} C_{e2}$$

$$s.t.\, C_E = \left[\omega_{C,e}^{\frac{1}{\upsilon_{C,E}}} C_{e1}^{\frac{\upsilon_{C,E-1}}{\upsilon_{C,E}}} + (1-\omega_{C,e})^{\frac{1}{\upsilon_{C,E}}} C_{e2}^{\frac{\upsilon_{C,E-1}}{\upsilon_{C,E}}} \right]^{\frac{\upsilon_{C,E}}{\upsilon_{C,E-1}}},$$

The respective first order conditions for C_{e1} and C_{e2} are the following:

$$Foc_{C_{e1}} :\ P_E \left[\omega_{C,e}^{\frac{1}{\upsilon_{C,E}}} C_{e1}^{\frac{\upsilon_{C,E-1}}{\upsilon_{C,E}}} + (1-\omega_{C,e})^{\frac{1}{\upsilon_{C,E}}} C_{e2}^{\frac{\upsilon_{C,E-1}}{\upsilon_{C,E}}} \right]^{\frac{-1}{\upsilon_{C,E-1}}} \omega_{C,e}^{\frac{1}{\upsilon_{C,E}}} C_{e1}^{\frac{-1}{\upsilon_{C,E}}} = P_{e1},$$

$$Foc_{C_{e2}} :\ P_E \left[\omega_{C,e}^{\frac{1}{\upsilon_{C,E}}} C_{e1}^{\frac{\upsilon_{C,E-1}}{\upsilon_{C,E}}} + (1-\omega_{C,e})^{\frac{1}{\upsilon_{C,E}}} C_{e2}^{\frac{\upsilon_{C,E-1}}{\upsilon_{C,E}}} \right]^{\frac{-1}{\upsilon_{C,E-1}}} \omega_{C,e}^{\frac{1}{\upsilon_{C,E}}} C_{e2}^{\frac{-1}{\upsilon_{C,E}}} = P_{e2},$$

Following a similar argument as before, we obtain:

$$C_{e1} = \omega_{C,E} \left(\frac{P_E}{P_{e1}} \right)^{\upsilon_{C,E}} C_E \tag{17}$$

$$C_{e2} = (1-\omega_{C,e}) \left(\frac{P_E}{P_{e2}} \right)^{\upsilon_{C,E}} C_E \tag{18}$$

Using these two equations we can obtain the relative prices between the two energy sectors:

$$\frac{P_{e1}}{P_{e2}} = \left(\frac{\omega_{C,E}}{1-\omega_{C,E}} \right)^{\frac{1}{\upsilon_{C,E}}} \left(\frac{C_{e1}}{C_{e2}} \right)^{\frac{1}{\upsilon_{C,E}}} C_E \tag{19}$$

and the price of the final consumption good becomes:

$$P_E = \left[\omega_{C,E} P_{e1}^{\frac{1}{\upsilon_{C,E}}} + (1-\omega_{C,E}) P_{e2}^{\frac{1}{\upsilon_{C,E}}} \right] \tag{20}$$

A.4 Non-energy consumption retailers

The non-energy retailer purchases the goods $C_s$from the 51 non-energy producer and aggregates them into a non-energy consumption bundle, C_{NE},using the following CES function:

$$C_{NE} = \left[\sum_{s=1}^{N_s} \omega_{C,s}^{\frac{1}{\upsilon_{C,s}}} C_s^{\frac{\upsilon_{C,s}-1}{\upsilon_{C,s}}} \right]^{\frac{\upsilon_{C,s}}{\upsilon_{C,s}-1}} \quad \sum_{s=1}^{N_s} \omega_{C,s} = 1, \tag{21}$$

where $\upsilon_{C,s} > 1$ is the elasticity of substitution across different non-enegery consumption goods.

The demand for the different non-energy consumption goods is determinated from the following profit maximization:

$$max_{C_s} P_{NE} C_{NE} - \sum_{s=1}^{N_s} P_s C_s$$

$$s.t.\; C_{NE} = \left[\sum_{s=1}^{N_s} \omega_{C,s}^{\frac{1}{\upsilon_{C,s}}} C_s^{\frac{\upsilon_{C,s}-1}{\upsilon_{C,s}}} \right]^{\frac{\upsilon_{C,s}}{\upsilon_{C,s}-1}},$$

The respective first order conditions for the s goods is the following:

$$Foc_{C_s} :\; P_{NE} \left[\sum_{s=1}^{N_s} \omega_{C,s}^{\frac{1}{\upsilon_{C,s}}} C_s^{\frac{\upsilon_{C,s}-1}{\upsilon_{C,s}}} \right]^{\frac{-1c}{\upsilon_{C,s}-1}} \omega_{C,s}^{\frac{1}{\upsilon_{C,s}}} C_s^{\frac{-1}{\upsilon_{C,s}}} = P_s,$$

Operating as in the previous CES functions, we reach the following condition (expressed in terms of prices):

$$P_s = \left(\omega_{C,s} \frac{C_{NE}}{C_s} \right)^{\frac{1}{\upsilon_{C,s}}} P_{NE} \tag{22}$$

A.5 Energy intermediate-input retailers

For each of the N_s non-energy sectors there is a retailer that purchases the energy intermediate goods e_e from the N_e energy sectors and aggregates them into an energy intermediate-input bundle for each non-energy sector, E_s, using a CES function as follows:

$$E_s = \left[\omega_{e,s}^{\frac{1}{\upsilon_{H,E}}} e_{e1,s}^{\frac{\upsilon_{H,E}-1}{\upsilon_{H,E}}} + (1 - \omega_{e,s})^{\frac{1}{\upsilon_{H,E}}} e_{e2,s}^{\frac{\upsilon_{H,E}-1}{\upsilon_{H,E}}} \right]^{\frac{\upsilon_{H,E}}{\upsilon_{H,E}-1}}$$

where $\upsilon_{H,E} > 1$ is the elasticity of substitution across different enegery intermediate goods.

The demand for the different energy types is determinated from the following profit maximization:

$$max_{e_{e1,s}, e_{e2,s}} \sum_{s=1}^{N_s} P_{E,s} E_s - P_{e1} \sum_{s=1}^{N_s} e_{e1,s} - P_{e2} \sum_{s=1}^{N_s} e_{e2,s}$$

$$s.t.\ E_s = \left[\omega_{e,s}^{\frac{1}{\upsilon_{H,E}}} e_{e1,s}^{\frac{\upsilon_{H,E}-1}{\upsilon_{H,E}}} + (1-\omega_{e,s})^{\frac{1}{\upsilon_{H,E}}} e_{e2,s}^{\frac{\upsilon_{H,E}-1}{\upsilon_{H,E}}} \right]^{\frac{\upsilon_{H,E}}{\upsilon_{H,E}-1}},$$

The respective first order conditions for $E_{e1,s}$ and $E_{e2,s}$ are the following:

$$Foc_{E_{e1,s}}:\ P_{E,s} \left[\omega_{e,s}^{\frac{1}{\upsilon_{H,E}}} e_{e1,s}^{\frac{\upsilon_{H,E}-1}{\upsilon_{H,E}}} + (1-\omega_{e,s})^{\frac{1}{\upsilon_{H,E}}} e_{e2,s}^{\frac{\upsilon_{H,E}-1}{\upsilon_{H,E}}} \right]^{\frac{-1}{\upsilon_{H,E}-1}} \omega_{e,s}^{\frac{1}{\upsilon_{H,E}}} e_{e1,s}^{\frac{-1}{\upsilon_{H,E}}} = P_{e1},$$

That operating becomes:

$$e_{e1,s} = \omega_{H,E} \left(\frac{P_{E,s}}{P_{e1}} \right)^{\upsilon_{H,E}} E_s \tag{23}$$

and in the case of $E_{e2,s}$:

$$e_{e2,s} = (1-\omega_{H,E}) \left(\frac{P_{E,s}}{P_{e2,}} \right)^{\upsilon_{H,E}} E_s \tag{24}$$

Using these two equations we can obtain the relative prices between the two energy sectors:

$$\frac{P_{e1}}{P_{e2}} = \left(\frac{\omega_{H,E}}{1-\omega_{H,E}} \right)^{\frac{1}{\upsilon_{H,E}}} \left(\frac{e_{e1,s}}{e_{e2,s}} \right)^{\frac{1}{\upsilon_{H,E}}} E_s \tag{25}$$

and the price of the energy intermediate-input good becomes:

$$P_{E,s} = \left[\omega_{H,E} P_{e1}^{\frac{1}{\upsilon_{H,E}}} + (1-\omega_{H,E}) P_{e2}^{\frac{1}{\upsilon_{H,E}}} \right] \tag{26}$$

A.6 Non-energy intermediate-input retailers

For each non-energy sector (N_s), there is a retailer that purchases the goods $H_{x,s}$ from each of the non-energy producers (N_s), and aggregates them into N_s non-energy intermediate-input bundle, H_s, using a Cobb-Douglas function with constant returns to scale:

$$H_s = \prod_{x=1}^{N_s} H_{x,s}^{\omega_{H,x,s}},\ \sum_{x=1}^{N_s} \omega_{H,x,s} = 1,$$

where $H_{x,s}$ is the intermediate input produced from sector x and then used by sector s ($N_s x N_s$).

The demand from sector x for the different non-energy intermediate inputs s is determined from the following profit maximization:

$$max_{H_{x,s}} P_{Hs}H_s - \sum_{s=1}^{N_s} P_{H,x,s}H_{x,s}$$

$$s.t.\ H_s = \prod_{x=1}^{N_s} H_{x,s}^{\omega_{H,x,s}}$$

The first order conditions for the intermediate producer x is:

$$\omega_{H,x,s}P_{Hs}H_{x,s}^{\omega_{H,x,s}-1} = P_{H,x,s}$$

Multipying both sides by $H_{x,s}$, we get:

$$H_{x,s} = \frac{P_{H,x,s}H_s}{\omega_{H,x,s}P_{Hs}} \tag{27}$$

and the price of the the 51 intermediate-input bundle, H_s, can be defined as:

$$P_{Hs} = \prod_{x=1}^{N_s} \left(\frac{P_{H,x,s}}{\omega_{H,x,s}}\right) \omega_{H,x,s} \tag{28}$$

A.7 Energy production sectors

There are N_e energy production sectors. In each of these sectors, denoted by the subscript e, production of energy is determined by the imported energy input from abroad,

$$Z_e = M_e. \tag{29}$$

The energy good Z_e is sold to the energy consumption retailer, C_e , and to each of the energy intermediate-input retailer, $E_{e,s}$ (N_s):

$$Z_e = C_e + \sum_{s=1}^{N_s} E_{e,s} \tag{30}$$

The maximization problem of the energy sector determines the labor demand to produce energy:

$$max_{L_e}(1-\tau_e)P_eM_e - P_{M,e}M_e$$

$$s.t.\ Z_e = M_e,$$

where the taxes are a function of the emissions $\tau_e = \frac{P_{CO_2}CO_{2,e}}{P_eZ_e}$.

The profit maximization yields to the following condition:

$$P_M = (1-\tau_e)P_e \tag{31}$$

We assume that the price of the imported good, $P_{M,e}$, is the same as that of the final good in the home economy.

A.8 Non-energy production sectors

The production of the non-energy sector is determinated from a CES function that uses energy, E_s, labor, L_s and non-energy intermediate input, H_s, that is sold to the non-energy consumption retailer, and the N_s non-energy intermediate-input retailers, $Z_s = C_s + \sum_{x=1}^{s} H_{x,s}$. The production function is the following:

$$Z_s = \left[\omega_{E,s}^{\frac{1}{\upsilon_E}} E_s^{\frac{\upsilon_E-1}{\upsilon_E}} + (1-\omega_{E,s})^{\frac{1}{\upsilon_E}} \left[L_s^{\alpha_s} H_s^{1-\alpha_s}\right]^{\frac{\upsilon_E-1}{\upsilon_E}}\right]^{\frac{\upsilon_E}{\upsilon_E-1}}$$

The maximization problem of the non-energy sector determines the demand for the different inputs:

$$\max_{E_s,L_s,H_s} P_s Z_s - P_{E,s}E_s - WL_s - P_{H,s}H_s$$

$$s.t.\ Z_s = \left[\omega_{E,s}^{\frac{1}{\upsilon_E}} E_s^{\frac{\upsilon_E-1}{\upsilon_E}} + (1-\omega_{E,s})^{\frac{1}{\upsilon_E}} \left[L_s^{\alpha_s} H_s^{1-\alpha_s}\right]^{\frac{\upsilon_E-1}{\upsilon_E}}\right]^{\frac{\upsilon_E}{\upsilon_E-1}}$$

The respective first order conditions for E_s becomes:

$$Foc_{E_s}:\ P_s \left[\omega_{E,s}^{\frac{1}{\upsilon_E}} E_s^{\frac{\upsilon_E-1}{\upsilon_E}} + (1-\omega_{E,s})^{\frac{1}{\upsilon_E}} \left[L_s^{\alpha_s} H_s^{1-\alpha_s}\right]^{\frac{\upsilon_E-1}{\upsilon_E}}\right]^{\frac{-1}{\upsilon_E-1}} \omega_{E,s}^{\frac{1}{\upsilon_E}} E_s^{\frac{-1}{\upsilon_E}} = P_{E,s},$$

That operating becomes:

$$E_s = \omega_E \left(\frac{P_s}{P_{E,s}}\right)^{\upsilon_E} Z_s \tag{32}$$

In the case of labor and the intermediate input, we split the problem into the following:

$$\max P_{va}V_a - WL_s - P_{H,s}H_s$$

$$s.t.\ V_{as} = L_s^{\alpha_s} H_s^{1-\alpha_s}$$

whose optimal conditions are:

$$L_s = \left[\frac{P_{H,s}}{(1-\alpha)P_{va}}\right]^{1/\alpha} H_s\ \ H_s = \left[\frac{P_{H,s}}{\alpha P_{va}}\right]^{1/(1-\alpha)}$$

so that

$$P_{va} = \frac{w^{\alpha} P_{H,s}^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \tag{33}$$

$$V_{as} = \frac{WL}{\alpha P_{va}} \tag{34}$$

and since the optimal condition for $E_{e,s}$ is equivalent for the “good” V_a, we have:

$$V_{as} = (1-\omega_E)\left(\frac{P_s}{P_{va}}\right)^{v_E} Z_s \tag{35}$$

Then

$$H_s = \frac{(1-\alpha)P_{va}}{P_{H,s}} \tag{36}$$

A.9 Closing the model

Labor Market

$$L = \sum_{s=1}^{N_s} L_s$$

Taxes

$$T = \sum_{s=1}^{N_e} \tau_e P_e Z_e$$

A.10 List of variables

The model contains $N_s = 51$ sectors and $N_e = 2$ energy sectors (electricity, gas, steam and air conditioning supply, and manufacture of coke and rened petroleum products). There is one final good C that is formed using the consumption energy good, C_E,from the energy consumption retailer and the non-energy consumption good, C_{NE}, the respective prices are P, P_E, P_{NE}. The consumption energy good is produced using the two kinds of intermediate inputs from energy producers (coke and refined petroleum and electrictiy and gas), C_{e1}, C_{e2} and their prices are $P_{e1}\, P_{e2}$, while the non-energy consumption good uses the inputs, C_s,from the 51 non-energy producers with the respetive

51 P_s prices.

The intermediate inputs produced from the energy producers come from the energy producer that use the imported energy input $M_{e1}$and M_{e2} respectively with prices $P_{M,e1}$ and $P_{M,e2}$. This input is also used for the production of the energy used in each of the 51 non-energy producers E_s, with prices $P_{E,s}$. This producer (the non-energy) also uses 51 inputs of labor L_s and 51 inputs of non-energy intermediate input, H_s in the production of the input to the non-energy consumption retailer (C_s to produce C_{NE}) and in the input for the non-energy intermediate retailer 51x51 $H_{x,s}$. The prices of his inputs are W (same wage for all kinds of labor) and 51x $P_{H,s}$. With respect to these two inputs used in the non-energy production sector, the labor comes from the final consumer, and the non-energy intermediate input comes from the non-energy intermediate retailer, that inputs of labor L_s and 51 inputs of non-energy intermediate input, H_s in the production of the input to the non-energy consumption retailer (C_s to produce C_{NE}) and in the input for the non-energy intermediate retailer 51x51 $H_{x,s}$. The prices of his inputs are W (same wage for all kinds of labor) and 51x $P_{H,s}$. With respect to these two inputs used in the non-energy production sector, the labor comes from the final consumer, and the non-energy intermediate input comes from the non-energy intermediate retailer, that uses input produced from sector x and then used by sector s ($N_s x N_s$, 51x51), $H_{x,s}$ with their respective (51x51) $P_{x,s}$ prices.

Therefore the model contains 1 final good C, 1 energy good C_E, 1 non-energy C_{NE},2 intermediate inputs from the energy producers C_{e1}, C_{e2}, 51 non-energy inputs from the non-energy producers C_s, 51x51 inputs for the intermediate non-energy retailer $H_{x,s}$, 2 imported energy input $M_{e1}$and M_{e2}. It also has, 51 energy inputs used in the non-energy producers E_s, 51 kind of labors input L_s and 51 non-energy intermediate input, H_s in the non-energy producer and 51x51 inputs $H_{x,s}$ in the non-energy intermediate retailer. All of these with the respective prices: $1xP$, $1xP_E$, $1xP_{NE}$, $1xP_{e1}$, $1xP_{e2}$, $51x51P_{H,x,s}1xP_{M,e1}$, $1xP_{M,e2}$, $51xP_{E,s}$, $51xP_{Hs}$.

A.11 Exercise strategy

Given the parameters and shares are calibrated from the data (see the Calibration section) the algorithm for finding the steady-state of the model is as follows:

a) Initial guess for the variables in the model: c_s, n_s, n_e, p_s,and P, the price that closes the labor market.

b) Equations 1 to 21 in the model are solved (see list of equations above) with the following order:

1. Fix wages to one as the numerary.
2. Define energy prices P_{e1}, P_{e2} as function of wages and taxes (for energy type and coverage, τ_{e1}, τ_{e1c} and τ_{e2}, τ_{e2c} respectively) and likewise for the non-energy producers $P_{E,s}$ (that includes either τ_{e1} or τ_{e2} and two specific taxes). These equations come from 31.
3. Adjust the shares of C_{e1}, C_{e2} to obtain $\tilde{\omega}_{C,E}$ and normalize them to obtain $\omega_{C,E}$. Note that from the data we get a share that doesn't correspond directly to the normalised share in the models and needs some computations.
4. Obtain the price of the energy aggregated good P_E from 20
5. Obtain the price of the non-energy aggregated good P_{NE} from 15.

6. Adjust the shares of C_s to obtain omega_tilde and then find the shares compatible with the data using equation 22.
7. Obtain the consumption of the aggregate non-energy good C_{NE}, using equation 21.
8. Obtain the consumption of the aggregate energy good C_E, using equation 14.
9. Find the price of non-energy consumption good P_s, from equation 22.
10. Define the production in each energy sector Z_{e1}, Z_{e2}, from equation 29.
11. Obtain the consumption of each energy good C_{e1}, C_{e2} from equations 17 and 18.
12. Find the prices of the non-energy intermediate input (51xP_{Hs}) from equation 28 .
13. Define the price of the value added good, P_{va} in the non-energy producers as in equation 33.
14. Define the value added, V_{as}, of the non-energy producers as in equation 34.
15. Obtain the production of the non-energy intermediate input,$H_s$from equation 36.
16. Then one can obtain the non-energy intermediate inputs of each sector by each non-energy sector, $H_{x,s}$ as in equation 27.
17. Adjust the shares of energy use in energy consumption retailer, $\widetilde{\omega}_{e,s}$ (51x2). This is defined by an aditional equation and the normalised to obtain $\omega_{e,s}$.
18. Define the price of the energy inputs used by the non-energy sector, $P_{E,s}$, as in equation 26.
19. Adjust the shares of non-energy consumption retailer, $\omega_{C,s}$ (51). This is defined by an aditional equation.
20. Obtain the total output of each non-energy sector, Z_s,from equation 35.
21. The energy inputs of each non-energy sector, E_s, are determined by equation 32.
22. The energy inputs of each kind for each non-energy sector $E_{e,s}$, come from equations 23 and24.
23. Additional equations are included to compute the labor supply, the tax revenue and labor tax .

24. Derive the new implied values of the guessed variables, c_s, n_s, n_e, p_s, from the market clearing conditions :
    (a) $C_{s,} = Z_{s,} - \sum_{x=1}^{s} H_{x,s,}$, from the resource constraint of the energy production sector
    (b) $n_s$from the production function of the non-energy producer
    (c) n_e from the production function of the energy producer
    (d) $p_s$from equation 22
25. Compute the residuals to be minimised.

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